

*Forthcoming Rand Journal of Economics*

## Mergers and the evolution of industry concentration: results from the dominant firm model

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Abstract:

*To what extent will an industry in which mergers are feasible tend towards monopoly? We analyze this question using a dynamic dominant firm model with rational agents, endogenous mergers and constant returns to scale production. We find that long-run industry concentration depends upon the initial concentration. A monopolistic industry will remain monopolized and a perfectly competitive industry will remain perfectly competitive. For intermediate concentration levels, the dominant firm may acquire or sell capital, depending on its ability to commit to future behavior. Industry evolution also depends on the elasticities of demand and supply and the discount factor.*

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Acknowledgements:

We thank Andy Atkeson, Joe Harrington, Narayana Kocherlakota, Kai-Uwe Kuhn, Jim Levinsohn, Matt Mitchell, Volker Nocke, Ariel Pakes, Leo Simon, two anonymous referees and seminar participants at various institutions for helpful comments. We acknowledge editorial assistance from Anita Todd and Jenni Schoppers and research assistance from Jackie Yuen. Holmes acknowledges financial support from the National Science Foundation through Grant SES-9906087. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

## 1. Introduction

A substantial literature in industrial organization has sought to determine the extent to which an industry in which mergers are feasible will tend toward monopoly. (See, for instance, Salant, Switzer, and Reynolds, 1983; Deneckere and Davidson, 1985; Perry and Porter, 1985; Gowrisankaran, 1999.) We feel that three key forces bear on this issue, for the case of a homogeneous goods industry with constant returns to scale. First, a monopoly maximizes industry profits, which provides an incentive to consolidate industry capital. Second, as pointed out by Stigler (1950), there is a free-rider problem in the merger process that limits consolidation. Small firms may have an incentive to stay out of a consolidated enterprise in order to free-ride on the attempts of large firms to raise prices. Third, large firms have a different incentive to invest in industry capital than do small firms, because large firms internalize the effect that their investment has on industry output and prices.<sup>1</sup> The purpose of this paper is to understand the equilibrium evolution of industry concentration within the context of a model that incorporates these three effects.

We develop our framework using the Perry and Porter (1985) model as our point of departure. Perry and Porter consider a static industry model with two stages. In the first stage, firms can merge in order to reallocate industry-specific capital. In the second stage, output is produced and sold to consumers and profits are realized. The industry structure features a dominant oligopoly and a continuum of infinitesimally small firms that form a competitive fringe. As in the textbook dominant firm model, in the production stage, the firms in the dominant oligopoly move first and the fringe firms move second.

Our model departs from this basic structure in three ways. First, our model is dynamic: the capital stock, rather than being fixed, depends upon investment decisions. Thus, concentration can change over time from mergers and from internal investment. In this latter respect the model is analogous to Kydland (1979). Second, we consider only the case where there is a single firm in the dominant oligopoly; i.e., there is one dominant firm. This simplifies our analysis while maintaining a structure that can address our fundamental question. Third, we endogenize the merger process. Perry and Porter (1985) seek to determine the

profitability of alternative hypothetical mergers. In contrast, we explicitly model the merger process.<sup>2</sup> In our model, the dominant firm moves first in the capital market, followed by the fringe.<sup>3</sup> Thus, the capital market parallels the output market.

We make four assumptions about the technology. The first two follow Perry and Porter (1985); the second two concern investment which is outside of the scope of the earlier paper. First, we assume that the dominant firm and the fringe share an identical constant returns to scale technology, which implies that mergers do not change the production possibility set. If the dominant firm were more efficient than the fringe or if there were increasing returns to scale, the dominant firm would tend to gain share at the expense of the fringe. We are interested in understanding the evolution of concentration abstracting from these two well-understood factors. Second, we assume that capital used in the production process is specific to the industry. If capital were not industry-specific, then capital could flow instantaneously into the industry from other industries and there would be no potential for monopolization. Third, we assume increasing adjustment costs for expanding industry capital. There is a long tradition in economics for studying this kind of environment (Lucas, 1967). Our conception of industry capital includes organization capital and relationship capital with customers.<sup>4</sup> Fourth, we assume that new capital and output are linked in fixed proportions, which simplifies the analysis by allowing for only one decision per firm at the output/investment stage. We make this assumption for convenience and show that our results are robust to relaxing this assumption.

We turn now to a discussion of our results. We find that industry evolution depends upon the initial concentration level. In particular, an industry starting with pure monopoly will remain a monopoly forever, and an industry starting with perfect competition will remain competitive forever. The reason for the monopoly result is that the industry profit force dominates: a monopoly maximizes total industry value; the monopolist completely internalizes this and would lose from any sell-off. The reason for the competition result is that the free-rider force dominates: for a given market structure, a fringe firm maximizes the value per unit of capital, while the dominant firm earns less per unit of capital. In order to acquire

capital, a dominant firm must pay the fringe its outside option, which is the value of being a fringe firm given the final industry structure. To acquire capital, an infinitesimally small dominant firm must pay the fringe a higher price than it can earn on each of its new units of capital, and, unlike a larger dominant firm, it does not have an existing base of capital over which to internalize the benefit from the acquisition.

If the industry starts out in between a monopoly and perfect competition, the dominant firm sometimes acquires capital in the merger stage and may merge to monopoly. However, the dominant firm sometimes sells off capital. Sell-offs occur because the dominant firm cannot commit to future behavior. Fringe firms invest, rationally anticipating that, in future periods, the dominant firm will hold back output and possibly buy capital.<sup>5</sup> The dominant firm would like to discourage fringe investment by committing to not holding back output and not acquiring future capital, but such commitments are not credible. By divesting capital, the dominant firm discourages fringe investment. Thus, sell-offs substitute for commitment.<sup>6</sup>

During the investment stage, the dominant firm generally invests at a lower rate than the fringe firms, so concentration declines. If there are sell-offs during the merger stage, then concentration decreases in both stages and the industry tends to converge to perfect competition in the long run. But if there are positive mergers, the merger stage offsets the investment stage and the net effect will vary. There can be stable steady states with an intermediate level of concentration where these two effects exactly offset each other.

Industry evolution also depends upon three key model parameters: the elasticities of fringe supply and industry demand and the discount factor. As fringe supply becomes more elastic, long-run concentration decreases. This occurs because an increase in the elasticity of supply is equivalent to a decrease in the importance of industry-specific capital, and without industry-specific capital, mergers are futile. In contrast, as demand becomes more *inelastic*, long-run concentration decreases. This may seem counterintuitive since monopolization raises industry profits the most for inelastic demand. However, inelastic demand causes an even larger free-rider effect, because the dominant firm reacts to decreases in demand elasticity by reducing output, which causes higher prices and, in turn, causes the fringe firms to *raise*

output, leading to an increased wedge between the dominant firm and fringe values. The increased wedge leads to less acquisition and less investment relative to the fringe, both of which lower the long-run concentration of the industry. A high discount factor means that the future is weighted heavily, which leads to a large commitment problem. Thus, sell-offs only occur when the discount factor and initial fringe market share are high enough for commitments to future behavior to be relevant.

The remainder of this paper is divided as follows. Section 2 presents the model. Section 3 presents results for a single-period variant of the model. Section 4 presents results for the general model with multiple periods and forward-looking agents. Section 5 provides robustness checks of the assumption that capital and output are produced in fixed proportions. Section 6 concludes.

## 2. The model

We adopt a discrete time model with  $T$  periods. In some cases it is convenient to work with an infinite horizon,  $T = \infty$ ; in other cases we work with a finite horizon,  $T < \infty$ . Firms choose actions to maximize their expected discounted value of profits. In each period there is a merger process followed by an output/investment process. We model the industry in partial equilibrium with a demand curve that is constant over time. We start by detailing the preferences and technologies and then define the equilibrium of the model.

### Preferences and technologies

Demand at price  $p$  is  $Q = D(p)$ . Assume demand is strictly decreasing, and let  $p = D^{-1}(Q) \equiv P(Q)$  denote the inverse demand curve. The discount factor is  $\beta$ .

We explain the technologies of the output/investment process (the second stage) and the merger process (the first stage). At the start of the output/investment process, there is a dominant firm and a competitive fringe, endowed with  $K_d$  and  $K_f$  units of capital stock, respectively. The capital stock is specific to the industry. The fringe is composed of a continuum of firms, each of which owns an infinitesimally small amount of  $K_f$  and hence is a price taker.

The dominant firm and the fringe firms all have access to the same technology, and they produce homogeneous products. Firms combine industry-specific capital  $K$  with non-industry-specific labor  $L$  in a production process  $F(K, L)$  that produces joint outputs, the consumption good  $Q$  and future capital  $K_{next}$ . Our base model assumes that the two outputs are produced in fixed proportions. In Section 5, we relax this assumption. Let  $Q = F(K, L)$  be the production of the consumption good and  $K_{next} = \sigma F(K, L)$  be the production of future capital given inputs  $K$  and  $L$ , where  $0 < \sigma < 1$ . We assume that current capital  $K$  is consumed in the production process. If we interpret  $Q$  as a measure of end-of-period capital and define  $\delta \equiv 1 - \sigma$ , then  $\delta$  can be interpreted as the depreciation rate, since  $K_{next} = (1 - \delta)Q$  and  $\sigma$  is the fraction of end-of-period capital that survives into the next period.

We make several assumptions about  $F(K, L)$ . First, we assume constant returns to scale. Second, we assume that  $F(0, L) = 0$ , implying that if the dominant firm were to obtain all the industry-specific capital, monopoly could be ensured forever. Third, we assume that  $F(K, L)$  is strictly concave and is strictly increasing in both arguments (for  $K > 0$  and  $L > 0$ ) and that the Inada conditions on  $L$ ,  $\lim_{L \rightarrow \infty} F_L(K, L) = 0$  and  $\lim_{L \rightarrow 0} F_L(K, L) = \infty$ , hold for  $K > 0$ .

In the analysis it will be convenient to utilize a labor cost function rather than the underlying production function. Let  $C(Q, K)$  be the labor cost to produce  $Q$  units of the consumption good (and  $\sigma Q$  of next-period capital); i.e.,  $C(Q, K) = \omega L'$  for the  $L'$  that solves  $Q = F(K, L')$ , given the competitive wage  $\omega$ . The assumptions on  $F(K, L)$  imply that  $C(Q, K)$  is homogeneous of degree 1, so that  $C(Q, K) = KC(Q/K, 1)$ . In the analysis we use lowercase  $q$  to denote output per unit of capital; i.e.,  $q = Q/K$ . Let  $c(q) = C(q, 1)$  denote the labor cost per unit of capital necessary to produce  $q$  units of output per unit of capital. The assumptions on  $F$  imply that  $c$  is strictly increasing and strictly convex and that  $c'(0) = 0$ .

The timing in the output/investment stage follows the textbook treatment of the dominant firm model (Carlton and Perloff, 1994). The dominant firm first sets an industry price. The fringe firms observe this price and simultaneously decide on production levels. The dominant firm supplies the residual demand at the price. Equivalent to our assumption that

the dominant firm chooses the price from a residual demand curve, we could assume that it chooses the quantity from the same residual demand curve. We will work with this alternate quantity formulation in the following sections because it is notationally more convenient; Perry and Porter (1985) do the same.

In the merger process, the dominant firm posts a price at which it commits to buy or sell all the capital that is supplied or demanded at this price. The fringe firms then simultaneously choose whether to sell to the dominant firm given the price and their expectation of the future state. This amount of capital purchased can be zero (corresponding to no merger) or negative (corresponding to a divestiture). It is again more convenient to work with the alternate but equivalent formulation where the dominant firm chooses a quantity of capital to purchase and then picks the price that would yield this quantity.

Our merger process is inconsistent with Perry and Porter (1985) in one regard. Perry and Porter (1985) assume that the fringe will merge if the total return to the merged entity exceeds what the firms would get separately. An extensive form that would generate this is that the dominant firm makes take-it-or-leave-it offers to all firms in the fringe sector; if any one fringe firm declines to tender its capital at the offered price, the deal falls through for all the fringe firms and no merger takes place. In contrast, we assume, as do Shleifer and Vishny (1986), that every individual fringe firm is an infinitesimally small player. With our assumption, there is a much stronger free-rider effect against merger compared to Perry and Porter (1985).<sup>7</sup> An appealing property of our assumption is that it extends to the capital market the assumption that fringe firms behave competitively in the output market.

