Fall 2005 Micro (Majors) Prelim

I. 2 (a) Let $\precsim$ be continuous and strictly increasing.

$\forall x \in \mathbb{R}^n_+$ define $A^+(x) = \{a \in \mathbb{R}_+ \mid ae \succsim x\}$

$A^-(x) = \{a \in \mathbb{R}_+ \mid ae \precsim x\}$

$\precsim$ strictly increasing $\Rightarrow \forall x \in \mathbb{R}^n_+ \quad x \succsim 0 \Rightarrow x \succsim 0e \Rightarrow 0 \in A^-(x) \; \forall x \in \mathbb{R}^n_+ \Rightarrow$

$A^-(x)$ nonempty $\forall x \in \mathbb{R}^n_+$

Let $y = (y_1, \dots, y_n) \in \mathbb{R}^n_+$ be given. Let $\bar{y} = \max_{i=\{1,\dots,n\}} y_i$. Since $\succsim$ strictly increasing,

$\bar{y}e \succsim y \Rightarrow \bar{y} \in A^+(y) \Rightarrow A^+(x)$ nonempty $\forall y \in \mathbb{R}^n_+$.

Continuity of $\precsim \Rightarrow A^+(x), A^-(x)$ closed.

Completeness of $\precsim \Rightarrow A^+ \cup A^- = \mathbb{R}_+$ $\quad \mathbb{R}_+$ is connected, A^+, A^- are closed and nonempty, and $A^+ \cup A^- = \mathbb{R}_+$, so $A^+ \cap A^- \neq \emptyset$.

Let $k(x) \in A^+(x) \cup A^-(x)$. Since $\succsim$ is strictly increasing, $k(x)$ is unique. Define $u(x) = k(x)$.

$u(x)$ then is well-defined.

CLAIM u so defined is a utility representation of $\succsim$.

Proof Let $x \succsim y$. By transitivity of $\succsim$, $k(x)e \sim x \succsim y \sim k(y)e$. $\quad u(x) = k(x)$, $u(y) = k(y)$.

$\succsim$ strictly increasing $\Rightarrow k(x) \geq k(y) \Rightarrow u(x) \geq u(y)$.

Let $u(\hat{x}) \geq u(\hat{y})$. Then $k(\hat{x}) \geq k(\hat{y})$. By $\succsim$ strictly increasing $k(\hat{x})e \succsim k(\hat{y})e$. Since $k(\hat{x})e \sim \hat{x}$ and $k(\hat{y})e \sim \hat{y}$, by transitive $\hat{x} \succsim \hat{y}$. ∎

Fall 2005 Micro (Majors) Prelim

I.2 (b)* Consider $\succsim$ defined such that $x \succ y \Leftrightarrow x_1 < y_1$, $x \sim y \Leftrightarrow x_1 = y_1$, $x \prec y \Leftrightarrow x_1 > y_1$.

$\succsim$ is transitive and complete

Let $\{x^n\}, \{y^n\}$ be such that $x^n \to x$, $y^n \to y$ and $x^n \succsim y^n$ $\forall n$.
Then $x_1^n \geq y_1^n$ $\forall n$ $\Rightarrow$ $x_1 \geq y_1$ $\Rightarrow$ $x \succsim y$. Thus $\succsim$ is continuous.

$(2,1,1,\ldots,1) \prec (1,0,0,\ldots,0)$, so $\succsim$ not strictly increasing

$u(2e) = 2$
$u(e) = 1$
$2e \prec e$, but $2 > 1$

(c) Consider lexigraphical preferences, specifically $x_1 \succ y_1 \Leftrightarrow x_1 > y_1$ or $(x_1 = y_1$ and $x_2 > y_2)$ or $\ldots$ or $(x_1 = y_1 \ \ldots \ x_{n-1} = y_{n-1}$ and $x_n > y_n)$

$\succsim$ is transitive, complete, and strictly increasing

$\succsim$ is not continuous since $(1-\frac{1}{n}, 1, 0, 0 \ldots) \prec (1,0,0,\ldots)$ $\forall n$ but $(1,1,0,0,\ldots) \succ (1,0,0,\ldots)$

$\not\exists k \in \mathbb{R}$ $(1,1,\ldots,0) \sim k(1,1,\ldots)$

$(1,1,\ldots,0) \succ 1(1,1,\ldots)$

$(1,1,\ldots,0) \prec (1+\varepsilon)(1,1,1,\ldots)$ $\forall \varepsilon > 0$

* preference suggested by Jacek Rothert