Macroeconomic Theory (8107)

Spring 2007, Mini 1

Problem set 3

Due Thursday, Feb 13, in class.

1) Consider a small open economy in which preferences of the representative agent are given by

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t|s_0) \frac{c^{1-\sigma}(s^t)}{1-\sigma}$$

In each period the agent receives a stochastic and exogenous endowment $e^{y(s^t)}$, $y(s^t) = \rho y(s^{t-1}) + \varepsilon(s^t)$, $\varepsilon \rightarrow N(0, \sigma_\varepsilon^2)$.

- Assume that the representative agent in the economy has initial wealth 0 and that she can trade internationally in each state s^t (including the initial state before her initial income realization is observed) a full set of AD securities with state contingent prices given by $\beta\pi(s^{t+1}|s^t)$. Solve exactly (i.e. without log-linearizing) for the equilibrium allocation.

- Log-linearize the economy above and solve for the log-linearized allocation. How is it different from the exact one? Explain

- Consider now the case in which the representative agent can only trade a non contingent bond $b(s^t)$ so its budget constraint is given by

$$c(s^t) + \bar{q}b(s^t) = b(s^{t-1}) + e^{y(s^t)}$$

  and $\bar{q} = \beta$ is its exogenous price.Write down the log-linearized equilibrium conditions around the point $b = 0$. Solve analytically for the parameters $\alpha_1, \alpha_2, \alpha_3$ and α_4 which characterize the decision rules

$$\begin{aligned} \hat{c}(s^t) &= \alpha_1 y(s^t) + \alpha_2 b(s^{t-1}) \\ b(s^t) &= \alpha_3 y(s^t) + \alpha_4 b(s^{t-1}) \end{aligned}$$

- Consider the polar cases of $\rho = 1$ and $\rho = 0$. Characterize the equilibrium statistical processes for bonds and consumption under these two cases. Compare the allocations with those under complete markets. Under which case is the bond a better instrument to share risk? Explain why.

- Now focus on the case $\rho = 1$. Compute the log-linearized equilibrium allocation. In the space $\sigma, \sigma_\varepsilon^2$ compare the ex-ante welfare from the log-linearized complete markets allocation with the welfare from the log-linearized bond allocation. Comment your results

2) There are two countries US and Italy. US is endowed with cheese (X) while Italy is only endowed with pasta (Y) but Italian and Americans like to eat both. American consumers' utility is given by

$$\begin{aligned} & \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t|s_0) \frac{c^{1-\sigma}(s^t)}{1-\sigma} \\ c(s^t) &= G(x(s^t), y(s^t)) \end{aligned}$$

where $x(s^t)$ and $y(s^t)$ is their consumption of cheese and pasta and $G(x,y) = (\omega x^{\frac{\gamma-1}{\gamma}} + (1-\omega) y^{\frac{\gamma-1}{\gamma}})^{\frac{\gamma}{\gamma-1}}$

Italian consumers are denoted by a star and their utility is given by

$$\begin{aligned} & \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t|s_0) \frac{c^{*1-\sigma}(s^t)}{1-\sigma} \\ c^*(s^t) &= G(x^*(s^t), y^*(s^t)) \end{aligned}$$

The endowment of cheese and pasta are random and given by $X(s^t), Y(s^t)$ Assume that Italians and Americans can only trade cheese and pasta and let $p(s^t)$ be the price of pasta in terms of cheese. Also assume that there is no trade in financial assets (financial autarky).

- Define an equilibrium for this economy and write down the equations that characterize it.

- Solve analytically for equilibrium allocations and prices

- Show that if $\gamma < 1$, US consumption falls relative to italian consumption when the endowment of cheese (the american good) increases. Explain

- Show that if $\gamma = 1$ (so that $G(x,y) = x^\omega y^{1-\omega}$) the equilibrium allocation under financial autarky is Pareto efficient. Explain the economic intuition behind this result.