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THE TIME CONSISTENCY OF OPTIMAL MONETARY AND FISCAL POLICIES

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We show that optimal monetary and fiscal policies are time consistent for a class of economies often used in applied work, economies appealing because they are consistent with the growth facts. We establish our results in two steps. We first show that for this class of economies, the Friedman rule of setting nominal interest rates to zero is optimal under commitment. We then show that optimal policies are time consistent if the Friedman rule is optimal. For our benchmark economy in which the time consistency problem is most severe, the converse also holds: if optimal policies are time consistent, then the Friedman rule is optimal.

KEYWORDS: Friedman rule, maturity structure, time inconsistency, sustainable plans.

A CLASSIC ISSUE IN MACROECONOMICS is whether or not optimal monetary and fiscal policies are time consistent. In a monetary economy, Calvo (1978) shows that the incentive for the government to inflate away its nominal liabilities leads to a time consistency problem for optimal policies. In a real economy, Lucas and Stokey (1983) show that the incentive for the government to devalue its real debt typically also leads to a time consistency problem for optimal policies. They show that, with a carefully chosen maturity structure for real government debt, optimal policies can be made time consistent in a real economy. But they conjecture that the analogous result does not hold for a monetary economy with both nominal and real debt. Contrary to their conjecture, we show that for a class of monetary economies typically used in applied work, optimal policies are time consistent.

Our benchmark model is an infinite horizon model with end-of-period money balances in the utility function of the representative consumer. In this model, the government has access to nominal and real debt of all maturities and must finance a given stream of government expenditures with a combination of consumption taxes and seigniorage. Following Lucas and Stokey (1983), we abstract from the well-understood problems arising from capital taxation by not including any kind of capital. We also discuss variations of our model, such as those with beginning-of-period balances in the utility function as well as cash-credit economies and shopping time economies.

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We establish our result in two steps. We first show that for a class of monetary economies typically used in applied work, the optimal policy under commitment is the *Friedman rule* of setting the nominal interest rate to zero. We then show that if the Friedman rule is optimal under commitment in such economies, then the optimal policy is time consistent. For the benchmark economy, we also show the converse, that if the optimal policy is time consistent, then the Friedman rule is optimal under commitment.

Our approach to the issue of time consistency is basically that of Lucas and Stokey (1983). To establish our benchmark for optimal policy, we begin by solving for *Ramsey policies*, namely, the optimal policies in an environment in which the initial government has a commitment technology that binds the actions of future governments. In this environment, therefore, the initial government chooses policy once and for all. Ramsey policies here consist of sequences of consumption taxes and money supplies.

We then turn to the environment of interest, in which no such commitment technology exists. Here each government inherits a maturity structure of nominal and real debt and decides on the current setting for the consumption tax and the money supply, as well as on the maturity structure of nominal and real debt that its successor will inherit. We ask whether a maturity structure of government debt can be chosen so that all governments carry out the Ramsey policies. If it can, we say that the Ramsey policies are *time consistent*, or equivalently, that the Ramsey problem is time consistent.

We find that if the Ramsey policies are to be time consistent, then the structure of the nominal bonds which a government in a particular time period leaves to its successor in the next period must be severely restricted. One of these restrictions is well understood: the present value of these nominal claims must be zero. If this present value is positive, then the successor will inflate the nominal claims away by setting the price level to be very high, while if the present value is negative, then the successor will make its claims on the public large by setting the price level to be very low.

In this sort of environment, if the Ramsey policies are to be time consistent, the government in any period must be able to induce its successor to carry out its plan even with these restrictions. The restrictions constrain the ability of any government to influence its successor primarily through the maturity structure of the real bonds. When the Friedman rule is optimal, consumers are satiated with money balances, and no seigniorage is raised—as if money has disappeared—so that the economy is equivalent to a real economy with one consumption good and labor. For such a real economy, we can use the same scheme for the maturity structure of real debt that Lucas and Stokey (1983) use to show that optimal policy is time consistent. We show that this result holds for our benchmark economy as well as some other commonly used monetary economies.