### **Equilibrium of the model**

We analyze the Markov-perfect equilibria (MPE) of our model in the sense of Maskin and Tirole (2001). This means that we examine equilibria where actions are a function solely of payoff-relevant state variables, which are the capital stocks in our case. Let  $T = \infty$ . and let  $(K_d^\circ, K_f^\circ)$  denote the capital stocks held by the dominant firm and fringe before the merger stage. Let  $(K_d, K_f)$  denote the capital stocks *after* the merger stage but *before* the

output/investment stage. (Throughout, the superscript “ $\circ$ ” will denote pre-merger values, and the absence of a superscript will signify post-merger values.) The total capital stock  $K$  is the same before and after the merger stage,  $K = K_d + K_f = K_d^\circ + K_f^\circ$ . It will be convenient to keep track of the state with the total capital stock and the share of the total held by the dominant firm. Let  $m^\circ = K_d^\circ/K$  be the industry concentration before the merger stage, and let  $m = K_d/K$  denote the concentration after the merger stage.

Define  $v_f(m, K)$  to be the discounted value to a fringe firm possessing one unit of capital at the output/investment stage when the aggregate state is  $(m, K)$ . Analogously, define  $v_d(m, K)$  to be the discounted value to the dominant firm *per unit of capital* possessed by the dominant firm. Since the dominant firm holds  $mK$  units of capital at this stage, its *total* return is  $w_d(m, K) = mKv_d(m, K)$ . Throughout the paper, a “ $v$ ” will denote a return per unit of capital, while a “ $w$ ” denotes a total return.

At the merger stage, the dominant firm with market share  $m^\circ$  chooses the post-merger market share  $m$ . Given  $m$ , the amount of capital purchased by the dominant firm is  $mK - m^\circ K$  and the equilibrium price of capital is

$$p_K(m, K) = v_f(m, K).$$

This is the price at which fringe firms are indifferent among buying, selling, or holding onto their capital. Thus, the dominant firm chooses  $m$  to solve

$$\begin{aligned} w_d^\circ(m^\circ, K) &= \max_m mKv_d(m, K) - (mK - m^\circ K)p_K(m, K) \\ &= \max_m mKv_d(m, K) - (m - m^\circ)Kv_f(m, K). \end{aligned} \tag{1}$$

The first term in the objective function is the dominant firm’s return entering the output/investment stage with a share of  $m$  and thus  $mK$  total units of capital. The second term subtracts the amount spent on the acquisition of capital (this subtracts a negative number in the event of a sell-off). Let  $\tilde{m}(m^\circ, K)$  be the solution to this problem.

Let  $v_f^\circ(m^\circ, K)$  be the return per unit of capital to the fringe before the merger stage.

Given the merger policy of the dominant firm, this satisfies

$$v_f^\circ(m^\circ, K) = v_f(\tilde{m}(m^\circ, K), K). \quad (2)$$

Now consider the output/investment stage. Let  $q_d$  and  $q_f$  denote output per unit of capital for each firm type, so total output in the dominant firm and fringe sectors is  $Q_d = mKq_d$  and  $Q_f = (1 - m)Kq_f$ . Recall that when fringe firms make their output/investment decision, the dominant firm has already made its move. Let  $\tilde{q}_f(q_d, m, K)$  be the equilibrium output choice (per unit of capital) in the fringe sector, given the choice  $q_d$  by the dominant firm and given  $m$  and  $K$ . This solves the following problem:

$$\tilde{q}_f(q_d, m, K) = \arg \max_{q_f} pq_f - c(q_f) + \beta v_{f,next}^\circ(1 - \delta)q_f \quad (3)$$

where :

$$\begin{aligned} p &= P(Q) \\ Q &= mKq_d + (1 - m)K\tilde{q}_f(q_d, m, K) \\ v_{f,next}^\circ &= v_f^\circ(m_{next}^\circ, K_{next}) \\ m_{next}^\circ &= \frac{mKq_d}{mKq_d + (1 - m)K\tilde{q}_f(q_d, m, K)} \\ K_{next} &= (1 - \delta)Q. \end{aligned}$$

Because it is infinitesimally small, an individual fringe firm takes the current price  $p$  and the future per-unit-of-capital value  $v_{f,next}^\circ$  as fixed when making its output/investment decision. A choice of  $q_f$  yields current revenues of  $pq_f$  and a current cost of  $c(q_f)$ . It also yields  $(1 - \delta)q_f$  units of capital next period, each unit of which will be worth  $v_{f,next}^\circ$ . The representative fringe assumes that the other firms in the fringe sector will behave according to  $\tilde{q}_f(q_d, m, K)$ , so the total fringe sector output will be  $Q_f = (1 - m)K\tilde{q}_f(q_d, m, K)$ . Given this anticipated fringe output and the observed dominant firm output, the representative fringe can calculate the current price and the future prices of capital and output.

Lastly, the dominant firm chooses  $q_d$  to maximize value per unit of initial capital, given the fringe reaction  $\tilde{q}_f(q_d, m, K)$ . Thus,

$$v_d(m, K) = \max_{q_d} p(q_d)q_d - c(q_d) + \frac{\beta w^\circ(m_{next}^\circ(q_d), K_{next}(q_d))}{mK} \quad (4)$$

subject to :

$$\begin{aligned} p(q_d) &= P(Q(q_d)) \\ Q(q_d) &= mKq_d + (1 - m)K\tilde{q}_f(q_d, m, K) \\ m_{next}^\circ(q_d) &= \frac{mKq_d}{mKq_d + (1 - m)K\tilde{q}_f(q_d, m, K)} \\ K_{next}(q_d) &= (1 - \delta)Q(q_d). \end{aligned}$$

(Note that this is equivalent to maximizing the dominant firm's total value since initial capital is fixed at  $mK$ .) Unlike a fringe firm, the dominant firm recognizes that its output/investment choice has an effect on the current and future prices and  $\tilde{q}_f(q_d, m, K)$ .

We now define an MPE of this model for the infinite horizon model. An MPE is a set of functions  $(v_f^\circ, v_f, v_d, w_d^\circ, \tilde{q}_f, \tilde{q}_d, \tilde{m})$  such that:

- (i) The per-unit-of-capital value  $v_f^\circ(m^\circ, K)$  solves (2).
- (ii) The per-unit-of-capital  $v_f(m, K)$  is the value of (3) for  $q_d$  evaluated at  $q_d = \tilde{q}_d(m, K)$ .
- (iii) The per-unit-of-capital value  $v_d(m, K)$  solves (4).
- (iv) The total value  $w_d^\circ(m^\circ, K)$  solves (1).
- (v) The policy function  $\tilde{q}_f(q_d, m, K)$  solves (3).
- (vi) The policy function  $\tilde{q}_d(m, K)$  solves (4).
- (vii) The policy function  $\tilde{m}(m^\circ, K)$  solves (1).

For the finite horizon case,  $T < \infty$ , the definition of equilibrium is analogous to the above, except that now policy functions and value functions must be indexed by time; e.g.,  $v_{f,t}(m_t, K_t)$ .

All of the analytical results that we present for the infinite horizon model are obtained

by first characterizing the finite horizon model and then taking limits as  $T$  goes to infinity. Following the results of Fudenberg and Levine (1986), if the sequence of equilibria defined for the finite horizon case has a limit, this limit is an equilibrium of the infinite horizon game.<sup>8</sup> Our analytic results for the infinite horizon case apply only to equilibria that can be constructed in this way.

### 3. The single-period model

This section characterizes the single-period version of our model. We start with this case primarily because it is more analytically tractable but still allows us to understand the impact of the three key forces noted in the introduction. In addition, our analysis of this version allows us to understand how the results with high  $\beta$  differ from the results with low  $\beta$  in the multi-period model, because in the limiting case where  $\beta = 0$ , merger and investment behavior is the same as in the single-period model.

It is notationally convenient in this section to normalize  $K = 1$ , as it allows us to eliminate  $K$  from the state space. When  $K = 1$ ,  $m^\circ$  and  $m$  are the dominant firm's pre-merger and post-merger capital levels as well as shares.

#### **Are perfect competition and monopoly absorbing states?**

Our first result considers industry evolution starting at the extreme points of monopoly or competition,  $m^\circ = 1$  or  $m^\circ = 0$ . As noted in the introduction, two general principles emerge here. If we begin the period with monopoly, we stay with monopoly. If we begin with perfect competition, we stay with perfect competition. Formally, we have

*Proposition 1.* The equilibrium merger policy function  $\tilde{m}(m^\circ)$  satisfies  $\tilde{m}(1) = 1$  and  $\tilde{m}(0) = 0$ .

*Proof.* Applying (1) to the case of  $m^\circ = 1$ , the dominant firm's problem is

$$\max_m \{mv_d(m) + (1 - m)v_f(m)\}. \quad (5)$$

The objective function here is the sum of the dominant firm profit plus the fringe sector profit,

which equals total industry profit. As total industry profit is maximized with monopoly,  $m = 1$  is a solution to (5). In Lemma 3 below, we prove that  $m = 1$  is the unique solution.

For the case of  $m^o = 0$ , applying (1) again, the dominant firm's problem is

$$\max_m \{mv_d(m) - mv_f(m)\}. \quad (6)$$

At  $m = 0$ , the value in (6) is zero. From (3) and (4), a fringe firm could always choose the dominant firm quantity choice and earn  $v_d(m)$ . However, since  $v_f(m)$  maximizes the fringe earnings, it must be at least as high as  $v_d(m)$ . Thus, for  $m > 0$ ,  $v_d(m) \leq v_f(m)$ . Hence,  $m = 0$  is a solution to (6). Below we show that  $v_d(m) < v_f(m)$  for  $m > 0$ , which implies uniqueness. *Q.E.D.*

It is worth noting that the logic of Proposition 1 suggests there is no incentive for a second dominant firm to emerge out of the fringe. Suppose the model were to allow for the possibility of multiple large firms and that there is “free entry” into becoming a large firm. If such a new entrant were to buy up a positive measure of capital, there would now be a dominant duopoly as in Perry and Porter (1985). But, analogous to the discussion above, such an entrant would have to pay the fringe value per unit of capital, which is greater than the dominant duopolist's value—a losing proposition.

### **Are there mergers or sell-offs?**

We now show that there are always positive mergers between the extremes of  $m^o = 0$  and  $m^o = 1$ . This requires an analysis of first-order conditions in the output/investment stage and the merger stage. With a single period, the first-order condition of the fringe firm's problem in the output/investment stage (3) reduces to

$$p - c'(q_f) = 0; \quad (7)$$

i.e., price equals marginal cost. The first-order condition of the dominant firm's problem (4)

can be written as

$$MR_d - c'(q_d) = 0, \quad (8)$$

where the dominant firm's marginal revenue is

$$MR_d \equiv p + q_d P' \left[ m + (1-m) \frac{\partial q_f}{\partial q_d} \right]. \quad (9)$$

To interpret (9), consider a decision by the dominant firm to expand its total output by one unit. It gets a price  $p$  for the extra unit. But this action will depress the price on the  $m q_d$  units that it is already selling. If the dominant firm were a monopoly, then the change in price would be  $P'$  and the bracketed term in (9) would equal one. This is the direct part of the investment effect. Now suppose that there is a non-zero fringe. Then, any increase in dominant firm quantity is mitigated by a decrease in fringe quantity. This is the strategic (Spence, 1977; Dixit, 1980) part of the investment effect that has the opposite sign from the direct part. Thus, the bracketed term is less than one for a dominant firm with a fringe. However, it is straightforward to show that the direct part dominates and hence that the bracketed term must be strictly positive.<sup>9</sup> In combination with the fact that  $P' < 0$ , we have shown

*Lemma 1.* A dominant firm with  $m > 0$  has  $MR_d < p$ , and hence  $q_d < q_f$ .

The difference between a fringe firm and the dominant firm is that a fringe firm's sales are infinitesimally small, and so the second term in (9) drops out. In Figure 1, we illustrate the dominant firm and fringe output decisions. The dominant firm produces at the level  $q_d$ , where  $MR_d$  equals marginal cost. The profit per unit of capital obtained by the dominant firm is illustrated by the lightly shaded area in the graph between the price line and the marginal cost curve up to  $q_d$ . A fringe firm could always choose to produce at  $q_d$  and obtain the dominant firm profit. But it can do even better by raising its output to  $q_f$ , where the price equals marginal cost. The fringe value equals the light gray area (the dominant firm value) plus the dark gray triangle between  $q_d$  and  $q_f$ . Thus,  $v_d(m) < v_f(m)$  for  $m > 0$ . But

note that at the extreme where  $m = 0$ , the dominant firm behaves like a perfectly competitive firm,  $q_d = q_f$ , and the return per unit of capital is the same,  $v_d(0) = v_f(0)$ .

