Problem Set 1
UMN, Macroeconomic Theory 8108, Spring 2007

- due March 29^{th} in lecture, each student submits his or her version of the homework

Problem 1

This problem is related to the model of asymmetric information in Ljungqvist, Sargent, 2^{nd} edition (from now on LS), the proof of concavity of P on page 666, but it is self contained.

Let b^0, b^1 and $\delta \in (0,1)$ be given. Let's define b by:

$$u(y+b) = \delta \cdot u(y+b^0) + (1-\delta) \cdot u(y+b^1) \tag{1}$$

Prove that if u has weakly decreasing absolute risk aversion, in that. $-u''(c)/u'(c)$ is weakly decreasing in c then for $x > y$:

$$u(x+b) \leq \delta \cdot u(x+b^0) + (1-\delta) \cdot u(x+b^1) \tag{2}$$

Note: prove means to write the proof not to cite somebody's theorem. Hint: think of b as $y+b$ as the certainty equivalent of a gamble that gives $y+b^0$ with probability δ and $y+b^1$ with probability $1-\delta$.

Problem 2: LS 19.1, 19.5

Problem 3

Consider the model with one sided lack of commitment in LS section 19.3.

(i) Fully justify, i.e. in more detail than LS the expressions (19.3.18), (19.3.19), (19.3.27a), (19.3.27b). In particular, justify the *max* operator in these expressions.

(ii) (thanks to Rene for inspiration) Consider the recursive specification of the money lender's problem as defined by equations (19.3.4) - (19.3.8). In SLP fashion, define the operator T that maps the space of real valued function on $[v_{aut}, \overline{v}]$ into itself, i.e. $T : R([v_{aut}, \overline{v}]) \to R([v_{aut}, \overline{v}])$. Use this to argue that $P(v)$, the fixed point of this operator is a concave function. In particular, I want you to show that T maps the space of concave functions into itself.[1] Please look at the proof that LS have in section 19.3.5 as well.

(iii) Prove that $P(v)$ is differentiable. Either use SLP techniques as in the previous exercise, or complete the proof of LS, section 19.3.5.

(iv) Suppose now that the endowment process is not i.i.d. but follows a Markov process with a transition matrix T. Write the money lender's problem both sequentially and recursively to account for this change. Discuss what effects of the Markovian property on the optimal contract, properties of P, profits of the money lender etc. you would expect. Try to characterize the optimal contract.

Problem 4

From Section 19.5 in LS. Suppose that $u(c)$ is concave and consider an incentive compatible allocation. Prove that if the local downward constraint binds, $C_{s,s-1} = 0$, and transfers are weakly decreasing in the state, $b_{s-1} \geq b_s$, then the upward constraint holds, $C_{s,s-1} \geq 0$.

[1] For your own fun (and I will look at it if you submit it), you can finish the proof. For that, you need to show that the fixed point exists and is unique. Also, you may want to try to prove other properties in this way.