Midterm exam
UMN, Macroeconomic Theory 8105, Fall 2006

You have 90 minutes and the total number of points is 90. Good luck!
This version includes what has been added during the exam.

Problem 1 - Two period economy (25 points)

Consider a 2 period-economy with a representative consumer and one firm. Technology available to the firm is given by $F(K_t, N_t) = AK_t^{\alpha} N_t^{1-\alpha}, \alpha \in (0,1), A > 0$. The consumer is endowed with $k_1 > 0$ units of capital at the beginning of the first period. Assume full depreciation of capital. Preferences are given by: $U(c_1, l_1, c_2, l_2) = \gamma \log(c_1) + (1-\gamma)\log(l_1) + \beta[\gamma \log(c_2) + (1-\gamma)\log(l_2)], \gamma \in (0,1), \beta \in (0,1)$. As usually, we assume $\forall t : l_t + n_t \leq 1$ and non-negativities on all variables.

(a) (10 points) State and solve the Social Planner's Problem for this economy.

Sketch of the solution:

Definition: the social planner's problem is the following:

$$\max_{(n_1,l_1,c_1,k_1,n_2,l_2,c_2,k_2)} U(c_1, l_1, c_2, l_2) \text{ s.t.} \tag{1}$$
$$n_1 + l_1 \leq 1 \tag{2}$$
$$n_2 + l_2 \leq 1 \tag{3}$$
$$c_1 + k_2 \leq F(k_1, n_1) \tag{4}$$
$$c_2 \leq F(k_2, n_2) \tag{5}$$
$$k_1 \leq \overline{k}_1 \text{ given} \tag{6}$$
$$(n_1, l_1, c_1, k_1, n_2, l_2, c_2, k_2) \geq 0 \tag{7}$$

The solution to this problem is:

$$l_1^* = \frac{1-\gamma}{\gamma(1+\alpha\beta)(1-\alpha) + (1-\gamma)} \tag{8}$$
$$n_1^* = 1 - l_1^* \tag{9}$$
$$k_1^* = \overline{k}_1 \tag{10}$$
$$c_1^* = \frac{A(k_1^*)^{\alpha}(n_1^*)^{1-\alpha}}{1+\alpha\beta}\left[\frac{\gamma(1+\alpha\beta)(1-\alpha)}{\gamma(1+\alpha\beta)(1-\alpha)+(1-\gamma)}\right]^{1-\alpha} \tag{11}$$
$$l_2^* = \frac{1-\gamma}{1-\gamma\alpha} \tag{12}$$
$$n_2^* = 1 - l_2^* \tag{13}$$
$$k_2^* = \alpha\beta c_1^* \tag{14}$$
$$c_2^* = A(k_2^*)^{\alpha}(n_2^*)^{1-\alpha} \tag{15}$$

A set of equations that define the solution (such as the above) is enough here.

(b) (10 points) Define an Arrow-Debreu equilibrium for this economy. Find the equilibrium prices and quantities.

Sketch of the solution: I skip the definition.

The quantities will be the same as before, the prices will be:

$$p_1 = 1 \ (normalization) \tag{16}$$

$$r_1 = \alpha A(k_1^*)^{\alpha-1}(n_1^*)^{1-\alpha} \tag{17}$$

$$w_1 = (1-\alpha)A(k_1^*)^{\alpha}(n_1^*)^{-\alpha} \tag{18}$$

$$r_2 = 1 \ (the\ no-arbitrage\ condition) \tag{19}$$

$$p_2 = \frac{r_2}{\alpha A(k_2^*)^{\alpha-1}(n_2^*)^{1-\alpha}} \tag{20}$$

$$w_2 = (1-\alpha)A(k_2^*)^{\alpha}(n_2^*)^{-\alpha} \tag{21}$$

(c) (5 points) Compare the results that you obtained in (a) and (b) and make the appropriate conclusion.

Solution: The solution to the SPP is PO. Since ADE = solution to SPP, ADE is PO (FWT).

