Problem Set 5
UMN, Macroeconomic Theory 8108, Spring 2007

- due May 12^{th}, each student submits his or her version of the homework

Problem 1

LS Problem 22.1

Problem 2

LS Problem 22.2

Problem 3

LS Problem 22.3

Problem 4 Extremal sustainable plans

Consider the economy defined in the second half of class on Tuesday, April 24^{th}. In particular consider the operator T defined as:

$$
\begin{aligned}
T(W): \quad = \quad & \{v = (1-\beta)u(K,\tau) + \beta \int w(\tau,\theta)d\theta \\
& s.t. \\
& (K,\tau,w(\tau,\theta)): \quad \begin{array}{c} w(\tau,\theta) \in W \\ K = k(K,\tau) \\ (1-\beta)u(K,\tau) + \beta \int w(\tau,\theta)d\theta \geq (1-\beta)u(K,\widetilde{\tau}) + \beta \int w(\widetilde{\tau},\theta)d\theta, \ \forall \widetilde{\tau} \in [0,\overline{\tau}] \end{array} \quad \}
\end{aligned}
$$

1. Prove the bang-bang property.
2. Suppose $W = [w_l, w_h]$. Define the operator $\mathbf{T} : \Re^2 \longrightarrow \Re^2$. $\mathbf{T}(w_h, w_l) = (\overline{T}(w_l, w_h), \underline{T}(w_l, w_h))$, where $\overline{T}, \underline{T}$ are defined as in lecture on April, 17^{th}. That is:

$$
\begin{aligned}
\overline{T}(w_l, w_h) := & \max_{\tau \in [0,\overline{\tau}], K, p \in [0,1]} (1-\beta)u(K,\tau) + \beta[pw_l + (1-p)w_h] \\
& s.t. \\
K = & \ k(K,\tau) \\
(1-\beta)u(K,\tau) + \beta[pw_l + (1-p)w_h] \geq & \ (1-\beta)u(K,\widetilde{\tau}) + \beta w_l, \ \forall \widetilde{\tau} \in [0,\overline{\tau}]
\end{aligned}
$$

$$
\begin{aligned}
\underline{T}(w_l, w_h) := & \min_{\tau \in [0,\overline{\tau}], K, p \in [0,1]} (1-\beta)u(K,\tau) + \beta[pw_l + (1-p)w_h] \\
& s.t. \\
K = & \ k(K,\tau) \\
(1-\beta)u(K,\tau) + \beta[pw_l + (1-p)w_h] \geq & \ (1-\beta)u(K,\widetilde{\tau}) + \beta w_l, \ \forall \widetilde{\tau} \in [0,\overline{\tau}]
\end{aligned}
$$

Characterize $\overline{T}, \underline{T}$ as precisely as you can.