Even though the *average* value per unit of capital is higher for the fringe, it can be seen from Figure 1 that the *marginal* value of *transferring* a unit of capital from the fringe to the dominant firm is higher than the fringe value of this capital. This result is analogous to the famous result by Gilbert and Newbery (1982).<sup>10</sup> The formal result here is

*Lemma 2.*

$$\frac{d[mv_d(m)]}{dm} = v_d(m) + m \frac{dv_d(m)}{dm} > v_f(m), \text{ for } m > 0.$$

*Proof.*

Consider the fringe policy function  $\tilde{q}_f(q_d, m)$ , implicitly defined by (7). Expanding (7) using (3) and differentiating yields

$$\frac{\partial \tilde{q}_f}{\partial m} = \frac{-P'K(q_f - q_d)}{c_f'' - P'(1-m)K} \text{ and } \frac{\partial \tilde{q}_f}{\partial q_d} = \frac{P'mK}{-(1-m)KP' + c_f''}. \quad (10)$$

The dominant firm value is

$$v_d = \max_{q_d} P(mKq_d + (1-m)K\tilde{q}_f(q_d, m))q_d - c(q_d).$$

Using the envelope theorem,

$$\begin{aligned} \frac{\partial v_d}{\partial m} &= P' \left[ K(q_d - q_f) + (1-m)K \frac{\partial \tilde{q}_f}{\partial m} \right] q_d \\ &= \frac{-P'K(q_f - q_d)c_f''}{c_f'' - P'(1-m)K}. \end{aligned}$$

Rearranging terms and substituting for  $MR_d$  using (9) and (10),

$$\begin{aligned}
\frac{d[mv_d(m)]}{dm} &= v_d(m) + m \frac{dv_d(m)}{dm} \\
&= pq_d - c(q_d) + m \frac{dv_d(m)}{dm} + (v_f(m) - pq_f - c(q_f)) \\
&= v_f(m) + p(q_d - q_f) - (c(q_d) - c(q_f)) + m \frac{-P'K(q_f - q_d)c''_f q_d}{c''_f - P'(1-m)K} \\
&= v_f(m) + [MR_d q_d - c(q_d)] - [MR_d q_f - c(q_f)].
\end{aligned}$$

The integral is strictly positive since  $q_d$  is the sole maximizer of  $MR_d q - c(q)q$ , and  $q_f \neq q_d$  for  $m > 0$ . *Q.E.D.*

To understand the intuition behind this result, it is useful to examine the impact of a (small) transfer  $\varepsilon$  of capital from the fringe to the dominant firm. If the dominant firm operates the new  $\varepsilon$  capital at the level  $q_f$ , it would earn  $pq_f - c(q_f)$  per unit of new unit of capital, which is the fringe profit  $v_f(m)$  (the light gray and dark gray areas in Figure 1). A decrease in output on the newly transferred capital from  $q_f$  to  $q_d$  raises the dominant firm's profit per new unit by the black triangle in Figure 1. Thus, the black triangle is the amount by which the marginal benefit of the unit to the dominant firm exceeds the value of the unit to the fringe. This triangle is equal to the difference between the dominant firm's total value from operating the capital at the rate  $q_f$  (which is  $MR_d q_f - c(q_f)$ ) and at the rate  $q_d$  (which is  $MR_d q_d - c(q_d)$ ).

We can use Lemma 2 to understand the dominant firm's incentive to merge. Define the *merger marginal benefit* to be the slope of the dominant firm's acquisition choice problem (1).

$$\text{merger marginal benefit} \equiv v_d(m) + m \frac{dv_d(m)}{dm} - v_f(m) - (m - m^\circ) \frac{dv_f(m)}{dm}. \quad (11)$$

If the dominant firm alters its acquisition decision to purchase one more unit of capital, it can earn a profit of  $v_d$  on this new unit (the first term), and acquiring this extra unit drives up the profit on the units of capital the dominant firm is taking into the output/investment

stage (the second term). But the dominant firm has to pay  $v_f$  for it (the third term), and buying one extra unit raises the price of capital in the capital market (the fourth term).

From the proof of Lemma 2, we know that the net effect of the first three terms is strictly positive. The fourth term limits the incentive of the dominant firm to buy all the capital because as it does so it raises the price of capital; the dominant firm recognizes its monopsony power in the capital market and factors this effect into its decision. (An analogous effect is in Lewis, 1983.)<sup>11</sup> But note that at  $m = m^\circ$ , this monopsony effect is zero since acquisitions are zero, so the fourth term of (11) is zero at this point. Thus, a small positive merger is always better than a zero merger. In order to rule out the case of  $m < m^\circ$ , we impose the following intuitive regularity condition:

*Assumption 1.* The demand function  $D(p)$  and the cost function  $c(q)$  are such that the equilibrium price  $\tilde{p}(m)$  is strictly increasing in  $m$ , for  $m > 0$ .

We can show that this assumption is satisfied under the constant elasticity parameterization for demand and cost given by (19) below. We then obtain

*Proposition 2.* Suppose  $m^\circ \in (0, 1)$  and Assumption 1 holds. Then the optimal capital purchase is strictly positive,  $\tilde{m}(m^\circ) > m^\circ$ . Moreover,  $\tilde{m}(m^\circ)$  is increasing in  $m^\circ$ .

*Proof.* Using (11) and Lemma 2,

$$\text{merger marginal benefit} > -(m - m^\circ) \frac{dv_f(m)}{dm}. \quad (12)$$

Now,  $\frac{dv_f}{dm} = q_f \frac{dp}{dm}$  by the envelope theorem. Assumption 1 implies that  $v_f(m)$  is increasing in  $m$ , and hence that the *merger marginal benefit* is strictly positive for  $m \in [0, m^\circ]$ . Thus, the optimal choice of  $m$  is strictly greater than  $m^\circ$ . Next note that the slope of the *merger marginal benefit* with respect to  $m^\circ$  equals  $\frac{dv_f}{dm}$ , which is strictly positive under Assumption 1. This implies that the optimum  $\tilde{m}(m^\circ)$  is monotonic. *Q.E.D.*

## Is there merger to monopoly?

While a positive merger is optimal, the dominant firm may not necessarily merge to complete monopoly because of its monopsony power in the capital market. The more capital it acquires, the higher the price it must pay per unit of capital. However, if the dominant firm's initial share  $m^\circ$  is close to one, the monopsony power effect is a second-order consideration and there is complete merger to monopoly, as long as the following assumption holds.

*Assumption 2.* The fringe value at pure monopoly is bounded,  $v_f(1) < \infty$ .

As  $\beta = 0$ , this assumption holds if and only if the pure monopoly price is bounded. We state the assumption in terms of the fringe value to make it comparable to the analysis in the next section. If demand is constant elasticity, Assumption 2 is satisfied if and only if the demand elasticity is strictly greater than one. We then obtain

*Proposition 3.* Suppose Assumptions 1 and 2 both hold. Then there exists a cutoff  $\hat{m}^\circ$  satisfying  $0 < \hat{m}^\circ < 1$  such that  $\tilde{m}(m^\circ) = 1$  if and only if  $m^\circ \geq \hat{m}^\circ$ . Thus, for  $m^\circ$  above the cutoff, the dominant firm buys the entire stock of fringe capital.

*Proof.* By Assumption 2, the monopoly price is finite. Thus, the first-order necessary condition (8) for the firm's output choice holds at  $m = 1$ , and Lemma 2 can be applied to the  $m = 1$  case. From Lemma 2, the first three terms of the *merger marginal benefit* are strictly positive at  $m = 1$ . If  $m^\circ = m = 1$ , the fourth term is zero, so the *merger marginal benefit* is strictly positive. Using Proposition 2 and continuity, for  $m^\circ$  close enough to one, the optimal merger must be the corner solution  $\tilde{m}(m^\circ) = 1$ . From Proposition 1, for  $m^\circ$  close enough to zero,  $\tilde{m}(m^\circ) < 1$ . Since  $\tilde{m}(m^\circ)$  is nondecreasing from Proposition 2, there must be an interior cutoff  $\hat{m}^\circ$  as claimed. *Q.E.D.*

Note that merger to monopoly will never occur with inelastic demand. Unlike Proposition 3, the results away from monopoly do not depend on whether or not demand is elastic.

### What happens with no merger to monopoly?

Suppose now that  $m^\circ$  is less than the cutoff  $\hat{m}^\circ$  where there is merger to monopoly. Define  $m_{next}^\circ(m)$  to be the dominant firm's share of new capital at the end of the period. (We use the subscript *next* because if there were an additional period added to this single-period model,  $m_{next}^\circ(m)$  would be the pre-merger dominant firm share in the next period.) Given the fixed proportions assumption for new capital and output, the end-of-period new capital share equals the output share,

$$m_{next}^\circ(m) = \frac{mq_d}{mq_d + (1-m)q_f}, \quad (13)$$

where  $q_d$  and  $q_f$  are the equilibrium levels of output and are implicitly a function of  $m$ . We find

*Proposition 4.* Suppose that  $m^\circ \in (0, \hat{m}^\circ)$ , so that there is a positive merger  $\tilde{m}(m^\circ) > m^\circ$  but no merger to monopoly,  $\tilde{m}(m^\circ) < 1$ . Then  $m_{next}^\circ(m)$  is strictly less than the current post-merger share; i.e.,  $m_{next}^\circ(\tilde{m}(m^\circ)) < \tilde{m}(m^\circ)$ .

*Proof.* From Lemma 1, we know that  $q_d < q_f$ ; i.e., the dominant firm invests at a lower rate than the fringe. The result is an immediate consequence. *Q.E.D.*

Note that if there were no merger stage, this investment force would be the only effect, and the dominant firm's market share would necessarily decline to zero in the long run (Holmes, 1996). With mergers, there are two offsetting effects on concentration. The merger stage increases concentration, while the output/investment stage decreases concentration. The net effect is, in general, ambiguous.

## 4. The multi-period model

We now turn to an analysis of the general multi-period model. We divide our discussion into the same four subsections as in Section 3.

### Are perfect competition and monopoly absorbing states?

We show here that perfect competition and monopoly are absorbing states for the multi-period model, and we characterize these states. Thus, this subsection extends Proposition 1.

We begin with the following lemma:

*Lemma 3.* Suppose the horizon is finite,  $t \in \{1, 2, \dots, T\}$ ,  $T < \infty$ . (i) For all  $t$ ,  $v_{f,t}(m, K) > v_{d,t}(m, K)$ , if  $m > 0$ , and  $v_{f,t}(m, K) = v_{d,t}(m, K)$ , if  $m = 0$ . (ii) For all  $t$ ,  $v_{d,t}(1, K) > mv_{d,t}(m, K) + (1 - m)v_{f,t}(m, K)$ , if  $m < 1$ .

*Proof.* (i) We prove here the weak inequality. The proof of the strict inequality, which relies on a lengthy technical detail, is in Gowrisankaran and Holmes (2003).

To show that

$$v_{f,t}(m, K) \geq v_{d,t}(m, K), \quad (14)$$

consider an initial state  $(m_1^\circ, K_1)$  and the resulting equilibrium path. Along this path, the dominant firm in period  $t$  grows at a rate  $e_{d,t}$  externally through acquisition of capital, where

$$e_{d,t} = \frac{m_t - m_t^\circ}{m_t^\circ},$$

and at rate  $q_{d,t}$  internally through investment. The value to the dominant firm in period  $t$  after the merger stage is

$$v_{d,t} = p_t q_{d,t} - c(q_{d,t}) + \beta(1 - \delta)v_{d,t+1}^\circ,$$

and the value before the merger stage is

$$v_{d,t}^\circ = e_{d,t} v_{d,t} - (e_{d,t} - 1) p_{K,t}.$$

A fringe firm behaves competitively and takes the output price sequence  $\{p_t\}$  and the capital price sequence  $\{p_{K,t}\}$  as given. A fringe firm could always choose to mimic the dominant firm path of external and internal growth and get the same payoff per unit of capital as the dominant firm. Since this path is in the fringe firm's choice set, inequality (14) must hold.