Problem 2 - Homothetic aggregation (30 points)
Consider an economy with agents of type $j = 1, 2$. Every agent of type 1 is indexed by $i \in I_1$ and every agent of type 2 is indexed by $i \in I_2$. (I_1 and I_2 are index sets and it is natural to assume $I_1 \bigcap I_2 = \emptyset$). Both sets are finite. There are L different goods and each agent $i \in I_1 \bigcup I_2$ receives an endowment $w^i \in \Re^L_{++}$; his preferences are represented by $u^i : \Re^L_+ \to \Re$ and u^i is str. concave, str. increasing and homothetic (that is, $u^i(x) = u^i(y) \Rightarrow u^i(\lambda x) = u^i(\lambda y) \;\; \forall x, y \in \Re^L_+, \forall \lambda \geq 0$). Finally, we assume that:

$$\forall i, j \in I_1 : u^i = u^j; \tag{22}$$
$$\forall i, j \in I_1 : w^i = w^j; \tag{23}$$
$$\forall i, j \in I_2 : u^i = u^j; \tag{24}$$
$$\forall i, j \in I_2 : w^i = w^j. \tag{25}$$

Note, that assumptions (23) and (25) are not needed in what follows.

(a) (5 points) Define an equilibrium for this economy.

Sketch of the solution: I will skip this.

(b) (10 points) Taking $p \in \Re^L_{++}$ as given, show that the demand function $x^i(p, pw^i)$ (i.e. the solution to the consumer's problem as a function of prices and the value of her endowment) is homogenous of degree one in the value of the endowment, i.e $\forall pw^i \in \Re^L_{++}, \forall \lambda > 0 : x^i(p, \lambda pw^i) = \lambda x^i(p, pw^i)$.

Solution:

Step 1: WTS: $u^i(x^i(p, \lambda pw^i)) = u^i(\lambda x^i(p, pw^i))$. Suppose WLOG $u^i(x^i(p, \lambda pw^i)) > u^i(\lambda x^i(p, pw^i)) \Longrightarrow$ (by homotheticity) $u^i(\frac{x^i(p,\lambda pw^i)}{\lambda}) > u^i(x^i(p, pw^i))$. Obviously $\frac{x^i(p,\lambda pw^i)}{\lambda}$ is affordable under p, w^i, i.e. $px^i(p, \lambda pw^i) \leq \lambda pw^i \Longrightarrow p\frac{x^i(p,\lambda pw^i)}{\lambda} \leq pw^i$ and it delivers strictly higher utility than $x^i(p, pw^i)$. Thus $x^i(p, pw^i)$ cannot be the optimal demand given p, w^i. Contradiction.

Step 2: Suppose now that $x^i(p, \lambda pw^i) \neq \lambda x^i(p, pw^i) \iff \frac{x^i(p,\lambda pw^i)}{\lambda} \neq x^i(p, pw^i)$. By the above these are both affordable given p, w^i. Thus also $x_\alpha := \alpha\frac{x^i(p,\lambda pw^i)}{\lambda} + (1 - \alpha)x^i(p, pw^i)$ is affordable $\forall \alpha \in (0, 1)$. By strict concavity $u^i(x_\alpha) > u^i(x^i(p, \lambda pw^i))$, thus $x^i(p, pw^i)$ cannot be the optimal demand given p, w^i. Contradiction.

Note that this works $\forall i \in I_1 \bigcup I_2$.

(c) (5 points) Show that $\sum_{i \in I_i} x^i(p, pw^i) = x^j(p, \sum_{i \in I_i} pw^i), \forall j \in I_i$.

Sketch of the solution:

Use $x^i(p, pw^i) = (pw^i) \cdot x^i(p, 1)$, which is a consequence of the above. You need to use assumptions (22) and (24) as well.

(d) (10 points) State and prove a theorem which says that an equilibrium in part (a) can be re-written as an equilibrium in an economy with two agents.

Solution:

Theorem: If $((x^i)_{(i \in I_1 \bigcup I_2)}, p)$ is an equilibrium of an economy with $I_1 \bigcup I_2$ agents with endowments $(w^i)_{(i \in I_1 \bigcup I_2)}$ then $(x_1 = \sum_{i \in I_1} x^i, x_2 = \sum_{i \in I_2} x^i, p)$ is an equilibrum of an economy with 2 agents with endowments $(w_1 = \sum_{i \in I_1} w^i, w_2 = \sum_{i \in I_2} w^i), p$.

Proof: Take the equilibrium price vector in the $I_1 \bigcup I_2$ economy. Given this price vector, the two agents in the 2 agent economy will demand $x_1 = \sum_{i \in I_1} x^i, x_2 = \sum_{i \in I_2} x^i$ by Part (iii). Market clearing in the $I_1 \bigcup I_2$ economy implies market clearing in the 2 agent economy.