We argue that economies for which the Friedman rule is optimal—and, hence, those for which optimal policies are time consistent—are of applied interest. This is because the preferences in these economies are the most frequently used in the applied literature. An appealing feature of them is that they are consistent with the growth facts.

For our benchmark economy, we also prove the converse of our main result: if the optimal policy is time consistent, then the Friedman rule is optimal under commitment. A critical step in proving this result is uncovering some subtle extra restrictions on nominal debt: if the Friedman rule does not hold in some period, then the present value of nominal bonds from that period on must be zero. In general, the government in any period is so restricted in influencing its successor that it cannot induce the government to carry out the continuation of its plan.

One way to get some intuition for our results is to count instruments and policies in a finite horizon version of our economy in a manner reminiscent of that of Tinbergen (1956). The basic idea is that in order for a government's policies to be time consistent, the government must have at least as many instruments of influence on its successor as its successor has policy choices. In particular, the period 0 government must have at least as many instruments as the period 1 government has policy choices. In a T-period economy, the period 1 government chooses $2T$ policies—the T taxes on consumption and the T taxes on real balances. The extra restrictions on nominal debt discussed above imply that the period 0 government has effectively only the T real bonds as instruments to influence the period 1 government. When the Friedman rule is optimal, consumers are satiated with money, and at the associated allocation, a small variation in interest rates has no effect on revenues. In this sense, money effectively disappears, and the period 1 government has only T policies. Hence, the period 0 government has sufficiently many instruments to induce its successor to carry out its plan, and the solution to the Ramsey problem is time consistent for period 1. A similar argument holds for other periods. When the Friedman rule is not optimal, the period 0 government does not have enough instruments to induce its successor to carry out its plan, and thus, the solution to the Ramsey problem is not time consistent.

For most of this study, we follow the original approach to time consistency used by Calvo (1978) and Lucas and Stokey (1983) in order to highlight the relation between our results and the earlier literature in the most transparent way. We also relate this approach to the approaches of sustainable plans used by Chari and Kehoe (1990) and credible policies used by Stokey (1991). Those approaches explicitly build the government's lack of commitment into the environment with an equilibrium concept in which governments explicitly think through how their choices of debt influence their successors' choices. We show that optimal policies are time consistent if and only if they are supportable as a Markov sustainable equilibrium. Relating the concepts of time consistency and sustainable plans is of interest in itself.

Our study is related to that of Persson, Persson, and Svensson (1987). They argue that with a sufficiently rich term structure of both nominal and real government debt, optimal policies can be made time consistent regardless of whether or not the Friedman rule is satisfied. Unfortunately, their result is not true. Calvo and Obstfeld (1990) sketch a variational argument which suggests that the solution proposed by Persson, Persson, and Svensson (1987) is not time consistent. Calvo and Obstfeld conjecture that the mistake of those researchers is that their proposed solution violates the second-order conditions. We formalize the Calvo–Obstfeld conjecture here and precisely characterize the conditions under which it applies. We find that the mistake of Persson, Persson, and Svensson has less to do with second-order conditions than with a lack of attention to endogenous restrictions on the nominal debt that a government can leave to its successor. More importantly, unlike Calvo and Obstfeld, we conclude that Ramsey policies are time consistent for an interesting set of economies.

1. THE RAMSEY PROBLEM AND THE FRIEDMAN RULE

We start by constructing our benchmark economy, describing its Ramsey problem, and demonstrating the conditions under which the Friedman rule is optimal in this economy. In so doing, we extend the results in the literature to a somewhat broader class of environments. We show that for preferences commonly used in applied work, the Friedman rule is optimal. These preferences are attractive because they are consistent with balanced growth. We also describe restrictions on government debt that are necessary for the existence of an interior solution to the Ramsey problem in this economy.

The Benchmark Economy

Consider an economy with money, nominal government debt, and real government debt. Time is discrete. The resource constraint is given by

$$(1) \qquad c_t + g_t = l_t,$$

where c_t, g_t, and l_t denote consumption, government spending, and labor in time period t. Throughout, the sequence of government spending is exogenously given.