(ii) We prove here the strict inequality, as the proof of the weak inequality is the same as in Proposition 1. Assume contradictorily that there exists  $m_t \in [0, 1)$  such that the discounted industry profit starting at  $(m_t, K_t)$  is equal to the discounted industry profit at pure monopoly  $(1, K_t)$ . The pure monopoly problem is concave so there is a unique solution. Thus, the sequences of quantities are identical and equal to the sequence of monopoly quantities, i.e.

$$q_f(m_t, K_t) = q_d(m_t, K_t) = q_d(1, K_t), \quad (15)$$

implying that the sequences of prices are equal to the monopoly sequence of prices. However, it is straightforward to show that the fringe output in a period given pure monopoly prices in every remaining period is strictly higher than the pure monopoly output in the period, which yields a contradiction. *Q.E.D.*

Using the same arguments as in the proof of Proposition 1, the above lemma immediately implies that if the dominant firm has zero capital in period  $t$ ,  $m_t^\circ = 0$ , it will purchase no capital. If it has all the capital,  $m_t^\circ = 1$ , it will sell no capital. Thus, perfect competition and pure monopoly are absorbing states.

In what follows it will sometimes be useful to refer to the perfect competition and pure monopoly steady states in the infinite horizon model in which capital stocks do not change over time. In a steady state, investment must exactly offset depreciation. Therefore, for both competition and monopoly the steady state investment rate must be

$$q_{com}^* = q_{mon}^* = \frac{1}{1 - \delta}, \quad (16)$$

where the asterisk denotes steady state values, “com” denotes competition, and “mon” denotes monopoly.

For the case of the perfect competition steady state, solving the fringe first-order

condition for the stationary equilibrium price, we obtain

$$p_{com}^* = (1 - \beta)c'(q_{com}^*) + \beta(1 - \delta)c(q_{com}^*). \quad (17)$$

The right side of (17) can be interpreted as dynamic marginal cost. Together, (16) and (17) uniquely define  $q_{com}^*$  and  $p_{com}^*$ . Note that the steady state is fully characterized by  $q_{com}^*$  and  $p_{com}^*$ , because these variables imply values for other steady state variables, such as  $Q_{com}^* = D(p_{com}^*)$  and  $K_{com}^* = (1 - \delta)Q_{com}^*$ .

It is straightforward to show that the monopoly stationary price is a markup over the stationary competitive price,

$$p_{mon}^* = \frac{\varepsilon_D}{\varepsilon_D - 1} p_{com}^*, \quad (18)$$

where  $\varepsilon_D$  is the elasticity of demand evaluated at the stationary output level. Together with (16), (18) defines the monopoly steady state.

We note one caveat about the competitive steady state here. In this limit, the dominant firm has measure zero, so it is infinitesimally small like the fringe. But the structure of moves is different; the (not very) dominant firm moves first, and the fringe firms move second. The fact that the structure of moves for the dominant firm does not converge to that of the fringe may be a cause for objection. Nevertheless, it is worth noting that in the limit where  $m = 0$  the *behavior* of the dominant firm converges to that of the fringe,  $q_d = q_f$ , since the dominant firm's marginal revenue converges to the discounted price. Thus our results are not an artifact of different behavior near the limit.

### **Are there mergers or sell-offs?**

We first examine how industry concentration and the discount factor influence whether or not there will be positive mergers. We show that there will be a positive merger for any  $\beta < 1$  provided that  $m^\circ$  is sufficiently close to one. In contrast, when the industry is sufficiently unconcentrated ( $m^\circ$  close to zero) there will be sell-offs for  $\beta$  sufficiently close to one. As noted in the introduction, these results are driven by commitment. By divesting itself of

capital, the dominant firm lowers the expectations of future prices, thereby inducing fringe firms to invest less.<sup>12</sup> When  $m^\circ$  is sufficiently close to one or  $\beta$  sufficiently close to zero, the commitment problem is negligible.

Formally, we show

*Proposition 5.* (i) Assume demand is elastic, and suppose there is a finite horizon,  $T < \infty$ . Fix an initial capital stock  $K_1$ . If  $m_1^\circ < 1$  is close enough to one, then  $\tilde{m}_1(m_1^\circ, K_1) > m_1^\circ$ ; i.e., there is a positive merger.

(ii) Suppose the horizon is infinite,  $T = \infty$ . There exists a  $\beta' < 1$  such that if  $\beta > \beta'$ ,  $K = K_{com}^*(\beta)$  and the dominant firm's initial market share  $m^\circ$  is positive but sufficiently small, then  $\tilde{m}(m^\circ, K) < m^\circ$ ; i.e., the dominant firm sells capital.

*Proof.* (i) See Appendix A. (ii) The proof is available in Gowrisankaran and Holmes (2003). We obtain the result by analytically evaluating the policy and value functions (first- and second-order terms) near the limit where  $m^\circ = 0$ . *Q.E.D.*

A couple of comments about Proposition 5 are worth noting. First, in (i), we assume elastic demand to avoid the complication with inelastic demand that price is infinite in the limit of pure monopoly. In our numerical work, we also include the case of inelastic demand and we find an analogous result. Similarly, in (ii), we assume that capital is evaluated at the steady state level  $K_{com}^*$  because it vastly simplifies the proof. In our numerical work, we have found that the merger function tends to be invariant to the level of the capital stock, suggesting that our results will likely extend to other levels of capital.

The logic of (i) can be seen by considering Lemma 2. There we showed that with  $\beta = 0$ , the *merger marginal benefit* at  $m = m^\circ$  is positive, because the dominant firm could break even by operating new capital at the rate  $q_f$  and would realize a first-order gain from a rate of  $q_d$ . When  $\beta > 0$ , the remaining fringe firms will in general behave differently following a transfer, even if the dominant firm were to operate the new capital at a rate  $q_f$  in the current period, because the remaining fringe firms expect the dominant firm to behave differently

from the fringe in the future. However, in the limit where  $m_1^\circ$  is close to one, the fringe sector is arbitrarily close to zero in size, and this future response of the fringe is a second-order consideration, implying that the form of the *merger marginal benefit* is as in Lemma 2.

The analytic results above consider the extreme cases where  $m^\circ$  is close to zero or  $m^\circ$  is close to one. We can numerically calculate the equilibrium for intermediate cases.<sup>13</sup> It is convenient to focus on the constant elasticity case:

$$\begin{aligned} D(p) &= p^{-\varepsilon_D} \\ c(q) &= q^{1+\frac{1}{\varepsilon_S}}. \end{aligned} \tag{19}$$

In (19),  $\varepsilon_D$  is the elasticity of demand and  $\varepsilon_S$  is the elasticity of fringe supply.<sup>14</sup>

Figure 2 shows the merger policy function  $\tilde{m}(m^\circ, K)$  for  $\varepsilon_D = .5$ ,  $\varepsilon_S = 2$ ,  $\delta = .2$ ,<sup>15</sup>  $K = \frac{1}{2}(K_{mon}^* + K_{com}^*)$ ,<sup>16</sup> and two values of  $\beta$ . Observe that for  $\beta = 0$ , the merger level lies above the 45-degree line for all interior values of  $m^\circ$ ; i.e., merger levels are positive. Proposition 2 implies that this must be true. In contrast, for high  $\beta$  (here  $\beta = .9$ ), the function lies strictly below the 45-degree line for small  $m^\circ$  and above it for high  $\beta$ , consistent with Proposition 5. The cutoff value here is roughly .60. Note that sell-offs occur here even at a relatively high level of concentration.

In addition to  $\beta$  and  $m^\circ$ ,  $\tilde{m}$  also depends upon the elasticities of demand and fringe supply. Figure 3 plots  $\tilde{m}$  for different values of  $\varepsilon_S$  with  $\varepsilon_D = 2$  and  $\beta = .3$ . The figure shows that an increase in  $\varepsilon_S$  shifts the merger function *down*. Figure 4 illustrates the impact of  $\varepsilon_D$  on  $\tilde{m}$ , holding  $\varepsilon_S = 1$  and  $\beta = .3$ . If we start at  $\varepsilon_D = 1$  and increase  $\varepsilon_D$  slightly, the function shifts up and continues to have the same shape. (This is not shown in the figure to keep the graph simple.) However, a large increase in  $\varepsilon_D$  changes  $\tilde{m}$  into the “saw-toothed” shape seen for  $\varepsilon_D = 4$ . Note that while the merger function for  $\varepsilon_D = 4$  is clearly higher than the function for  $\varepsilon_D = 1$  for most initial shares  $m^\circ$ , the  $\varepsilon_D = 4$  function dips just below the  $\varepsilon_D = 1$  function at kink points such as at  $m^\circ = .77$ .

The comparative statics findings discussed here are representative of our findings with

the other numerical examples we calculated. The same saw-toothed pattern occurs whenever  $\varepsilon_D$  is large or  $\varepsilon_S$  is low, provided that  $\beta$  is sufficiently high. We can also show analytically that the merger function increases in  $\varepsilon_D$  and decreases in  $\varepsilon_S$  near the limit where  $m^\circ$  is close to zero.<sup>17</sup>

### Is there merger to monopoly?

The next question is when we will obtain immediate merger to monopoly. If such a merger does occur, the evolution of industry concentration is complete since monopoly is an absorbing state.

Our result is

*Proposition 6.* Suppose the horizon is infinite,  $T = \infty$ . Fix  $\varepsilon_D > 1$ ,  $\varepsilon_S > 0$ , and  $K > 0$ .

(i) There exists a cutoff  $\bar{\beta} < 1$  such that  $\beta > \bar{\beta}$  implies  $v_f(1, K) = \infty$ , while  $\beta \leq \bar{\beta}$  implies  $v_f(1, K) < \infty$ . The cutoff is the unique  $\bar{\beta} \in (0, 1)$  that satisfies

$$0 = \varepsilon_D \left( 1 + \frac{1}{\varepsilon_S} \right) - \frac{\varepsilon_D}{\varepsilon_S} \bar{\beta} - (\varepsilon_D - 1) \bar{\beta}^{-\frac{1}{\varepsilon_S}}. \quad (20)$$

(ii) Suppose  $\beta \leq \bar{\beta}$ . If  $m^\circ$  is close enough to one then  $\tilde{m}(m^\circ, K) = 1$ ; i.e., there is immediate merger to monopoly.

(iii) Suppose  $\beta > \bar{\beta}$ . There is never merger to monopoly.

(iv) The cutoff  $\bar{\beta}$  decreases in  $\varepsilon_S$ , increases in  $\varepsilon_D$ , and is constant in  $\delta$ .

*Proof.* (i) and (ii) See Appendix A. (iii) This follows immediately from (i). (iv) This can be shown by implicitly differentiating (20). *Q.E.D..*

Thus, we find merger to monopoly only when  $\beta$  is below the cutoff  $\bar{\beta}$ . Moreover, the cutoff is decreasing in  $\varepsilon_S$  and increasing in  $\varepsilon_D$ , analogous to our results from the previous subsection. Table 1 presents the cutoffs  $\bar{\beta}$  for various values of  $\varepsilon_D$  and  $\varepsilon_S$ , and illustrates this point as well. Note that Proposition 6 considers only the case of elastic demand, because with inelastic demand,  $v_f(1, K) = \infty$ , and mergers to monopoly will never occur.

To understand the logic of Proposition 6, it is useful to digress and consider for a moment an extreme case of the single-period version of our model where the elasticity of supply is infinite,  $\varepsilon_S = \infty$ . Here the cost function per unit of capital is linear,  $c(q) = q$ . If the dominant firm had a pure monopoly, it would set a price strictly above the marginal cost. A fringe firm in this environment, behaving competitively and facing a price above its constant marginal cost, would earn infinite profits per unit of capital. Thus, a dominant firm faced with a non-zero fringe would never attempt to purchase the entire stock of fringe capital because the purchase price would be infinite while the monopoly payoffs would be bounded.<sup>18</sup> Similarly to  $\varepsilon_S = \infty$ , a fringe firm with high  $\beta$  would earn infinite profits per unit of capital under monopoly. The only difference is that fringe profits can be unbounded even when  $\beta < 1$ , because there are an infinite number of periods over which to earn profits.

### **What happens with no merger to monopoly?**

Lastly, we would like to understand how industry concentration evolves for the case in which merger to monopoly does not occur in a given period. In such a case, the investment stage will also influence the evolution of concentration, and we will move on to the subsequent period where the merger stage will begin anew.

Our analytical result for the investment stage is

*Proposition 7.* Suppose the horizon is finite and  $m_t > 0$ . Then  $q_{d,t} < q_{f,t}$  if  $m_t$  is either close to one or close to zero.

It is straightforward to show that  $q_{d,t} < q_{f,t}$  when  $m_t = 1$ , because the monopoly dominant firm has no strategic considerations in this case. Gowrisankaran and Holmes (2003) prove this result for  $m_t$  close to zero as part of the proof of Proposition 5 (ii).