Notes: it is straight forward to formulate a converse of this Theorem, but it seems that this is not what the question asks. Do it as an exercise. Let me remind you again that assumptions (23) and (25) were not used at all.

Problem 3 - Habit persistence problem (35 points)

Consider the following sequential problem:

$$v^*(k_0, c_{-1}) := \max_{\{c_t, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t (\log c_t + \gamma \log c_{t-1}) \text{ s.t.}$$

$$k_{t+1} + c_t \leq A k_t^{\alpha}, c_t \geq 0, k_t \geq 0, \forall t,$$

$$k_0, c_{-1} \text{ given}$$

$$\beta \in (0,1), \gamma > 0, A > 0, \alpha \in (0,1)$$

(i) (5 points) Write the problem in canonical form.

Solution:

$$v^*(k_0, c_{-1}) := \max_{\{c_t, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t (\log c_t + \gamma \log c_{t-1}) \text{ s.t.}$$

$$(k_{t+1}, c_t) \in \Gamma(k_t, c_{t-1}), \forall t$$

$$\Gamma(k_t, c_{t-1}) = \{(k_{t+1}, c_t) \in \mathbf{R}_+^2 : k_{t+1} + c_t \leq A k_t^{\alpha}\}$$

$$k_0, c_{-1} \text{ given}$$

(ii) (15 points) Write down the corresponding functional equation. Use the guess and verify method to obtain the value function and the policy functions. (Hint: guess that $v(k,c) = E + F \log k + G \log c$ and derive the constants E, F, G.)

Solution:

The functional equation is:

$$v(k,c) = \max_{(k',c') \in \Gamma(k,c)} \log c' + \gamma \log c + \beta v(k', c') \tag{26}$$

Note that we have replaced k_t with k, k_{t+1} with k', c_{t-1} with c, c_t with c'. I also dropped the k_0, c_{-1} given.

Now, we make use of the guess, plug into (26) and write:

$$E + F \log k + G \log c = \max_{(k',c') \in \Gamma(k,c)} \log c' + \gamma \log c + \beta(E + F \log k' + G \log c') \tag{27}$$

Now we solve for the RHS. FOC are sufficient by concavity of the return function and convexity of the constraint set. Note also that the constraint in the correspondence Γ will bind since the return function is strictly increasing. Now, we can rewrite the RHS of equation (27) as follows:

$$\max_{(k',c')} \log c' + \gamma \log c + \beta(E + F \log k' + G \log c') \text{ s.t.} \tag{28}$$

$$k' + c' = A k^{\alpha} \tag{29}$$

The Lagrangian for this problem is (alternatively you may plug in for one of the variables from the constraint and solve the unconstrained maximization problem that you get):

$$L = \log c' + \gamma \log c + \beta(E + F \log k' + G \log c') + \lambda(A k^{\alpha} - k' - c') \tag{30}$$

The FOC are:

$$
\begin{aligned}
(c'): \ \frac{1+\beta G}{c'} &= \lambda && (31)\\
(k'): \ \frac{\beta F}{k'} &= \lambda && (32)\\
(\lambda): \ k' + c' &= Ak^{\alpha} && (33)
\end{aligned}
$$

(31) and (32) imply:

$$
(1+\beta G)k' = \beta F c' \Longrightarrow c' = \frac{(1+\beta G)k'}{\beta F} \qquad (34)
$$

Plugging this into (33) yields:

$$
\frac{(1+\beta F+\beta G)k'}{\beta F} = Ak^{\alpha} \Longrightarrow k' = \frac{Ak^{\alpha}\beta F}{1+\beta F+\beta G} \Longrightarrow \qquad (35)
$$

$$
c' = \frac{(1+\beta G)\frac{Ak^{\alpha}\beta F}{1+\beta F+\beta G}}{\beta F} = \frac{Ak^{\alpha}(1+\beta G)}{1+\beta F+\beta G} \qquad (36)
$$

These are our optimal policy functions. Plugging them back into (27):

$$
E+F\log k+G\log c = \log(\frac{Ak^{\alpha}(1+\beta G)}{1+\beta F+\beta G})+\gamma\log c+\beta[E+F\log(\frac{Ak^{\alpha}\beta F}{1+\beta F+\beta G})+G\log(\frac{Ak^{\alpha}(1+\beta G)}{1+\beta F+\beta G})]
$$