In this economy, consumers have preferences over sequences of consumption c_t, real money balances m_t, and labor l_t given by

$$(2) \qquad \sum_{t=0}^{\infty} \beta^t U(c_t, m_t, l_t)$$

with the discount factor $0 < \beta < 1$, where $m_t = M_t/p_t$ is defined as end-of-period nominal money balances M_t divided by the nominal price level p_t. We assume that the period utility function $U(c, m, l)$ is concave, twice continuously differentiable, increasing in c, and decreasing in l. We also assume that consumers are satiated at a finite level of real balances, so that for each value of c and l, there is a finite level of m such that $U_m(c, m, l) = 0$, where here and throughout, we let U_m and U_{mm} denote the partial derivatives $\partial U/\partial m$ and $\partial^2 U/\partial m^2$, respectively. We use analogous notation for other partial derivatives throughout.

In terms of assets, we assume that the government issues both nominal and real bonds for every maturity. For the nominal bonds, for each period t and s with $t \leq s$, we let $Q_{t,s}$ denote the price of one dollar in period s in units of dollars in period t, and we let $B_{t,s}$ denote the number of such nominal bonds. Similarly, for the real bonds, we let $q_{t,s}$ denote the price of one unit of consumption in period s in units of consumption in period t and let $b_{t,s}$ denote the number of such real bonds. We let $B_t = (B_{t,t+1}, B_{t,t+2}, \ldots)$ denote the vector of nominal bonds purchased by consumers in t which pay off $B_{t,s}$ in s for all $s \geq t+1$. We use similar notation for the real bonds b_t and the nominal and real debt prices Q_t and q_t. For later use, note that arbitrage among these bonds implies that for all $t \leq r \leq s$, their prices satisfy $Q_{t,s} = Q_{t,r}Q_{r,s}$, $q_{t,s} = q_{t,r}q_{r,s}$, and $Q_{t,s} = q_{t,s}p_t/p_s$. By convention, $Q_{t,t} = 1$ and $q_{t,t} = 1$.

Each consumer's sequence of budget constraints in period t can be written as

$$(3) \qquad p_t(1+\tau_t)c_t + M_t + \sum_{s=t+1}^{\infty} Q_{t,s}B_{t,s} + p_t \sum_{s=t+1}^{\infty} q_{t,s}b_{t,s}$$
$$= p_t l_t + M_{t-1} + \sum_{s=t}^{\infty} Q_{t,s}B_{t-1,s} + p_t \sum_{s=t}^{\infty} q_{t,s}b_{t-1,s}.$$

Thus, in period t, each consumer has a nominal wage income of $p_t l_t$, nominal money balances M_{t-1}, a vector of nominal bonds B_{t-1}, and a vector of real bonds b_{t-1}. Consumers purchase consumption c_t, new money balances M_t, and new vectors of nominal bonds B_t and real bonds b_t. Purchases of consumption are taxed at the rate τ_t. In period 0, consumers have initial money balances M_{-1}, together with initial vectors of nominal and real bonds B_{-1} and b_{-1}. We assume that in each period, the real values of both nominal and real bonds, or debt, purchased are bounded by a constant. This constant can be chosen sufficiently large so that the constraint does not bind.

For convenience, we will work with the consumers' problem in period 0 form. The sequence of budget constraints (3) can be collapsed to the period 0 budget constraint:

$$(4) \qquad \sum_{t=0}^{\infty} q_{0,t}[(1+\tau_t)c_t + (1 - Q_{t,t+1})m_t]$$
$$= \sum_{t=0}^{\infty} q_{0,t} l_t + \frac{M_{-1}}{p_0} + \sum_{t=0}^{\infty} Q_{0,t}\frac{B_{-1,t}}{p_0} + \sum_{t=0}^{\infty} q_{0,t}b_{-1,t}.$$

We can interpret the term $(1-Q_{t,t+1})m_t$ as the effective tax on real balances paid by consumers. Notice that this effective tax is positive when $Q_{t,t+1}<1$ and zero when $Q_{t,t+1}=1$. The consumers' problem in period 0 is to choose sequences of consumption, real balances, and labor to maximize (2) subject to (4).