While we have no analytic result for  $m_t$  in the intermediate case, in our numerical simulations with constant elasticity, we found that the dominant firm invested at a lower rate than the fringe for all positive values of  $m_t$ . We conclude that concentration declines during the output/investment stage.

We have found computationally that an increase in  $\varepsilon_S$  and a decrease in  $\varepsilon_D$  and  $\beta$  all increase  $q_{f,t}$  relative to  $q_{d,t}$  and thus decrease concentration at the investment/output stage. As we discussed in the introduction, these factors cause a greater wedge between fringe and dominant firm values, which is why they lead both to less mergers and to more fringe investment relative to dominant firm investment. As these factors work in the same direction at the merger and output/investment stages, they imply unambiguous, and potentially testable, predictions about the impacts of elasticities and the discount factor on the long-run evolution of an industry.

Let  $F(m^\circ, K)$  be the composite of these two stages, i.e., next period's pre-merger share given the current period's pre-merger share and capital,

$$F(m^\circ, K) = m_{next}^\circ(\tilde{m}(m^\circ, K), K). \quad (21)$$

In Figure 5, we plot  $F(m^\circ, K)$  for an example parameter vector.<sup>19</sup> Observe that  $F(m^\circ, K)$  crosses the 45-degree line several times. This indicates that industries can have steady states with intermediate levels of concentration.<sup>20</sup> If  $F(m^\circ, K)$  approaches the 45-degree line from above (below), the steady state will be stable (unstable) in the sense that an industry that starts with a concentration near the steady state level will converge toward (away from) it. Thus, for this example,  $m^\circ = .49$  is a stable steady state. The existence of multiple stable steady states implies that initial conditions in an industry may determine long-run concentration.

Figure 6 displays the composite effect  $F(m^\circ, K)$  for the saw-toothed  $\tilde{m}$  function from Figure 4. The composite function is also saw-toothed, and the lower envelope of the saw teeth just touches the 45-degree line. There are multiple steady states here that are stable from below but unstable from above. This example illustrates a type of “limit merger” behavior. In this example, if the initial share  $m^\circ$  is in the range of approximately .64 to .75, the dominant firm merges to a share of .80. The dominant firm's share declines somewhat during the investment/output phase, the dominant firm merges next period to .80, and the

process repeats itself. For  $m^\circ$  even slightly above this range, the dominant firm merges to monopoly within two periods.

## 5. The assumption on investment and output

In formulating the model, we made the assumption that output and new capital are produced in fixed proportions. This greatly simplifies the analysis because a firm has only one choice variable at the output/investment stage. In this section, we show that our results are robust when we relax this assumption.

A natural generalization of our base model is one where the investment and output decisions are separate and take place in different stages. Suppose that there are *three* stages: a merger stage, an investment stage, and an output stage. As before, let  $K$  denote old capital and  $Q$  denote new capital. To allow for the possibility that output may be different from the level of new capital, let  $Y$  denote output. Since we have assumed throughout that the dominant firm moves first in the merger and investment stages, we assume that the dominant firm moves first in the output stage, as well.

In this new environment we need to specify the output technology in the third stage. A very simple such technology is one where capital is the only input and output can be produced at zero marginal cost up to capacity; i.e.,  $Y_t \leq Q_t$ . This is different from our base model in that it allows for a firm to costlessly dispose of part of its output instead of bringing it to market. Fringe firms, as price takers, will always produce up to capacity. There are no dynamic considerations from production conditional on  $Q_t$ . Hence, if industry demand is elastic ( $\varepsilon_D > 1$ ), the dominant firm's static residual demand will be elastic and it will produce up to capacity as well. Thus, for the elastic demand case, separating the output choice from the investment choice leads to no change in the equilibrium dynamics.

Suppose, now, that industry demand is inelastic,  $\varepsilon_D < 1$ . If, at the output stage, the dominant firm's share of industry capital is small enough, its *residual* demand will still be elastic and again it will produce up to capacity. But if the dominant firm's share of industry capital is sufficiently high, its residual demand will be inelastic and the dominant firm will

produce at less than capacity. This outcome will affect the earlier investment and merger stages.

For the single-period version of this model with constant elasticities (19), we can show that if the dominant firm's initial capital share  $m^\circ$  is sufficiently high, the dominant firm *sells* capital in the merger stage. Moreover, for  $m^\circ$  close enough to one, the change in the dominant firm's market share during the merger stage (as a percentage of the fringe post-merger share) is approximately

$$\frac{m - m^\circ}{1 - m} = (\varepsilon_D - 1) \frac{\varepsilon_S}{\varepsilon_S + 1}, \quad (22)$$

which is negative since  $\varepsilon_D < 1$ . From (22), we can see that our basic results are robust to relaxing the assumption that investment equals output. In particular, net merger activity increases with the elasticity of demand and decreases with the elasticity of supply, just as in our base model. Moreover, the dominant firm uses sell-offs at the merger stage as a substitute for commitment in the same way that it does in the multi-period model when  $\beta$  is high. To see the parallel with the multi-period model, observe that in this generalized single-period model there are now two stages to the output decision, first investment, second output per unit of investment. When making its investment decision, the dominant firm would benefit from being able to commit to its later stage output decision—because such commitment would influence fringe investment behavior in a way that would benefit the dominant firm. But commitment to future behavior is infeasible, just as it is in the multi-period version of the base model.

We can also consider richer models of the output stage in which capital is combined with variable factors to determine output. A natural specification is a Cobb-Douglas technology  $Y = Q^\alpha L^{1-\alpha}$ , where  $L$  is the variable input. We examined the implications of this Cobb-Douglas specification for the constant elasticity demand case (19) and a single period. For this case, equilibrium concentration rises as we increase  $\alpha$ , because a higher  $\alpha$  implies a greater ability to commit. Moreover, equilibrium concentration increases when  $\varepsilon_D$  increases and  $\varepsilon_S$  decreases, exactly as in our base model.

Thus, it appears that our basic results on the impact of elasticities on concentration are not dependent on our simplifying assumption that new capital and output are in fixed proportions. The reason is that the same three key forces will affect the industry in the same ways even for this new model. Moreover, separating the investment and output processes simply lessens the ability of the dominant firm to commit to its actions and, in that way, is similar to increasing the discount factor.

## 6. Conclusions

At the beginning of a typical industrial organization textbook, there is often a discussion of the underlying conditions in an industry that would tend to result in a competitive market structure for the industry in the long run. This list of conditions usually includes an absence of scale economies and an absence of entry barriers. Our results suggest that three additional variables can be added to this list. The competitive outcome is more likely when: (1) firms are more forward-looking, (2) the supply of new capital is *more* elastic, and (3) demand is *less* elastic. Moreover, the results show that the initial conditions of the industry can affect the long-run concentration to the point where competitive industries might never become monopolized and monopolistic industries might never become competitive.

We have obtained these results using a model that highlights three key forces influencing industry evolution when mergers are allowed: the fact that monopolization allows firms to raise prices, the free-rider effect that limits the ability of firms to merge, and the fact that the dominant firm and the fringe have different incentives to invest in new industry capital. We assume constant returns to scale and increasing adjustment costs for expanding capital, which we view as the “textbook” assumptions of a classic benchmark case. We use the dominant firm model, but there is nothing in the logic of our results that would suggest that these results are specific to models with a single strategic agent.

We have emphasized the “textbook” nature of our model rather than relate our model to specific real world industries. In any real world context, factors that we have left out, such as differential efficiencies across firms and scale economies, would undoubtedly play a

role in the evolution of industry structure. However, we think that the forces that we have emphasized in our model have important real world counterparts. Consider the “textbook” example of monopolization: Standard Oil in the late 19th century. It is well-known that John D. Rockefeller consolidated the petroleum industry by buying out rivals. It is less appreciated that he went to great effort to keep on as employees the owners of the firms that he bought. He recognized that the knowledge of these owners was part of the capital stock of the industry, and he did not want this capital to be free to start creating new capital (Chernow, 1998). This kind of industry-specific human capital is similar to the concept of capital in our model.<sup>21</sup> Moreover, Rockefeller had to continually buy out new capacity from fringe firms—capacity often built with the express purpose of being bought out.<sup>22</sup> We think of this as being analogous to the continual buyouts that occur in our model when there is a stationary equilibrium with intermediate concentration. Rockefeller complained that the new capacity meant to be bought out was “blackmail” (Leeman, 1956). This illustrates the loss to the dominant firm from not having the ability to make commitments about its future acquisition behavior. Rockefeller tried to keep his acquisitions secret (Chernow, 1998). One interpretation of this fact is that Rockefeller was aware that  $p_K = v_f(m)$  is increasing in  $m$ , in words, that the price of capital is increasing in the amount of capital that the dominant firm purchases, since it is equal to the fringe value at the resulting state.

There is much talk today that with the “new economy,” the issues that were important in the late 19th century have less relevance today. Much attention is placed on industries such as software with high fixed costs, low marginal costs, and product differentiation. We readily admit that our model has little to say about such industries. However, there remain many industries where the technology for producing industry-specific capital is closer to our model than it is to that in the software industry. In industries such as banking, an important part of the industry-specific capital stock is the customer relationship base. Firms in this industry can expand this base by internal investment or external investment.

Thus, the forces that we have highlighted potentially play a role in the evolution of concentration in many industries, along with other forces which we do not allow, such as scale

economies and firm efficiency differences. A natural avenue for future research would be to understand the impact of antitrust policies on outcomes and welfare in a model with a richer industry specification.

## Appendix A

This appendix contains proofs of results from the paper.

*Proof of Proposition 5* (i) From Section 3, the result holds for  $t = T$ . Fix  $t$  and assume that the result holds for  $t' > t$  for all  $K$ . We will show that the result holds at time  $t$  for all  $K$ . By induction the proof will be complete.

The merger problem at time  $t$  is

$$W = \max_m m v_{d,t}(m, K) - (m - m_t^\circ) v_{f,t}(m, K).$$

implying that

$$\text{merger marginal benefit} = \frac{\partial W}{\partial m} \Big|_{m^\circ=m=1} = v_{d,t}(1, K) - v_{f,t}(1, K) + \frac{\partial v_{d,t}}{\partial m}(1, K). \quad (\text{A1})$$

By continuity, the dominant firm's discounted profits at  $m_t^\circ$  are close to the monopoly discounted profits. From Lemma 3, industry profit is at a unique maximum at  $m_t = 1$ . These two facts imply that  $\tilde{m}_t(m_t^\circ, K_t)$  is in the neighborhood of 1. We show below that (A1) is strictly positive, which implies that  $\tilde{m}_t(m_t^\circ, K_t) > m_t^\circ$ , for  $m_t^\circ$  close enough to 1.

