Spring 2005 Micro (Prelim)

Q1.1 (i) $\max_{a \geq 0} \pi \cdot u(w-L+a-\rho a) + (1-\pi)\, u(w-\rho a)$

From FOC $\pi u'(w-L+a-\rho a)\cdot(1-\rho) = (1-\pi)\, u'(w-\rho a)\cdot \rho$

$$\frac{u'(w-L+a-\rho a)}{u'(w-\rho a)} = \frac{1-\rho}{\rho}\,\frac{\pi}{1-\pi}$$

$$\rho > \pi \Rightarrow \frac{1-\rho}{\rho}\,\frac{\pi}{1-\pi} < 1 \Rightarrow u'(w-L+a(1-\rho)) > u'(w-\rho a)$$

strictly risk averse $\Rightarrow$ u strictly concave $\Rightarrow$ $w-\rho a > w-L+a(1-\rho)$

$\Rightarrow \quad L > a$

(ii) FOC $\pi u'(w-L+a-\rho a)\cdot(1-\rho) = (1-\pi)\, u'(w-\rho a)\cdot\rho$

$\frac{\partial FOC}{\partial L}$ $\quad \pi u''(w-L+a-\rho a)(1-\rho)\left(\frac{\partial a}{\partial L} - \rho\frac{\partial a}{\partial L} - 1\right) = -(1-\pi)\, u''(w-\rho a)\,\rho^2 \frac{\partial a}{\partial L}$

$$\pi u''(w-L+a-\rho a)(1-\rho)\left(\frac{\partial a}{\partial L} - \rho\frac{\partial a}{\partial L} - 1\right) + (1-\pi)u''(w-\rho a)\rho^2\frac{\partial a}{\partial L} = 0$$

$$\frac{\partial a}{\partial L}\Big[\underbrace{\pi u''(w-L+a-\rho a)(1-\rho)}_{neg} - \underbrace{\pi\rho u''(w-L+a-\rho a)(1-\rho)}_{neg} + \underbrace{(1-\pi)u''(w-\rho a)\rho^2}_{neg}\Big] = \underbrace{\pi u''(w-L+a-\rho a)(1-\rho)}_{neg}$$

Assume $\rho < 1$

Thus $\frac{\partial a}{\partial L} > 0$.

If $\rho > 1$, self-insurance is optimal, and $a = 0$.

Solved with Wyatt Brooks and others.