In period t, the government inherits the nominal money M_{t-1}, the nominal debt vector B_{t-1}, and the real debt vector b_{t-1}. To finance government spending g_t, the government collects consumption taxes $\tau_t c_t$ and issues new money M_t, new nominal debt B_t, and new real debt b_t. The government's sequence of budget constraints is analogous to that of the consumer. We collapse this constraint into the period 0 budget constraint:

$$(5)\qquad \sum_{t=0}^{\infty} q_{0,t}[\tau_t c_t + (1-Q_{t,t+1})m_t - g_t] = \frac{M_{-1}}{p_0} + \sum_{t=0}^{\infty} Q_{0,t}\frac{B_{-1,t}}{p_0} + \sum_{t=0}^{\infty} q_{0,t} b_{-1,t}.$$

We use the notation ${}_tc=(c_t, c_{t+1},\dots)$ for consumption and similar notation for real balances, labor, prices, and taxes. For given initial conditions M_{-1}, B_{-1}, and b_{-1}, then, a *period* 0 *competitive equilibrium* is a collection of sequences of consumption, real balances, and labor $({}_0c, {}_0m, {}_0l)$ together with sequences of prices $({}_0p, Q_0, q_0)$ and taxes ${}_0\tau$ that satisfy the resource constraint in each period and consumer maximization. The government budget constraint is then implied.

In any equilibrium, nominal interest rates are nonnegative, so that the one-period bond price $Q_{t,t+1}=1+(U_{mt}/U_{lt})\le 1$. Since $U_{lt}<0$, for interest rates to be nonnegative, the marginal utility of money must satisfy

$$(6)\qquad U_{mt}\ge 0,$$

which we refer to as the *nonnegative interest rate constraint*. As is well known, the allocations in a competitive equilibrium are characterized by three simple conditions: the resource constraint (1), the nonnegative interest rate constraint (6), and the implementability constraint,

$$(7)\qquad \sum_{t=0}^{\infty}\beta^t R(c_t, m_t, l_t) = -\frac{U_{l0}}{p_0}\left(M_{-1} + \sum_{t=0}^{\infty} Q_{0,t}B_{-1,t}\right) - \sum_{t=0}^{\infty}\beta^t U_{lt} b_{-1,t},$$

where $R(c_t, m_t, l_t) = c_tU_{ct} + m_tU_{mt} + l_tU_{lt}$ is the government surplus $\tau_t c_t + (1-Q_{t,t+1})m_t - g_t$ expressed in marginal utility units and $Q_{0,t} = \Pi_{s=0}^{t-1}[1+(U_{ms}/U_{ls})]$. This implementability constraint should be thought of as the period 0 budget constraint of either the consumers or the government, where the consumers' first-order conditions have been used to substitute out prices and policies.

The Ramsey Problem

Given the initial conditions M_{-1}, B_{-1}, and b_{-1}, the *Ramsey problem* in period 0 is to choose ${}_0c$, ${}_0m$, ${}_0l$, and p_0 to maximize consumers' utility (2) subject to the economy's resource constraint (1) for all t, the nonnegative interest rate constraint (6) for all t, and the implementability constraint (7).

Our results will depend critically on whether or not the allocations that solve this problem satisfy the *Friedman rule*, in that $Q_{t,t+1}=1$, for all t, so that nominal interest rates are zero in every period. Since $Q_{t,t+1}=1+(U_{mt}/U_{lt})$ and $U_{lt}<0$, the Friedman rule holds if and only if $U_{mt}=0$, for all t.

We now discuss the initial conditions for both nominal and real government debt that we choose for the Ramsey problem. To make the problem interesting, we want initial conditions for which distortionary taxes are necessary. A sufficient condition for this to be true is that in each period t, the sum of government spending and its real initial debt maturing in period t is positive. That is, $g_t+b_{-1,t}\geq 0$ for all t with strict inequality for some t. We assume that this condition holds throughout.