Since the proposition holds for  $t + 1$  and by continuity, we know that for  $m_t$  close to 1, the dominant firm will merge to monopoly at time  $t + 1$ . Thus, the total value of the dominant firm in the post-merger stage (aside from a multiplicative factor  $K_t$ ) is

$$\begin{aligned} m_t v_{d,t} &= m_t [P(Q_t)q_{d,t} - c(q_{d,t}) + \beta\sigma q_{d,t}v_{d,t+1}] + (1 - m_t)\beta\sigma q_{f,t}v_{d,t+1} \\ &\quad - (1 - m_t)\beta\sigma q_{f,t}v_{f,t+1}, \end{aligned} \quad (\text{A2})$$

where the second to last term of (A2) is the value to the dominant firm of purchasing the residual capital in  $t + 1$  and the last term is the cost of this capital. Dividing by  $m_t$  and rearranging yields

$$v_{d,t} = P(Q_t)q_{d,t} - c(q_{d,t}) + \beta\sigma q_{d,t}v_{d,t+1} + \beta\sigma \frac{1 - m_t}{m_t} q_{f,t} (v_{d,t+1} - v_{f,t+1})$$

Using the envelope theorem,

$$\begin{aligned}\left.\frac{\partial v_{d,t}}{\partial m}\right|_{m=1} &= [P'q_{d,t} + \beta\sigma^2 q_{d,t}v'_{d,t+1}] \frac{dQ_t}{dm}|_{q_d \text{ fixed}} - \beta\sigma q_{f,t} (v_{d,t+1} - v_{f,t+1}) \\ &= [P'q_{d,t} + \beta\sigma^2 q_{d,t}v'_{d,t+1}] (q_{d,t} - q_{f,t}) K - \beta\sigma q_{f,t} (v_{d,t+1} - v_{f,t+1}),\end{aligned}$$

where

$$v'_{d,t+1} \equiv \frac{\partial v_{d,t+1}}{\partial K_{t+1}}.$$

Substituting from (A1),

$$\begin{aligned}\left.\frac{\partial W}{\partial m}\right|_{m^\circ=m=1} &= p_t q_{d,t} - c(q_{d,t}) + \beta\sigma q_{d,t} v_{f,t+1} + [P'q_{d,t} + \beta\sigma^2 q_{d,t}v'_{d,t+1}] (q_{d,t} - q_{f,t}) K \\ &\quad - [p_t q_{f,t} - c(q_{f,t}) + \beta\sigma q_{f,t} v_{f,t+1}] \\ &= MR_{d,t} q_{d,t} - c(q_{d,t}) - [MR_{d,t} q_{f,t} - c(q_{f,t})],\end{aligned}\tag{A3}$$

where  $MR_{d,t} \equiv p_t + P'_t q_{d,t} K + \beta\sigma v_{d,t+1} + \beta\sigma^2 q_{d,t} K v'_{d,t+1}$  is dynamic marginal revenue. For the same reason as in Lemma 2, (A3) is strictly positive. *Q.E.D.*

*Proof of Proposition 6* (i) Let  $\theta = \frac{1}{\varepsilon_S}$  for ease of notation. At the stationary monopoly outcome, fringe profit with  $q = \frac{1}{\sigma\beta}$  is

$$\begin{aligned}\pi(\beta) &= pq - c\left(\frac{1}{\beta\sigma}\right) \\ &= \frac{\varepsilon_D}{\varepsilon_D - 1} \left[ (1 - \beta)c'\left(\frac{1}{\sigma}\right) + \beta\sigma c\left(\frac{1}{\sigma}\right) \right] \frac{1}{\beta\sigma} - c\left(\frac{1}{\beta\sigma}\right) \\ &= \frac{\varepsilon_D}{\varepsilon_D - 1} \left[ (1 - \beta)(1 + \theta)\sigma^{-\theta} + \beta\sigma\sigma^{-1-\theta} \right] \frac{1}{\beta\sigma} - \beta^{-1-\theta}\sigma^{-1-\theta} \\ &= \sigma^{-\theta-1}G(\beta),\end{aligned}\tag{A4}$$

where

$$G(\beta) \equiv \frac{\varepsilon_D}{\varepsilon_D - 1} \frac{(1 - \beta)}{\beta} (1 + \theta) + \frac{\varepsilon_D}{\varepsilon_D - 1} - \beta^{-1-\theta}.\tag{A5}$$

The second line of (A4) substitutes the stationary monopoly price (18) while the third line

uses the constant elasticity cost function (19).

Note that (A5) defines the same cutoff as in (20), the statement of the proposition. It is straightforward to see that  $G(1) > 0$ , and to verify using l'Hôpital's rule that  $\lim_{\beta \rightarrow 0} G(\beta) < 0$ . To show that there is a unique  $\bar{\beta}$  such that  $G(\bar{\beta}) = 0$  and  $G(\beta) < 0 \iff \beta < \bar{\beta}$ , we differentiate  $G(\beta)$  and obtain

$$G'(\beta) = \frac{1}{\beta^2} [-\omega + \beta^{-\theta}]$$

for

$$\omega \equiv (1 + \theta) \frac{\varepsilon_D}{\varepsilon_D - 1} > 1.$$

Define  $\beta' \equiv \omega^{-\frac{1}{\theta}}$  to be the unique  $\beta$  such that  $G'(\beta') = 0$ . As  $G'(\beta) < 0$  if and only if  $\beta > \beta'$ ,  $G(\beta) > 0$  for  $\beta \geq \beta'$ . For  $\beta \leq \beta'$ , the positive slope of  $G(\beta)$ , and opposite signs of the endpoints ensure the existence of a unique  $\bar{\beta}$  that satisfies the desired conditions.

If  $\beta > \bar{\beta}$ , then an industry at state  $(1, K)$  will eventually approach  $(1, K_{mon}^*)$ , and  $v_f(1, K)$  will be infinite. Now consider  $\beta \leq \bar{\beta}$ . As  $G(\beta) \leq 0$  in this range, profit from  $q = \frac{1}{\beta\sigma}$  is not positive. Concavity of the profit function and that fact that profit is positive for small enough  $q$  then implies that profit is not positive for  $q \geq \frac{1}{\beta\sigma}$ . Thus we need only consider  $q < \frac{1}{\beta\sigma}$ , and for such  $q$ , discounted fringe value is bounded. *Q.E.D.*

*Proof of Proposition 6 (ii)* Fix  $t = 1$  and let  $T$  go to infinity. Since  $\beta \leq \bar{\beta}$ ,  $v_{f,1}(1, K)$  is bounded which implies that  $v_{d,1}(m, K)$  is continuous at  $m = 1$ . Thus, (A3) from Proposition 5 (i) holds and  $\frac{\partial W}{\partial m}\big|_{m^\circ=m=1}$  is positive. By continuity,  $\frac{\partial W}{\partial m}$  is positive for  $m_1^\circ$  close to 1 and  $m_1 = 1$ . The argument from Proposition 5 (i) that  $\tilde{m}_1(m_1^\circ, K_1)$  is in the neighborhood of 1 is valid, implying immediate merger to monopoly for  $m_1^\circ$  close to 1. *Q.E.D.*

## Appendix B

This appendix provides details of the computational algorithm. We experimented with a variety of methods for computing the equilibrium of the model and ended up using a finite grid approximation method, as this yielded the best results. For this method, we discretized the state space  $(K, m)$  into a finite rectangular grid and then iterated on the fringe and

dominant firm policy and value functions (1)–(4) until reaching a fixed point. We generally used a  $200 \times 200$  grid for the state space and evaluated all state variables over the ranges  $m^o \in [0, 1]$  and  $K \in [\frac{1}{2}K_{mon}^*, Q_{comp}^*]$ . For the parameters where  $\varepsilon_D \leq 1$ , as  $K_{mon}^* = 0$  we chose a low positive cutoff instead of zero to avoid dividing by zero. For  $\beta \geq \bar{\beta}$ ,  $v_f(1, K) = \infty$  (see Proposition 6), which makes it hard to compute the fringe value function at the monopoly steady state. Thus, for these parameters, we truncated  $m$  slightly below one, while for the other parameters, we allowed a maximum  $m$  of one. We specified a minimum  $m$  of zero for all the models.

We experienced convergence problems with the estimation, because the finite grid approximation does not have smooth value functions. In order to make the problem smooth, we added a tiny logistic smoothing error to the payoffs at each grid point. We then evaluated values and policy functions assuming that the actual choices of  $m$  and  $Q_d$  were made as though the perceived payoffs were equal to the payoffs as specified by the model plus the smoothing error.

It is easy to see that the larger the grid size, the smaller the smoothing error necessary to make the reaction functions smooth up to computer precision. Conceptually, we would like to examine the results of the model as the smoothing error goes to zero and the grid size goes to infinity. We let the size of the smoothing error be  $10^{-3}$  times a standard logistic error or smaller and verified by examining even smaller values that the logistic error had little perceptible impact on the equilibrium at this magnitude.

With the smoothing error, the merger and investment decisions are a weighted sum of the decisions at grid points near the true optimum, where the weights are larger the closer the value at a grid point is to the true optimum. The weights are easy to compute using the standard multinomial logit formulas. We also experimented with using spline approximations of the value functions, which will similarly weight values at grid points to create differentiable approximations. However, we had much better results with the logistic smoothing error.

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## Notes

<sup>1</sup>Because increased investment lowers prices, the immediate effect is that the dominant firm invests less than fringe firms. Stigler (1968) described this as an “inverted umbrella” (p. 108) held by the dominant firm that protects and encourages the growth of fringe firms. However, investment by the dominant firm also has the strategic effect of deterring investment by the fringe (Spence, 1977; Dixit, 1980), which will counteract the immediate effect.

<sup>2</sup>Other papers that have attempted to endogenize the merger process include Cheong and Judd (1992), Kamien and Zang (1990), Gowrisankaran (1999), and Nocke (2000). These papers are limited in the results that they can obtain, because of the complexity of modeling a merger process with many strategic actors.

<sup>3</sup>Our formulation of the merger process is similar to Shleifer and Vishny (1986), who look at the analogous problem of a large shareholder buying shares from a competitive fringe of shareholders.

<sup>4</sup>Prescott and Visscher (1980) provide microfoundations for increasing adjustment costs in such environments.

<sup>5</sup>Rasmusen (1988) argues that firms may invest in new capital in an industry precisely with the intention that this capital be bought out in later periods by large rivals. Our model captures this force.

<sup>6</sup>One way for a dominant firm to commit to high output would be for it to organize itself into divisions with managers competing against each other. See Kamien and Zang (1990) and Lehto and Tombak (1999).

<sup>7</sup>It is straightforward to see that immediate merger to monopoly will always result if we use a take-it-or-leave-it merger process with our model.

<sup>8</sup>Formally, our game has an infinite number of agents, and the Fudenberg and Levine (1986) result only applies to games with a finite number of agents. However, our game is equivalent to a game with three agents—a fringe, a dominant firm, and a market maker—where the dominant firm moves first, followed by simultaneous moves by the fringe and market

maker. For this variant, the dominant firm behavior and values are equivalent to our original game, the market maker sets  $\tilde{q}_f$  and  $\tilde{m}$  and gets its maximal payoff when conditions (v) and (vii) hold (i.e., the markets clear), and the fringe earns payoffs given that aggregate industry evolution is characterized by  $\tilde{q}_f$  and  $\tilde{m}$ . It is straightforward to show that the set of equilibria is the same for the two games and hence that the games are equivalent.

<sup>9</sup>If the bracketed term were not strictly positive, then an increase in dominant firm quantity would not raise total industry output and hence not lower price. But, if price does not fall, then the fringe must not be contracting output. As the dominant firm is expanding output, total industry output is rising, which yields a contradiction.

<sup>10</sup>They showed that an additional unit of capital was worth more to an incumbent monopolist than to a new entrant.

<sup>11</sup>Lewis (1983) shows that an incumbent firm faced with a decision to simultaneously purchase *multiple* lumps of capital will purchase some but not necessarily all of them.

<sup>12</sup>The difference between our results for the single-period and multi-period models is analogous to the difference between Gilbert and Newbery (1982) and Krishna (1993). While Gilbert and Newbery (1982) find that new capital will be purchased by the incumbent, Krishna (1993) finds that when multiple units of the capital are to be sold sequentially, the capital may be purchased by the oustider.

<sup>13</sup>Appendix B provides details of the computational algorithm. The programs to compute this model are available on the web at [http://www.econ.umn.edu/~gautam/Dominant\\_Programs](http://www.econ.umn.edu/~gautam/Dominant_Programs).

<sup>14</sup>Observe that if the discounted present value of capital to a fringe firm is  $v$ , then the firm will invest at a rate  $q$  that maximizes  $vq - c(q)$ . It is straightforward to calculate that  $\varepsilon_S$  is the elasticity of fringe supply with respect to changes in discounted value  $v$ .

<sup>15</sup>We use the same  $\delta$  in all the numerical examples as we found that the merger policy functions were essentially invariant to  $\delta$  both for the analytic results near the limits and for the numerical examples.

<sup>16</sup>We use this expression for  $K$  in all the numerical examples. Note that the numerical value of  $K$  will vary across parameter values.

<sup>17</sup>Gowrisankaran and Holmes (2003) show this property using the same method as the proof of Proposition 5 (ii).

<sup>18</sup>One potential concern is that this result is an artifact of our assumption that each fringe firm is infinitesimally small. However, we can show that the same result will hold for a given fringe mass divided into a large but finite number  $n$  of firms. The reason for this is that the production stage game with finite  $n$  and  $\varepsilon_S = \infty$  is equivalent to Stackleberg with constant marginal costs and  $n$  followers. Thus, the dominant firm would have to pay each fringe firm the Stackleberg follower profits if it only deviated from merger to monopoly, i.e. the Stackleberg follower profits with  $n = 1$ . As the purchase price per fringe firm does not vary with  $n$ , the total purchase cost to achieve monopoly grows proportionally in  $n$ , implying that for large  $n$ , it will exceed the monopoly profits.

