

# A Williamson Hold-up Model

Econ 8601 Fall 2002

## 1. Description of the Model

There are two kinds of individuals, type  $A$  and type  $B$ . The measure of type  $A$  individuals is  $N_A$  and the measure of type  $B$  individuals is  $N_B$ , where  $N_A < .5N_B$ .

There are two periods,  $t = \{1, 2\}$ . Each individual is endowed with a single labor unit in each period. The discount factor is  $\beta = 1$ .

There are two technologies for producing the final good, the regular technology and the special technology. With the regular technology, a type  $j$  individual can produce  $q_j$  units of the final good with a single unit of time. Assume that  $q_A > q_B$  so that a type  $A$  individual has an absolute advantage in production with the regular technology.

The special technology works as follows. In period 0, a type  $A$  individual builds a factory (type  $B$  individuals cannot build factories). Factories vary in quality  $i$ . To build a factory of quality  $i$  requires  $i$  units of time in period 0. A type  $A$  individual building a factory of quality  $i$  uses the balance  $1 - i$  of his or her time endowment in period 0 to make the final good (and thus produce  $(1 - i)q_A$  units in period 0).

A factory produces no final good in period 0. A factory will produce output in period 1 if an individual uses his or her unit time endowment to manage the factory in period 1. When a factory is built in period 0, it is customized to be managed in period 1 by a *particular* individual. If this particular individual manages the factory in period 1, the output in period 1 is  $f(i) + q_S$ , where  $f(0) = 0$ ,  $f'(0) > 0$ , and  $f''(0) < 0$ . It does not matter whether this particular individual is a type  $A$  or type  $B$  person; the output is the

same in either case. It also makes no difference whether the particular individual is the type  $A$  person who built the plant or whether the particular individual is someone else.

But there is a difference if the person who manages the factory is different from the person the factory was customized for. In this case, the output is  $q_S$  instead of  $f(i) + q_S$ ; i.e., the investment  $i$  is wasted.

## 2. Complete Contracting Case

Suppose that contracts are complete and can specify a publicly observable investment level  $i$ .

The questions to be addressed are: What is an equilibrium in this economy? Under what conditions is the special technology used? Under what condition is there specialization where one worker customizes a factory for a different worker?

## 3. Incomplete Contracting Case

Suppose contracts are incomplete. Investment decisions must be made in period 0 before any contracts can be written. Suppose in period 0 one agent customizes a factory for another agent. Model the bargaining in period 1 between these two agents as Nash bargaining, with weight  $\alpha$  the person doing the customizing and weight  $1 - \alpha$  on the person the factory is customized for.