The solution to the Ramsey problem also depends critically on the structure of the value of the government's initial nominal liabilities—the initial money supply M_{-1} and the vector of initial nominal debt B_{-1}—through the term $-U_{l0}(M_{-1}+\sum_{t=0}^{\infty}Q_{0,t}B_{-1,t})/p_0$ in the implementability constraint. The term $(M_{-1}+\sum_{t=0}^{\infty}Q_{0,t}B_{-1,t})$ is the present value of the government's nominal liabilities in units of dollars in period 0. Dividing by p_0 converts this value into period 0 consumption good units, and multiplying by $-U_{l0}$ converts the result into units of period 0 utility.

We assume that *initial nominal government liabilities are all zero*, in that

$$(8)\qquad M_{-1}+B_{-1,0}=0 \quad\text{and}\quad B_{-1,t}=0,\quad \text{for all}\quad t\geq 1.$$

Under (8), the present value of nominal liabilities in (7) is identically equal to zero, and the Ramsey problem is independent of p_0.

The Friedman Rule

In the next section, we will show that the Ramsey problem is time consistent if and only if the Friedman rule is optimal in each period. Here we establish sufficient conditions for the Friedman rule to be optimal in each period in an economy that satisfies (8).

If we let γ_t, η_t, and λ_0 denote the multipliers on the resource constraint (1), the nonnegative interest rate constraint (6), and the implementability constraint (7), then for $t\geq 1$, the first-order conditions for c_t, m_t, and l_t are

$$(9)\qquad U_{ct}+\lambda_0(R_{ct}+b_{-1,t}U_{lct})+\eta_t U_{mct}+\frac{\lambda_0 U_{l0}}{p_0}Q_{ct,t+1}\sum_{s=t+1}^{\infty}Q_{t+1,s}B_{-1s}=\gamma_t,$$

$$U_{mt}+\lambda_0(R_{mt}+b_{-1,t}U_{lmt})+\eta_t U_{mmt}+\frac{\lambda_0 U_{l0}}{p_0}Q_{mt,t+1}\sum_{s=t+1}^{\infty}Q_{t+1,s}B_{-1s}=0, \tag{10}$$

$$U_{lt}+\lambda_0(R_{lt}+b_{-1,t}U_{llt})+\eta_t U_{mlt}+\frac{\lambda_0 U_{l0}}{p_0}Q_{lt,t+1}\sum_{s=t+1}^{\infty}Q_{t+1,s}B_{-1s}=-\gamma_t, \tag{11}$$

where $Q_{t,s}=\Pi_{r=t}^{s}[1+(U_{mr}/U_{lr})]$ and where $Q_{it,t+1}$ are the derivatives of $1+(U_{mt}/U_{lt})$ with respect to $i=c,m,l$. For $t=0$, we add $\lambda_0 U_{lc0}(M_{-1}+\sum_{t=0}^{\infty}Q_{0,t}B_{-1t})/p_0$ to the left side of (9) and analogous terms to (10) and (11). Finally, the first-order condition for the initial price level is

$$-\frac{\lambda_0 U_{l0}}{p_0^2}\left(M_{-1}+B_{-10}+\sum_{t=1}^{\infty}Q_{0,t}B_{-1t}\right)=0. \tag{12}$$

We can use these first-order conditions to establish circumstances under which the Friedman rule is optimal. Consider an economy with preferences that are *separable and homothetic*, in that $U(c,m,l)=u(w(c,m),l)$, where the function w is homothetic in c and m and for which initial nominal government liabilities are all zero. Preferences that are separable and homothetic include commonly used preferences in monetary models like $U=w(c,m)^{1-\sigma}v(l)/(1-\sigma)$, where w is homogeneous of degree one. Such preferences are consistent with some basic facts of economic growth: hours worked per person have been approximately constant, and consumption, real balances, and income have grown at approximately the same rate. (See the work of Lucas (2000).) The following proposition, proved in the Appendix, is related to but not covered by the results of Chari, Christiano, and Kehoe (1996).