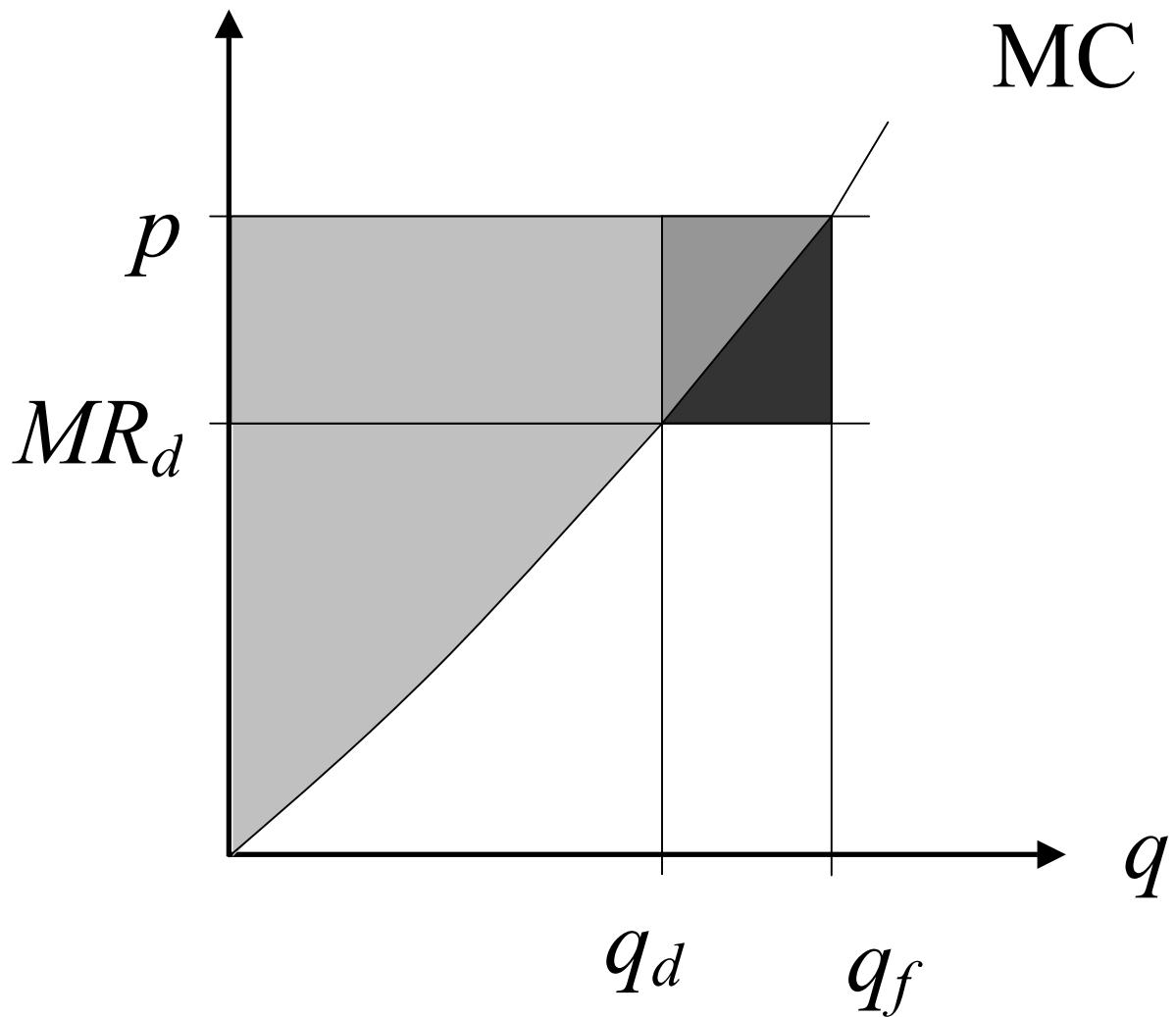
<sup>19</sup>See the discussion surrounding Figures 2-4 for details.

<sup>20</sup>To be at a steady state, we need a steady state level of capital in addition to a point where  $F(m^\circ, K) = m^\circ$ . The figures present  $F$  for a particular level of  $K$  rather than a steady state level. We have found that  $F(m^\circ, K)$  varies little with  $K$  and, therefore, that the fixed points in the figures are approximately steady levels of concentration.

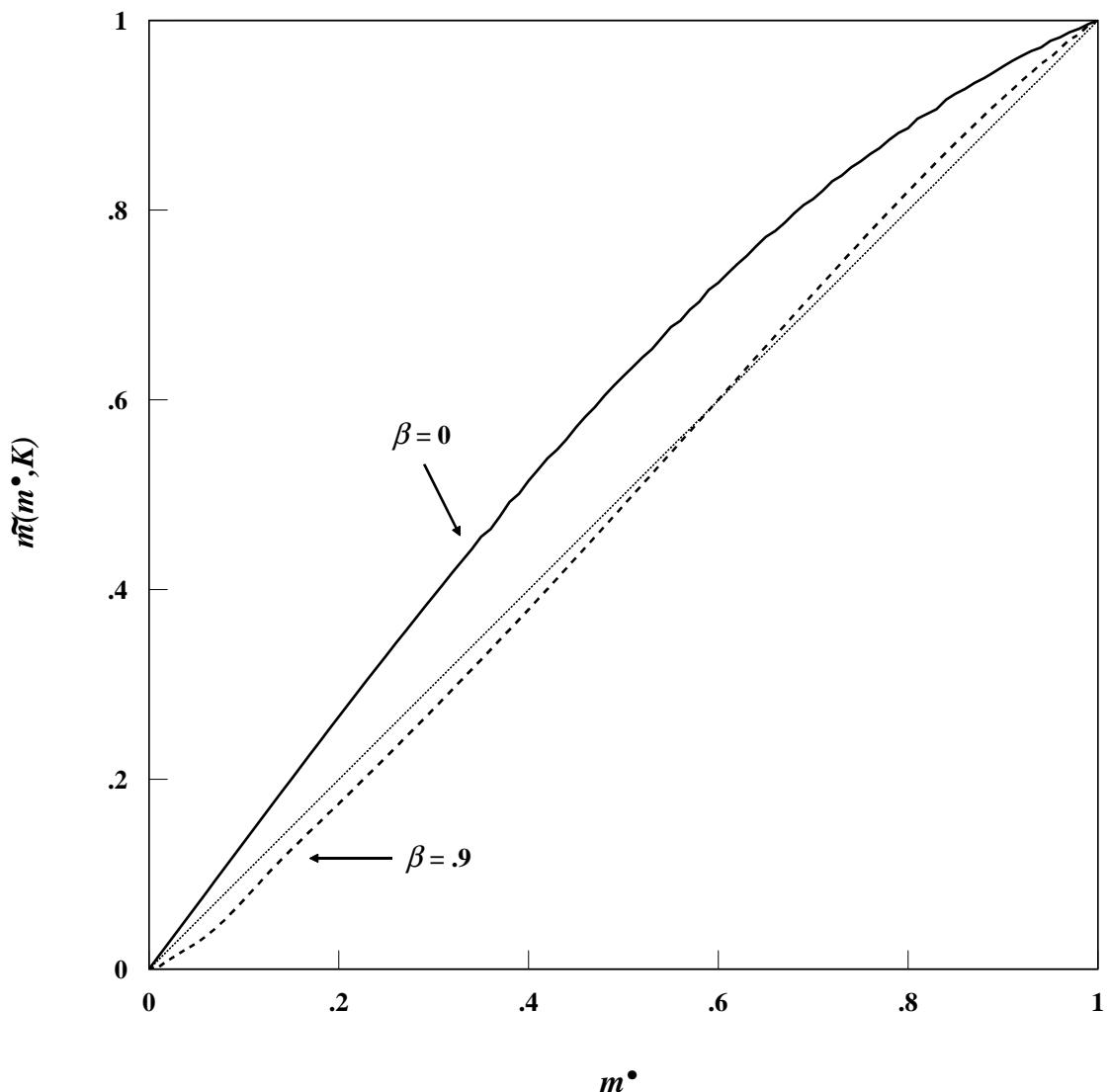
<sup>21</sup>Our thinking here is that the knowledge of how to start a refinery is embodied in an individual who has worked for many years in the industry. To start twice as many refineries, one would need twice as many individuals with this kind of human capital. This kind of knowledge is different from a blueprint that can be costlessly disseminated and used to start an arbitrary number of refineries.

<sup>22</sup>See Leeman (1956) for a discussion of the petroleum industry. The situation was similar in the sugar industry (Zerbe, 1969).

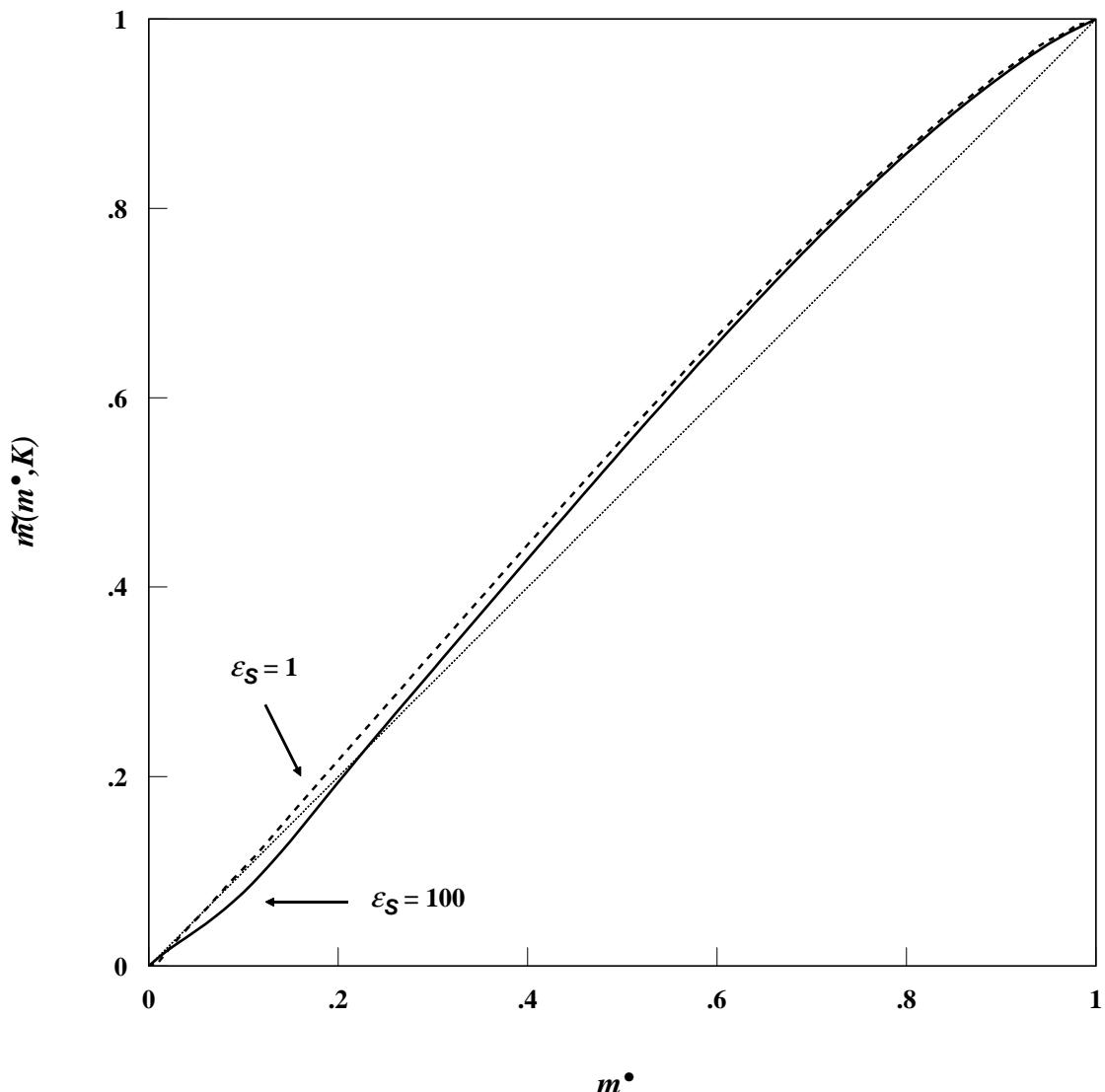
**Figure 1**  
**Profit Maximizing Quantities**  
**For Dominant Firm and Fringe**