Thus the following 3 equations determine the parameters E, F, G:

$$
\begin{aligned}
G &= \gamma && (37)\\
F &= \alpha + \beta\alpha F + \beta\alpha G && (38)\\
E(1-\beta) &= \log(\frac{A(1+\beta G)}{1+\beta F+\beta G}) + \beta[F\log(\frac{A\beta F}{1+\beta F+\beta G}) + G\log(\frac{A(1+\beta G)}{1+\beta F+\beta G})] && (39)
\end{aligned}
$$

This would be enough for full score on this question. Anyways, plugging in from the first into the second and then from both into the third yields:

$$
G = \gamma \qquad (40)
$$

$$
F = \alpha + \beta\alpha F + \beta\alpha\gamma \Longrightarrow F = \frac{\alpha(1+\beta\gamma)}{1-\alpha\beta} \qquad (41)
$$

$$
E = \frac{1}{(1-\beta)}\{\log(\frac{A(1+\beta\gamma)}{1+\beta(\frac{\alpha(1+\beta\gamma)}{1-\alpha\beta})+\beta\gamma})+\beta[(\frac{\alpha(1+\beta\gamma)}{1-\alpha\beta})\log(\frac{A\beta(\frac{\alpha(1+\beta\gamma)}{1-\alpha\beta})}{1+\beta(\frac{\alpha(1+\beta\gamma)}{1-\alpha\beta})+\beta\gamma})+\gamma\log(\frac{A(1+\beta\gamma)}{1+\beta(\frac{\alpha(1+\beta\gamma)}{1-\alpha\beta})+\beta\gamma})]
$$

$$
\Longrightarrow E = \frac{1}{(1-\beta)}\{(1+\beta\gamma)\log(A(1-\alpha\beta)) + \frac{\alpha\beta(1+\beta\gamma)}{1-\alpha\beta}\log\alpha\beta A\} \qquad (42)
$$

What a beautiful expression, isn't it? We can also rewrite the policies (35) and (36) and the value function in terms of the parameters of the problem.

$$
\begin{aligned}
k' &= \alpha\beta Ak^{\alpha} && (43)\\
c' &= (1-\alpha\beta)Ak^{\alpha} && (44)\\
v(k,c) &= \frac{1+\beta\gamma}{(1-\beta)}\{\log(A(1-\alpha\beta)) + \frac{\alpha\beta}{1-\alpha\beta}\log\alpha\beta A\} + \frac{\alpha(1+\beta\gamma)}{1-\alpha\beta}\log k + \gamma\log c && (45)
\end{aligned}
$$

Note that they are the same as in the standard growth model without habit persistence. To get more insight, let us to compare the value functions as well. Denote the solution to the growth model without habit persistence $V(k)$:

$$V(k) = \frac{1}{(1-\beta)}[\log A(1-\alpha\beta) + \frac{\alpha\beta}{1-\alpha\beta}\log \alpha\beta A] + \frac{\alpha}{1-\alpha\beta}\log k. \tag{46}$$

Now, the solution to our habit persistence can be written as:

$$v(k, c) = (1 + \beta\gamma)V(k) + \gamma \log c \tag{47}$$

Is it good news or not? It is, because if we set $\gamma = 0$ we are in the model without habit persistence. So we better get the appropriate value function, right? Plus, there is a nice intuition for (47). In the habit persistence model, consumption enters twice in the objective function (i.e. $\sum_{t=0}^{\infty} \beta^t(\log c_t + \gamma \log c_{t-1})$), once multiplied by 1 (as in the standard model) and once multiplied by $\beta\gamma$. So it enters just like this in the value function. Think about it.

(iii) (5 points) Recall Theorem 4.3 (using canonical form notation):

(a) If $\Gamma(x)$ is non-empty $\forall x \in X$,

(b) $\forall$ feasible sequence $\{x_t\}_{t=0}^{\infty} : \lim_{n\to\infty} \sum_{t=0}^{n} \beta^t F(x_t, x_{t+1}) \in \bar{R} := R \bigcup \{-\infty, \infty\}$,

(c) $\forall$ feasible sequence $\{x_t\}_{t=0}^{\infty} : \lim_{n\to\infty} \beta^n v(x_n) = 0$,

then $v = v^*$, i.e. the solution to the functional equation equals to the solution to the sequential problem. Can the theorem be applied here? Why or why not?