PROPOSITION 1: *If preferences are separable and homothetic and the initial nominal government liabilities are all zero, then the Friedman rule is optimal.*

Again, many preferences commonly used in applied work are covered by Proposition 1. Sometimes, however, applied work uses nonseparable homothetic preferences. In Proposition 2, we show that the Friedman rule is also optimal for such preferences as long as at the point of satiation, real balances and leisure are Pareto substitutes, in the sense that $U_{ml}\geq 0$. This proposition uses a much weaker form of homotheticity than Proposition 1:

$$\frac{U_m(\alpha c,\alpha m,l)}{U_c(\alpha c,\alpha m,l)}=\frac{U_m(c,m,l)}{U_c(c,m,l)} \tag{13}$$

for any positive α, which implies that real balances and consumption grow at the same rate along a balanced growth path. Notice that the case in which $U_{ml}>0$ at the point where $U_m=0$ corresponds to the case in which the nonnegativity constraint on interest rates binds at the Friedman rule.

PROPOSITION 2: *Assume that initial nominal government liabilities are all zero and that* $U_{ml} \geq 0$ *whenever* $U_m = 0$. *If preferences are consistent with balanced growth in the sense of* (13), *then the Friedman rule solves the Ramsey problem.*

In the proof, given in the Appendix, we show that if $U_{ml} \geq 0$ and preferences are consistent with balanced growth, then we can construct a feasible allocation and nonnegative multipliers that satisfy the first-order conditions and the implementability condition. We obtain this allocation by solving the Ramsey problem for a corresponding real economy without money. This real economy has utility function $\bar{U}(c, l) = U(c, m^*(c, l), l)$ where $m^*(c, l)$ is the satiation level of money balances so that $U_m(c, m^*, l) = 0$. The constructed allocation solves the Ramsey problem in the monetary economy because it is as if money has disappeared in that economy: consumers are satiated with money, seigniorage is zero, and $R_m = 0$, so that the marginal effect on revenues from a change in real balances is zero.

An immediate application of this proposition lets us extend our results to cash-credit and shopping time economies. Also, the proposition is related to, but not covered by, the results of Correia and Teles (1999).

Propositions 1 and 2 cover many preferences commonly used in applied work. These preferences are chosen to be consistent with the growth facts. For the sake of completeness, note that not all preferences that satisfy the growth facts imply that the Friedman rule is optimal. In particular, preferences such as $U = c^{1-\sigma} v(m/c, l)/(1-\sigma)$ satisfy balanced growth, but with them the Friedman rule is not optimal if $v_{12}/v_{11} > 0$. For such preferences, U_{ml} is negative at $U_m = 0$.

Endogenous Restrictions on Nominal Debt Left to Future Governments

In the next section, we will carefully define our notion of time consistency. The intuitive idea is that for each period t, the period t government must be able to leave vectors of nominal and real debt to its successor, the period $t+1$ government, so that the successor has the incentive to carry out the continuation of the plan of the period t government. Under our assumptions, the Ramsey problem in period 0 will entail an interior solution for p_{t+1} for all t. Thus, for any vector of nominal debt left to a government at $t+1$ to be part of a time consistent solution, the successor government must, having inherited such a vector, choose an interior solution for the price level p_{t+1}. Here we develop necessary restrictions on initial nominal debt vectors for such an interior solution to exist.

In the lemma below, we show that the necessary restrictions for an interior solution to exist are that if the Friedman rule does not hold in some period, then from that period on, the present value of nominal debt must be zero. Although useful in the analysis of time consistency, the lemma is simply about the type of restrictions that initial nominal debt vectors must satisfy in order for an interior solution to a Ramsey problem to exist. (We use this lemma later in Proposition 4, when we develop some necessary conditions for the solution to a Ramsey problem to be time consistent.)

For notational simplicity, we focus on the restrictions that the nominal debt inherited by the period 0 government must satisfy; the same logic applies to any period t. (Of course, for the period 0 government, the inherited debt is part of the environment, while for any period t government with $t \geq 1$, the inherited debt is part of the endogenous construction of a potentially time consistent plan. Regardless, the lemma applies to any period $t \geq 0$.)

