

### Assignment 3

- due November 16<sup>th</sup> in lecture

#### Problem 1 - Uncertainty in a finite, pure endowment economy

Consider the model discussed in class. That is, there are 2 consumers (denoted by 1 and 2) and 2 periods (1 and 2) and a single non-storable consumption good. Preferences of both consumers are ordered by  $Eu(c_1, c_2)$ , where  $E$  denotes the expectations operator and  $u(c_1, c_2) = \log c_1 + \log c_2$ . The endowments for the 2 consumers are:

- in period 1:  $\omega_1 = (1, 1)$ ,
- in period 2:  $\omega_2 = (1, 0)$  with probability  $p$  and  $(0, 1)$  with probability  $(1 - p)$ .

In the following questions, define an equilibrium for this economy and compute it.

- (i) Agents are allowed to trade the consumption good among themselves within a given period and state but not across different periods and states, i.e. contracts of the type: I will give you something today, you will give me something tomorrow or I will give you something if state 1 occurs you will give me something if state 2 occurs are ruled out.

In what follows assume agents can trade within period and state of the world as in (i).

- (ii) Agents are also allowed to trade a riskless bond at period 1. This is an asset that costs  $q$  at period 1 and gives payoff 1 in terms of the consumption good in period 2.
- (iii) Assume now that there is also a storage technology that the agents can use with return  $R > 0$ , i.e. if they save 1 unit of consumption in period 1, they will get  $R$  units of consumption in period 2 no matter what the state is. Compare the equilibrium allocations to those of (i) and (ii).
- (iv) Agents are able to trade state contingent bond in period 1.
- (v) Arrow-Debreu markets are opened. Compare the equilibrium allocation of consumption to that of question (iv). Define *perfect insurance*. Show that we will get it with  $p = 0.5$ .
- (vi) In addition to (v) assume there is a storage technology as in (iii). Compare the results that you get to those of (v).

# 1 Problem Set 3

2. Consider Problem 2 from Problem set 2. Now assume that the production function is of the form

$$(1) \quad F(k, l, z) = zk^{0.25}l^{0.75}$$

where  $z$  is an i.i.d. technology shock with mean 1 and variance  $\sigma^2$ . Linearize the intertemporal Euler equation and solve for the decision rule of capital of the form

$$(2) \quad k_{t+1} = \lambda_0 + \lambda_k k_t + \lambda_z z_t$$

where  $\lambda_0, \lambda_k, \lambda_z$  are parameters. Show that  $\lambda_k$  has the same value as in the deterministic case.

Now suppose  $z_t$  follows and AR(1) stochastic process of the form

$$(3) \quad z_{t+1} = \bar{z} + \rho z_t + \varepsilon_{t+1}$$

where the random variable  $\varepsilon_t$  is i.i.d. with mean zero and variance  $\sigma_\varepsilon^2$ . Show that  $\lambda_k$  is the same in this case as well. What is  $\lambda_z$  now? In the AR(1) case, consider the impulse responses associated with a one-standard deviation shock to  $\varepsilon$ . That is compute capital, output, employment, and consumption associated with starting at the steady state and a sample path of  $\varepsilon$  of the form  $\sigma_\varepsilon$  in period zero, and zero in all periods  $t \geq 1$ .

3. Sargent-Ljungqvist: Problems 8.2, 8.3, 8.4, 8.5 and 8.8.

4. Consider a growth model with an equal number of two types of agents. Their utility functions are the same and are  $\sum \beta^t U(c_t)$ . The underlying state  $s_t$  can take on one of two values  $\{H, L\}$ . If  $s_t = H$ , type 1 agents have an endowment of 2 units of labor and type 2 agents have an endowment of 0 units of labor. Assume initial endowments of capital are the same.

Define a competitive equilibrium with complete markets. What social planning problem does this equilibrium solve? Show that this planning problem is identical to an economy with just one type of agent.

Define a competitive equilibrium in which the two types of agents can only use capital to save across time. Is an equilibrium allocation with complete markets an equilibrium allocation here? Why not? (Hint: Conjecture that it is in the case where the complete markets economy is at a steady state. Then describe the evolution of capital stocks assuming a sample path that consist of a very long string of  $s_t = H$ .)

Extra Credit: Suppose now that the initial endowments of capital are not the same. Describe how you would compute the weights on the social planner's problem. (You do not have to compute these weights.)