

Problem Set 3
UMN, Macroeconomic Theory 8108, Spring 2007

- due on Tuesday April 17th in lecture, each student submits his or her version of the homework

Problem 1 - Atkeson-Lucas model (1995)

- (1) Set up the planning problem and characterize the solution of a one period version of that model—with an arbitrary initial entitlement. Let

$$\bar{x}_0(w_0, z_0) \text{ and } \bar{h}_0(w_0, z_0)$$

denote the solutions.

- (2) Draw the indifference curves in the (c,l) space and indicate the set of incentive feasible allocations.
 (3) Discuss briefly what the solution to part (1) looks like if all agents have equal entitlements.
 (4) Set up the planning problem for a two period version of that model in which at the beginning of the first period all agents have the same entitlement. Let

$$\hat{x}_0(w_0, z_0) , \hat{h}_0(w_0, z_0), \hat{x}_1(w_1, z_0, z_1) , \hat{h}_1(w_1, z_0, z_1)$$

denote the solution to the two period problem.

- (5) Is the repetition of the solution to the one period problem with equal entitlements optimal for the two period allocation problem? Explain.

That is, consider the two period allocation obtained by simply repeating the solution to the one period problem with equal entitlements. That is consider the allocations defined by

$$x_0^*(w_0, z_0) = \bar{x}_0(w_0, z_0) , h_0^*(w_0, z_0) = \bar{h}_0(w_0, z_0),$$

$$x_1^*(w_1, z_0, z_1) = \bar{x}_0(w_0, z_1) , h_1^*(w_1, z_0, z_1) = \bar{h}_0(w_0, z_0)$$

Do the allocations with the stars equal the allocations with the carets? How do they differ? Explain both in words and with mathematics.

Problem 2 - A two period Cole-Kocherlakota model

Consider trying to prove part *i* of the lemma which states that any allocation (c, 0, b, K) that solves (P3) then the aggregate resource constraint

$$K_t + \sum_{h_t} \pi(h_t) b_t(h_t) \leq RK_{t-1} \quad \forall t$$

holds with equality in both periods and $K_2 = 0$.

Now suppose by way of contradiction that in an allocation that solves (P3)

$$K_1 + \sum_{h_1} \pi(h_1) b_1(h_1) + \Delta_1 = 0$$

$$K_2 + \sum_{h_2} \pi(h_2) b_2(h_2) + \Delta_2 = RK_1$$

with Δ_1, Δ_2 , and K_2 all strictly positive.

- (1) Consider a new allocation that gives the extra resources in the first period Δ_1 to agents in the first period with the lowest endowment in that period, and the extra resources in the second period $\Delta_2 + RK_1$ to agents who in the second period have the lowest endowment in that period. Will this new allocation improve welfare relative to the original allocation? Will it be incentive compatible?
- (2) Consider a new allocation in which the planner take the discounted value of the extra resources

$$K_1 + \frac{K_2 + \Delta_2}{R}$$

and divides them equally among all agents in the first period. Will this new allocation improve welfare relative to the original one? Will it be incentive compatible?

- (3) Suppose the planner takes the discounted value of extra resources and gives it to the agents in the first period who have the highest endowments. Will this new allocation improve welfare and will it be incentive compatible?
- (4) Finally, suppose that the planner saves the extra resources in the first period and in the second period gives those resources plus the extra resources in the second period, a total of

$$RK_1 + K_2 + \Delta_2$$

to the agents in the second period with the highest endowments in that period. Will this new allocation improve welfare and will it be incentive compatible?

Problem 3 - Another problem on Cole and Kocherlakota

Let c^* be the solution to problem $P3$. Use c^* to construct a candidate solution to problem $P1$. Prove that the constructed allocation solve problem $P1$.

Problem 4 - Revelation principle

State and prove the revelation principle.

Problem 5 - A bonus problem

Consider the problem of hidden storage of section 19.6 with 2 periods and 2 income realizations only. Suppose the planner can store at rate R_p and the agents at rate R_a . Analyze the cases of $R_p > R_a$ and $R_p < R_a$.