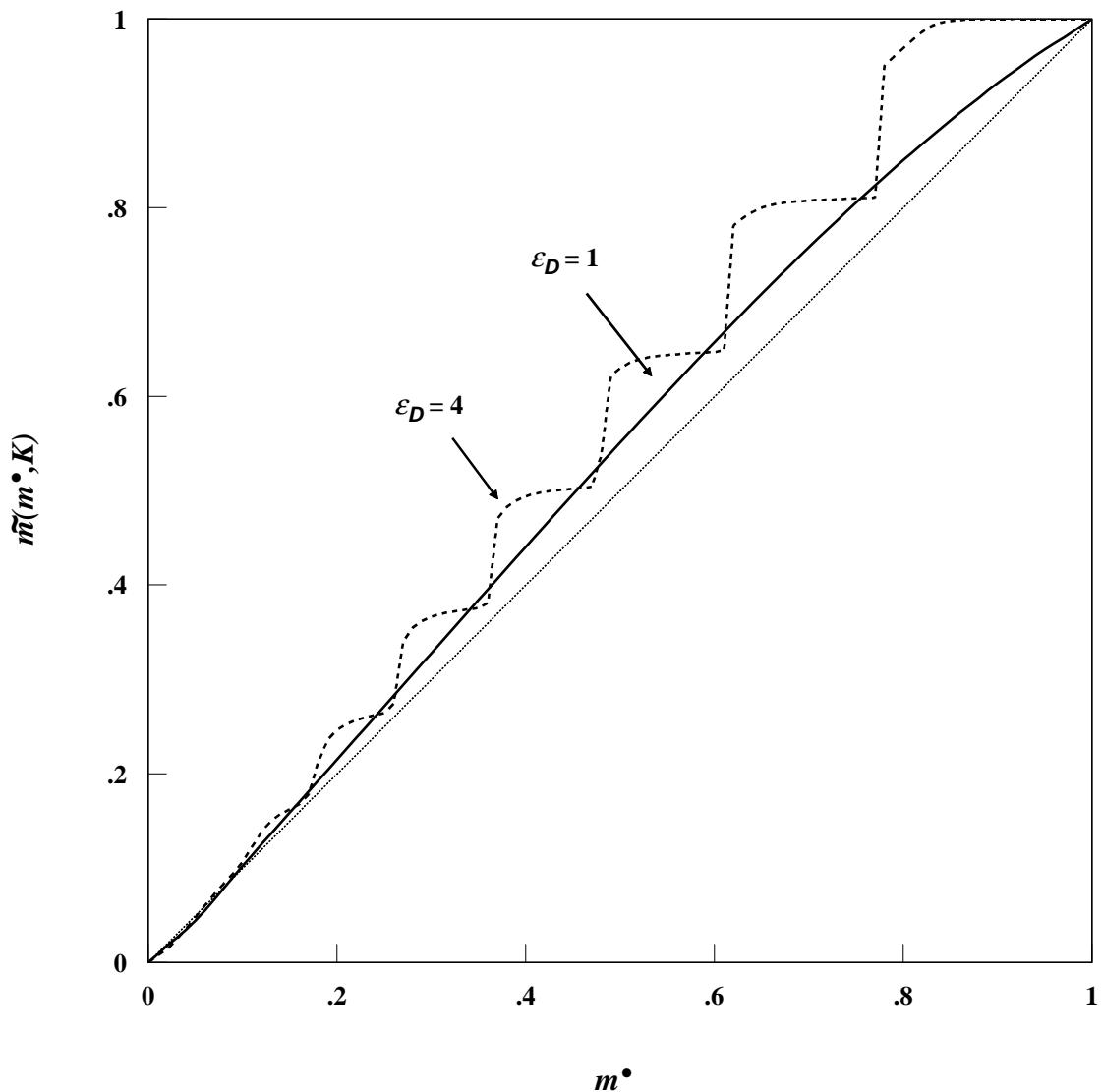
**Figure 2**  
**Effect of  $\beta$  on Merger Policy Function**  
 $\varepsilon_D = .5, \varepsilon_S = 2$



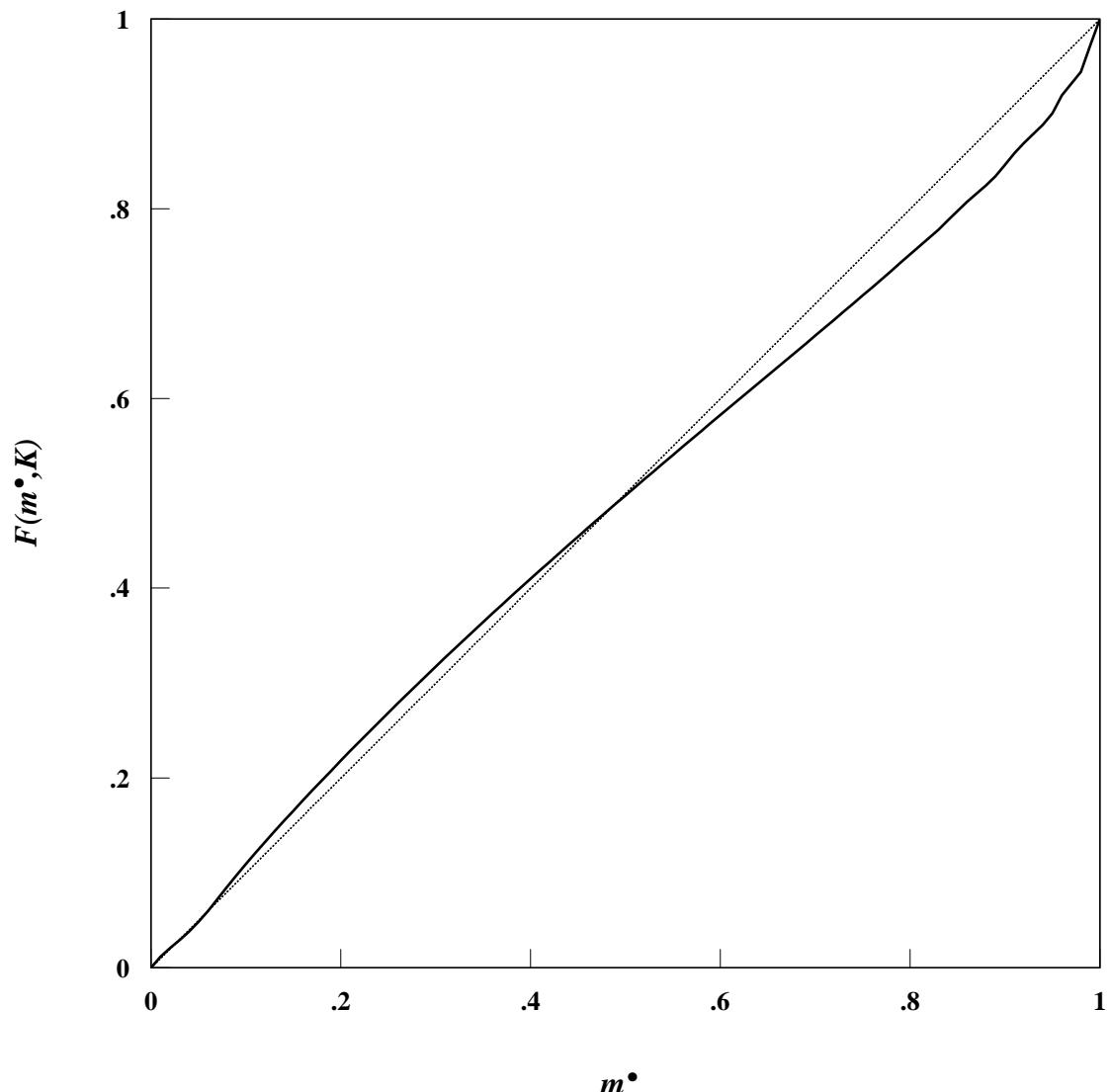
**Figure 3**  
**Effect of  $\varepsilon_S$  on Merger Policy Function**  
 $\varepsilon_D = 2, \beta = .3$



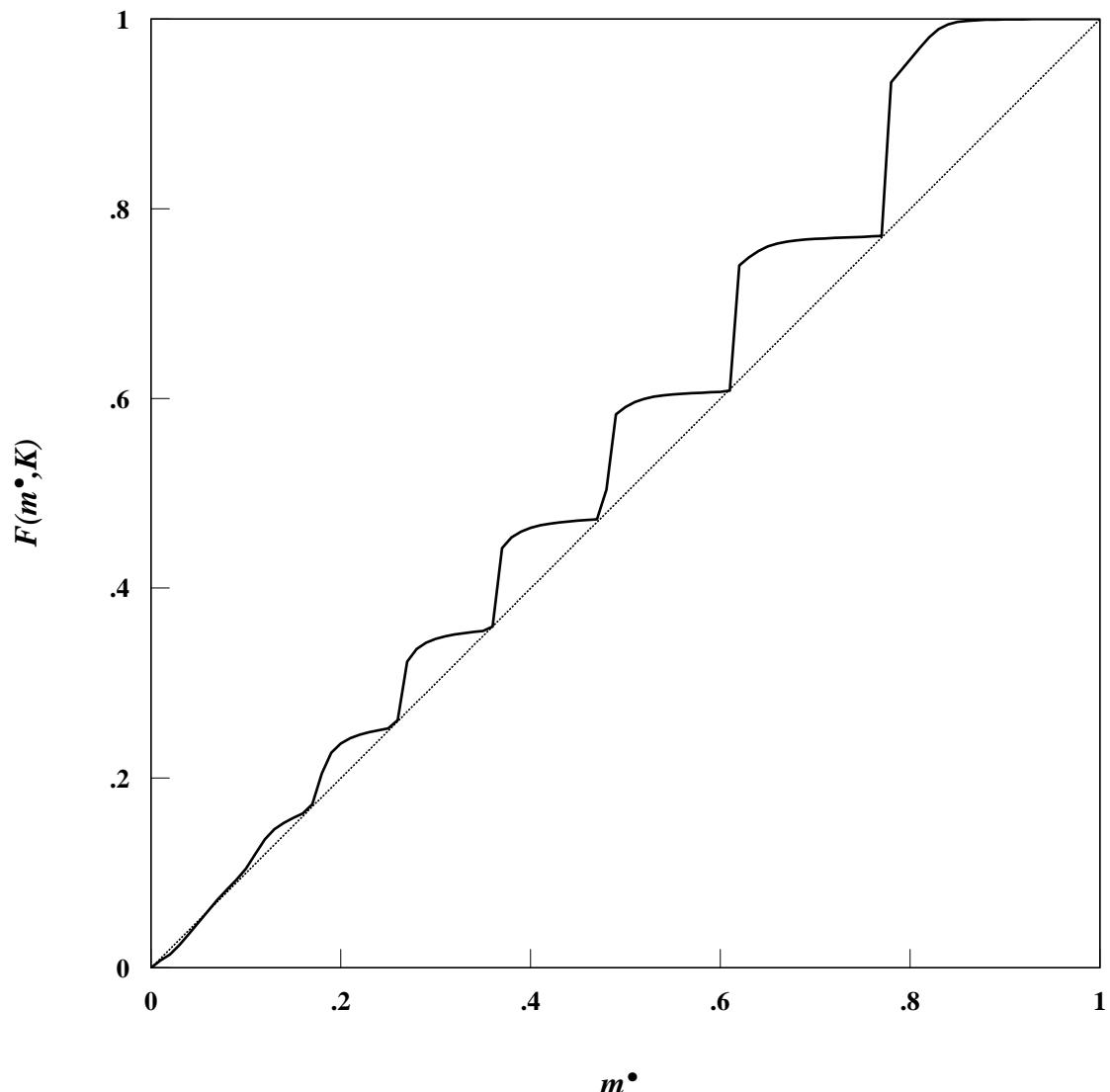
**Figure 4**  
**Effect of  $\varepsilon_D$  on Merger Policy Function**  
 $\varepsilon_S = 1, \beta = .3$



**Figure 5**  
**Industry Transition Function**  
 $\varepsilon_D = 1.1, \varepsilon_S = 1, \beta = .1$



**Figure 6**  
**Industry Transition Function**  
 $\varepsilon_D = 4, \varepsilon_S = 1, \beta = .3$



**Table 1: Critical Values  $\bar{\beta}(\varepsilon_D, \varepsilon_S)$**

Elasticity of Demand $\varepsilon_D$	Elasticity of Supply $\varepsilon_S$	Cutoff $\bar{\beta}$
2	.1	.852
2	1.0	.293
2	3.0	.055
10	.1	.951
10	1.0	.684
10	3.0	.434

---

Note: Monopoly is stable if and only if  $\beta \leq \bar{\beta}(\varepsilon_D, \varepsilon_S)$ .