Solution: This was a little tricky and I am not completely sure that what I have done bellow is correct. Comments are welcome.

We will show that condition (c) is not satisfied, thus the Theorem cannot be applied.

Solution 1: Define the feasible sequence: $\forall t > 0 : k_{t+1} = 0, c_t = 0, c_0 = Ak_0^\alpha$. Then $\forall t > 0 : v(k_t, c_{nt-1}) = -\infty \Longrightarrow \lim_{t\longrightarrow\infty} \beta^t v(k_t, c_{nt-1}) = -\infty \neq 0$.

Now, you may argue that this is not "correct" because we (and SLP) define the return function to be $F : X \longrightarrow \mathbf{R}$. This is clearly not satisfied here, since we take $\log 0 = -\infty \notin \mathbf{R}$. To address this issue we need to assume $X = (0, \infty)^2$, which is what we will do bellow.

Solution 2: Assume $X = (0, \infty)^2$, $k_0 > 0, c_1 > 0$. Note that the solution to Part (ii) is not affected, because the non-negativity condition was not binding anyways.

Case 1:

$$k_0 < \min\{A^{\frac{\beta}{1-\alpha\beta}}, 1\} \tag{48}$$

Then we will define:

$$k_{t+1} = k_t^{\frac{1}{\beta}} \tag{49}$$

$$c_t = Ak_t^\alpha - k_{t+1} = k_t^\alpha[A - k_t^{\frac{1-\alpha\beta}{\beta}}] \tag{50}$$

Rewriting the recursive equation (49) and (50):

$$k_{t+1} = k_0^{\frac{1}{\beta^{t+1}}} \tag{51}$$

$$c_t = (k_0^{\frac{1}{\beta^t}})^\alpha [A - (k_0^{\frac{1}{\beta^t}})^{\frac{1-\alpha\beta}{\beta}}] = (k_0^{\frac{\alpha}{\beta^t}})[A - k_0^{\frac{1-\alpha\beta}{\beta^{t+1}}}] \tag{52}$$

Note that this is feasible because (48) and (49) guarantees that k_t is a monotonically decreasing sequence. This together with (48) implies that $\forall t : Ak_t^\alpha > k_{t+1}$.

Plugging into condition (c) above; $\forall$ feasible sequence $\{k_t, c_t\}_{t=0}^{\infty}$:

$$\lim_{t\to\infty} \beta^{t+1} v(k_{t+1}, c_t) = \lim_{t\to\infty} \beta^{t+1}[E + F \log k_{t+1} + G \log c_t] =$$

$$\lim_{t\to\infty} \beta^{t+1}[E + F \log k_0^{\frac{1}{\beta^{t+1}}} + G \log(k_0^{\frac{\alpha}{\beta^t}})\{A - k_0^{\frac{1-\alpha\beta}{\beta^{t+1}}}]\} =$$

$$\lim_{t\to\infty} \beta^{t+1} \frac{1}{\beta^{t+1}} F \log k_0 + \lim_{t\to\infty} \beta^{t+1} \frac{\alpha}{\beta^t} \log k_0 + \lim_{t\to\infty} \beta^{t+1} \log\{A - k_0^{\frac{1-\alpha\beta}{\beta^{t+1}}}\} =$$

$$F \log k_0 + \beta\alpha \log k_0 + \lim_{t\to\infty} \beta^{t+1} \log\{A - k_0^{\frac{1-\alpha\beta}{\beta^{t+1}}}\} =$$

$$(F + \beta\alpha) \log k_0 + \lim_{t\to\infty} \beta^{t+1} \lim_{t\to\infty} \log\{A - k_0^{\frac{1-\alpha\beta}{\beta^{t+1}}}\} =$$

$$(F + \beta\alpha) \log k_0 < 0$$

This limit is < 0 since $F, \beta, \alpha > 0$ and $\log k_0 < 0$. We have used the fact that $\lim_{t\to\infty} \beta^{t+1} K = 0$ for an arbitrary constant K, plus several other properties of limits extensively (and hopefully correctly).

