Special Problem #3
Math 5616

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Special Problem #3:Continuation of 7.26, Verify the following assertions:
(b) $\{f_n\}$ is equicontinuous on $[0,1]$, since $|f_n'| \leq M$.
(c) Some $\{f_{n_k}\}$ converges to some f, uniformly on $[0,1]$.

(b) Claim: $|\frac{f_n(t)-f_n(s)}{t-s}| \leq M$ for all t, s. If this didn't hold we could use the Mean Value Theorem to find a ξ, say between s and t so that $|f_n'(\xi)| > M$ which would contradict the facts established in part (a).
Now our claim holds which means that for any $\epsilon > 0$ we see that letting $\delta := \epsilon/M$ we get that $|t-s| < \delta$ implies $|f_n(t) - f_n(s)| < |t-s|M < \delta M < (\epsilon/M)M = \epsilon$.

(c) All the assumptions of Theorem 7.25 in Rudin are satisfied. The sequence of f_n's are defined on the compact set $[0,1]$, they are uniformly bounded from part (a) and thus pointwise bounded, and they are equiconintuous from part (b); thus, some $\{f_{n_k}\}$ converges to some f, uniformly on $[0,1]$. $\square$