Spring 2006 Macro Prelim

1.1 (a) A TDCE is a set of prices $\{(p_t, w_t, r_t)\}_{t=0}^{\infty}$, a set of allocations to consumers $\{\{(c_t^i, x_t^i, k_t^i, l_t^i)\}_{i=1}^{I}\}_{t=0}^{\infty}$, a set of plans for firms $\{\{k_t^j, l_t^j, y_t^j\}_{j=1}^{J}\}_{t=0}^{\infty}$, and a set of policies $\{p_{ct}, p_{xt}, p_{kt}, p_{lt}\}_{t=0}^{\infty}$, such that

given $\{(p_t, w_t, r_t)\}_{t=0}^{\infty}$, $\forall i$ consumer i solve $\max_{\{c_t^i, l_t^i, x_t^i\}_{t=0}^{\infty}} U^i(c_t^i, l_t^i)$ subject to

$$\sum_{t=0}^{\infty} k_t^i r_t (1-\tau_{kt}) + l_t^i w_t (1-\tau_{lt}) \geq \sum_{t=0}^{\infty} p_t (1+\tau_{ct}) c_t^i + p_t (1+\tau_{xt}) x_t^i$$

$$k_{t+1}^i = x_t^i + (1-\delta) k_t^i$$

given $\{(p_t, w_t, r_t)\}_{t=0}^{\infty}$, $\forall j, \forall t$ Firm j solves $\max_{y_t^j} \; p_t y_t^j - r_t k_t^j - w_t l_t^j$ subject to $y_t^j \leq F^j(k_t^j, l_t^j)$

markets clear, i.e.

$$\sum_{i=1}^{I} k_t^i = \sum_{j=1}^{J} k_t^j \quad \forall t$$

$$\sum_{i=1}^{I} l_t^i = \sum_{j=1}^{J} l_t^j \quad \forall t$$

$$g_t + \sum_{i=1}^{I} x_t^i + \sum_{i=1}^{I} c_t^i = \sum_{j=1}^{J} y_t^j \quad \forall t$$

and government budget balances $\sum_{t=0}^{\infty} p_t g_t = \sum_{t=0}^{\infty}\sum_{i=1}^{I} \tau_{ct} p_t c_t^i + \tau_{xt} p_t x_t^i + \tau_{wt} w_t l_t^i + \tau_{kt} r_t k_t^i$

(b) Assume $\sum_{t=0}^{\infty} k_t^i r_t (1-\tau_{kt}) + l_t^i w_t (1-\tau_{lt}) = \sum_{t=0}^{\infty} p_t (1+\tau_{ct}) c_t^i + p_t (1+\tau_{xt}) x_t^i \quad \forall i$

$$y_t^j = F^j(k_t^j, l_t^j) \quad \forall j, \forall t$$

Assume firms make zero profit, which follows from CRS production functions.

Then $\sum_{j=1}^{J} p_t y_t^j = \sum_{j=1}^{J} (r_t k_t^j + w_t l_t^j)$

$$p_t \sum_{j=1}^{J} y_t^j = r_t \sum_{j=1}^{J} k_t^j + w_t \sum_{j=1}^{J} l_t^j$$

by market clearing $p_t g_t + p_t \sum_{i=1}^{I} x_i^t + p_t \sum_{i=1}^{I} c_t^i = r_t \sum_{i=1}^{I} k_t^i + w_t \sum_{i=1}^{I} l_t^i$

(cont'd)

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1.1 (b) (cont'd)

From last page $$p_t g_t + p_t \sum_{i=1}^{I} x_i^t + p_t \sum_{i=1}^{I} c_t^i = r_t \sum_{i=1}^{I} k_t^i + w_t \sum_{i=1}^{I} l_t^i \qquad \forall t$$

$$p_t g_t = \sum_{i=1}^{I} \left(r_t k_t^i + w_t l_t^i - p_t c_t^i - p_t x_t^i \right) \qquad \forall t$$

$$\sum_{t=0}^{\infty} p_t g_t = \sum_{t=0}^{\infty} \sum_{i=1}^{I} \left(r_t k_t^i + w_t l_t^i - p_t c_t^i - p_t x_t^i \right)$$

From consumer BC $$\sum_{t=0}^{\infty} \left(r_t k_t^i - \tau_{kt} r_t k_t + w_t l_t^i - \tau_{lt} w_t l_t^i \right) = \sum_{t=0}^{\infty} \left(p_t c_t^i + \tau_{ct} p_t c_t^i + p_t x_t^i + \tau_{xt} p_t x_t^i \right) \qquad \forall i$$

$$\sum_{t=0}^{\infty} \left(r_t k_t^i + w_t l_t^i - p_t c_t^i - p_t x_t^i \right) = \sum_{t=0}^{\infty} \left(\tau_{ct} p_t c_t^i + \tau_{xt} p_t x_t^i + \tau_{kt} r_t k_t^i + \tau_{lt} w_t l_t^i \right) \quad \forall i$$

$$\sum_{i=1}^{I} \sum_{t=0}^{\infty} \left(r_t k_t^i + w_t l_t^i - p_t c_t^i - p_t x_t^i \right) = \sum_{i=1}^{I} \sum_{t=0}^{\infty} \left(\tau_{ct} p_t c_t^i + \tau_{xt} p_t x_t^i + \tau_{kt} r_t k_t^i + \tau_{lt} w_t l_t^i \right)$$

Thus $$\sum_{t=0}^{\infty} p_t g_t = \sum_{i=1}^{I} \sum_{t=0}^{\infty} \left(\tau_{ct} p_t c_t^i + \tau_{xt} p_t x_t^i + \tau_{kt} r_t k_t^i + \tau_{lt} w_t l_t^i \right)$$