Case 2:

$$k_0 \geq \min\{A^{\frac{\beta}{1-\alpha\beta}}, 1\} \tag{53}$$

Just set $k_1 : k_1 < \min\{A^{\frac{\beta}{1-\alpha\beta}}, 1, Ak_0^\alpha\}$, $c_1 = Ak_0^\alpha - k_1$ and then proceed as above.

Notes: as an exercise you may try to define a feasible sequence for which the limit is $-\infty$. Note that what we are doing here is essentially decreasing the capital level so fast so that we offset the effect of β^n. In other words $\lim_{t\to\infty} \beta^t \log(x_t) = 0$ for many sequences $\{x_t\}$, unless they increase or decrease really fast. Increasing is not possible here, so we have to go for the other option. I.e. we need to construct a sequence of capital that is feasible and decreasing really fast. Saying something like this last sentence would be enough for a non-zero score, I guess. It is quite obvious that it can be done.

(iv) (10 points) Write down the 2 Euler equations and the 2 transversality conditions. Check whether the policy functions that you obtained in Part (ii) satisfy them. (Educational note: if it is so, then by Theorem 4.15, the solution to the functional equation - i.e. the sequence of allocations - also solves the sequential problem.)

Sketch of the solution: Now, I agree this one was a little unfair and not very well phrased, but if you got all the way here, that probably means that you were able to do this at least partially as well.

In other words, I don't think that this question being unfair was really a binding constraint. Think of it as of a bonus question.

To get the Euler equation, we may e.g. decide to go back the original sequential problem:

$$v^*(k_0, c_{-1}) := \max_{\{c_t, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t (\log c_t + \gamma \log c_{t-1}) \text{ s.t.}$$

$$k_{t+1} + c_t = Ak_t^{\alpha}$$

FOC:

$$(c_t): \frac{\beta^t}{c_t} = \lambda_t \tag{54}$$

$$(c_{t+1}): \frac{\beta^{t+1}}{c_{t+1}} = \lambda_{t+1} \tag{55}$$

$$(k_{t+1}): \lambda_t = \lambda_{t+1}\alpha A k_{t+1}^{\alpha-1} \tag{56}$$

Rewriting these:

$$\frac{\beta^t}{c_t} = \frac{\beta^{t+1}}{c_{t+1}} \alpha A k_{t+1}^{\alpha-1} \tag{57}$$

This is the Euler equation (seems like there is just 1). Note that we could plug in for c_t and c_{t+1} from the feasibility constraint above to get a second order difference equation in capital. Now we will verify that optimal policies satisfy the EE. Plugging in from the policies into the EE yields:

$$\frac{\beta^t}{\frac{Ak_t^{\alpha}(1+\beta G)}{1+\beta F+\beta G}} = \frac{\beta^{t+1}}{\frac{Ak_{t+1}^{\alpha}(1+\beta G)}{1+\beta F+\beta G}} \alpha A k_{t+1}^{\alpha-1} \Longleftrightarrow \frac{1}{k_t^{\alpha}} = \frac{\beta\alpha A}{k_{t+1}} \Longleftrightarrow k_{t+1} = \alpha\beta A k_t^{\alpha}$$

But this is our policy function (43). So given this policy, the EE is satisfied.

Note that in steady state the level of capital will be the same as in the standard model without habit persistence, since assuming $c_t = c^*$ and $k_t = k^*$ the EE above reduces to (f denotes the production function):

$$1 = \beta\alpha A(k^*)^{\alpha-1} = \beta f'(k^*) \tag{58}$$

The **transversality condition** will be added and discussed later.

Notes: compare this to section 4.5 in SLP. The problem obviously is that in our specification with $(k_{t+1}, c_t) \in \Gamma(k_t, c_{t-1})$, the optimal (k_{t+1}^*, c_t^*) is on the boundary of the set $\Gamma(k_t, c_{t-1})$. This is so since $k_{t+1}^* + c_t^* = Ak_t^{\alpha}$. Thus Theorem 4.15 cannot be directly applied. This is a problem, because based on the results in (iii) and (iv), we cannot say whether the solution to the functional equation solves the sequential problem. Comments on this welcome as well.

A good practice is to apply the same procedure as in (iv) to the same model without habit persistence (i.e. set $\gamma = 0$). The result should be that the solution to the functional equation is interior and it satisfies both the EE and TVC; thus it solves the sequential problem.