Spring 2006 Micro (Majors) Prelim

1.2 (a) U has state-separable representation if there exist functions $V_s : \mathbb{R} \to \mathbb{R}$ for all s such that $U(c_1, \dots, c_S) \geq U(\tilde{c}_1, \dots, \tilde{c}_S)$ IFF $\sum_{s=1}^{S} V_s(c_s) \geq \sum_{s=1}^{S} V_s(\tilde{c}_s)$.

Given that U is continuous and strictly increasing, U has state-separable representation IFF it obeys the independence axiom.

(b) CLAIM U has state-separable representation $\Rightarrow$ U obeys the independence axiom.

Proof Let U have state-separable representation. Suppose for contradiction that U does not obey the independence axiom. Then $\exists\, c, d \in \mathbb{R}^S$ and $\exists\, y, w \in \mathbb{R}$ such that $U(c_1, \dots, c_{s-1}, y, c_{s+1}, \dots, c_S) \geq U(d_1, \dots, d_{s-1}, y, d_{s+1}, \dots, d_S)$ and $U(c_1, \dots, s_{s-1}, w, c_{s+1}, \dots, c_S) < U(d_1, \dots, d_{s-1}, w, d_{s+1}, \dots, d_S)$. Then

$$\sum_{i \neq s} V_i(c_i) + V_s(y) \geq \sum_{i \neq s} V_i(d_i) + V_s(y) \quad \text{and} \quad \sum_{i \neq s} V_i(c_i) + V_s(w) < \sum_{i \neq s} V_i(d_i) + V_s(w)$$

$$\Rightarrow \sum_{i \neq s} V_i(c_i) \geq \sum_{i \neq s} V_i(d_i) \quad \text{and} \quad \sum_{i \neq s} V_i(c_i) < \sum_{i \neq s} V_i(d_i). \quad \text{CONTRADICTION}$$

So U obeys the independence axiom. ■

(c) The condition remains necessary, but not sufficient when $S=2$.

Consider $U(c_1, c_2) = c_1 + c_2 + c_1 c_2^2$

Note that
$$
\begin{aligned}
u(c_1, y) \geq u(d_1, y) \Rightarrow\ & c_1 + y + c_1 y^2 \geq d_1 + y + d_1 y^2 \\
& c_1 (1+y^2) \geq d_1 (1+y^2) \\
& c_1 \geq d_1 \\
& c_1 (1+w^2) \geq d_1 (1+w^2) \\
& c_1 + w + c_1 w^2 \geq d_1 + w + d_1 w^2
\end{aligned}
$$

Likewise $\forall w, y \in \mathbb{R}_+$ $u(y, c_2) \geq u(y, d_2) \Rightarrow y + c_2 + y c_2^2 \geq y + d_2 + y d_2^2$
$\Rightarrow u(w, c_2) \geq u(w, d_2)$

$U(c_1, c_2)$ obeys the independence axiom.

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1.2 (c) (cont'd)

Suppose for contradiction $\exists\, V_1 : \mathbb{R} \to \mathbb{R}$ and $V_2 : \mathbb{R} \to \mathbb{R}$ such that

$$U(g_1, g_2) \geq U(h_1, h_2) \Leftrightarrow V_1(g_1) + V_2(g_2) \geq V_1(h_1) + V_2(h_2).$$

Consider $g = (0,2)$, $h = (2,0)$

$$U(0,2) = 2 = U(2,0) \Rightarrow V_1(0) + V_2(2) = V_1(2) + V_2(0)$$

$$U(1,2) = 7 > 5 = U(2,1) \Rightarrow V_1(1) + V_2(2) > V_1(2) + V_2(1)$$

together $\quad V_1(1) - V_1(0) > V_2(1) - V_2(0)$

$$V_1(1) + V_2(0) > V_1(0) + V_2(1) \Rightarrow U(1,0) > U(0,1)$$

$$U(1,0) = 1 = U(0,1)$$

CONTRADICTION, so

$U(c_1, c_2)$ has no state-separable representation