Clearly, for the Ramsey problem in period 0 to have an interior solution for p_0 (that is, for $0 < p_0 < \infty$), M_{-1} and B_{-1} must satisfy the condition that

$$(14) \qquad M_{-1} + \sum_{t=0}^{\infty} Q_{0,t} B_{-1,t} = 0.$$

This condition, however, is not enough to eliminate the nominal forces that lead to the nonexistence of an interior solution. We show that the following stronger conditions are necessary. If in some period s, the Friedman rule does not hold, so that $Q_{s,s+1} < 1$, then the present value of government debt from period $s+1$ on must be zero, so that

$$(15) \qquad 0 = \sum_{t=s+1}^{\infty} Q_{0,t} B_{-1,t} = Q_{0,s} Q_{s,s+1} \sum_{t=s+1}^{\infty} Q_{s+1,t} B_{-1,t}.$$

Given our assumptions on $g_t + b_t$, there must be some period—say, r—in which consumption taxes are being levied, so that $-U_{cr}/U_{lr} = 1 + \tau_r > 1$. We will assume that in this period r, the second derivatives satisfy the conditions that

$$(16) \qquad U_{mm} + U_{lm} < 0 \quad \text{if} \quad U_m > 0, \quad U_{ll} + U_{lc} \geq 0, \quad \text{and} \quad U_{mc} + U_{ml} \geq 0.$$

The following lemma determines restrictions that inherited nominal debt must satisfy for an interior solution to that government's problem to exist.

LEMMA: *Assume that an interior solution to the Ramsey problem in period* 0 *with* $0 < p_0 < \infty$ *exists, that there is some period s in which the Friedman rule does not hold, that there is some period r in which consumption taxes are levied, and that in period r the conditions* (16) *hold. Then the value of initial nominal government debt from* $s+1$ *on is zero, so that* (15) *holds.*

The proof of the lemma (provided in the Appendix) is rather intricate and is related to the informal variational argument suggested by Calvo and Obstfeld (1990). We summarize it here. The proof proceeds by contradiction. We suppose that at the solution, the Friedman rule does not hold, and the present value of the nominal government liabilities from period 0 on is zero, but the present value from period t on is not zero. We then show that no such solution can exist by constructing an alternative allocation that gives higher utility. In the construction, we perturb the original allocation in two steps. The first step is a small variation in nominal interest rates, which may entail lowering them. This variation will make the present value from period 0 on strictly *negative*, so that the consumers owe the government some nominal amount. Once the consumers owe the government any nominal amount, say, one dollar, the government can raise any amount of revenues it desires in a lump-sum fashion by making the initial price level low enough. The second step of the perturbation is to reduce distortionary taxes and thus increase welfare while keeping the nominal interest rates unchanged from the first step to ensure that the consumers owe the government some nominal amount.

The lemma implies that when the Friedman rule is violated, the nominal debt must be severely restricted if an interior solution is to exist. For example, if the Friedman rule does not hold in every period, then the nominal liabilities must be zero in every period so that (8) holds. We will use this lemma to show that if the Friedman rule does not hold, the nominal debt is so restricted that the Ramsey problem cannot be made time consistent. (We later discuss how this analysis is altered in other common monetary economies.)

2. TIME CONSISTENCY AND THE FRIEDMAN RULE

Now we give a version of Lucas and Stokey's (1983) definition of *time consistency* and establish, for our benchmark economy, that the Ramsey problem is time consistent if and only if the Friedman rule solves the Ramsey problem.

