Fall 2007 Prelim - Micro

I.2 (a) $E[v(Y+Z)] = \pi_1 E[v(Y+Z) | Y=y_1] + \pi_2 E[v(Y+Z) | Y=y_2]$

$\forall$ concave v by Jensen's Inequality $v[E(Y+Z | Y=y_1)] \geq E[v(Y+Z) | Y=y_1]$

$v(y_1 + E(Z|Y=y_1)) \geq E[v(Y+Z) | Y=y_1]$

$v(y_1) \geq E[v(Y+Z) | Y=y_1]$

similarly $v(y_2) \geq E[v(Y+Z) | Y=y_2]$

$\pi_1 v(y_1) + \pi_2 v(y_2) \geq \pi_1 E[v(Y+z) | Y=y_1] + \pi_2 E[v(Y+z) | Y=y_2]$

$E[v(Y)] \geq E[v(Y+Z)]$ $\forall$ concave v

Y SSD $Y+Z$ so $Y+Z$ is more risky than Y

(b) $Y+Z = \frac{1}{2}(Y+2Z) + \frac{1}{2}Y$

$v(Y+Z) \geq \frac{1}{2} v(Y+2Z) + \frac{1}{2} v(Y)$ $\forall$ concave v

$E[v(Y+Z)] \geq \frac{1}{2} E[v(Y+2Z)] + \frac{1}{2} E[v(Y)]$

$\frac{1}{2} E[v(Y+Z)] + \underbrace{\frac{1}{2} E[v(Y+Z)] - \frac{1}{2} E[Y]}_{>0 \text{ by part a}} \geq \frac{1}{2} E[Y+2Z]$

$\frac{1}{2} E[v(Y+z)] \geq \frac{1}{2} E[Y+2Z]$

$E[v(Y+Z)] \geq E[Y+2Z]$ $\forall$ concave v

$Y+Z$ SSD $Y+2Z$ so $Y+2Z$ is more risky than $Y+Z$