We begin with a definition of time consistency. For convenience, define the *Ramsey problem in period t*, given inherited values for money balances M_{t-1}, nominal debt B_{t-1}, and real debt b_{t-1}, to be the problem of choosing allocations from period t onward, ${}_tc$, ${}_tl$, and ${}_tm$, and the price level p_t (by choosing M_t) to maximize

$$\sum_{s=t}^{\infty} \beta^{s-t} U(c_s, m_s, l_s)$$

subject to the resource constraint (1) and the nonnegative interest rate constraint (6) for each $s \geq t$ and the implementability constraint in t,

$$\sum_{s=t}^{\infty} \beta^{s-t} R_s = -\frac{U_{lt}}{p_t}\left(M_{t-1} + \sum_{s=t}^{\infty} Q_{t,s} B_{t-1,s}\right) - \sum_{s=t}^{\infty} \beta^{s-t} U_{ls} b_{t-1,s}, \tag{17}$$

where $Q_{t,s} = \Pi_{r=t}^{s-1}[1 + (U_{mr}/U_{lr})]$.

The Ramsey problem in period t is said to be *time consistent for* $t+1$ if values exist for nominal money balances M_t, nominal debt B_t, and real debt b_t that satisfy two conditions. First, the nominal money balances M_t are consistent with the period t allocation in that $M_t = p_t m_t$. Second, the continuation of the Ramsey allocations in period t from $t+1$ on—namely, ${}_{t+1}c$, ${}_{t+1}m$, and ${}_{t+1}l$ together with the price level p_{t+1}—solve the Ramsey problem in $t+1$, where the price level p_{t+1} is a function of the allocations and the nominal money supply according to

$$p_{t+1} = \frac{Q_{t,t+1} p_t}{q_{t,t+1}} = \beta \frac{U_{lt+1}}{U_{lt}} \frac{M_t / m_t}{1 + (U_{mt}/U_{lt})}.$$

The Ramsey problem in period 0 is *time consistent* if the Ramsey problem in period t is time consistent for $t+1$ for all $t \geq 0$.

Given this definition, the way to establish that a Ramsey problem in, say, period 0 is time consistent for period 1 is to show how the initial conditions for the period 1 problem—M_0, B_0, and b_0—can be chosen so as to give incentives for the government in period 1 to continue with the allocations chosen by the government in period 0.

To keep the proofs simple, we assume here that the utility function is weakly increasing in m; hence, we can drop the nonnegative interest rate constraint in the Ramsey problem. For later use, however, recall from the proof of the lemma (in the Appendix) that under this assumption, if $U_m = 0$, then

$$(18) \qquad U_{mm} = U_{mc} = U_{ml} = 0.$$

A simple example illustrates the main ideas behind two propositions to come, Propositions 3 and 4, which give sufficient and necessary conditions under which the Ramsey problem is time consistent.

EXAMPLE: Let the utility function U be additively separable in its three arguments. Let g_t be zero in all periods, so that $c_t = l_t$. Let initial government debt have $b_{-1,0} = b > 0$ and be zero in all other periods, and let nominal government debt satisfy (8). For the period 0 Ramsey problem, consider the combined first-order conditions for c_t and l_t for $t \geq 1$,

$$(19) \qquad (1+\lambda_0)(U_{ct} + U_{lt}) + \lambda_0 c_t (U_{cct} + U_{llt}) = 0,$$

and the first-order condition for m_t for $t \geq 0$,

$$(20) \qquad (1+\lambda_0) U_{mt} + \lambda_0 m_t U_{mmt} = 0.$$

Clearly, c_t and l_t are constant for $t \geq 1$, and m_t is constant for $t \geq 0$. It is easy to show that a constant level of positive taxes is levied in each period $t \geq 1$, so that the sum $U_{ct} + U_{lt} > 0$ and is constant for each $t \geq 1$.

To make this allocation time consistent for period 1, we must be able to choose new government debt $B_{0,t}$ and $b_{0,t}$, new nominal money balances M_0, and a new multiplier λ_1 that support the continuation of the period 0 allocations. To be able to do that, these constructed objects must satisfy the first-order conditions for the Ramsey problem in period 1, that is, the combined first-order conditions for c_t and l_t for all $t \geq 1$,

$$(21) \qquad (1+\lambda_1)(U_{ct}+U_{lt})+\lambda_1 c_t(U_{cct}+U_{llt})+\lambda_1 U_{llt} b_{0t}=0,$$

and the first-order condition for m_t for $t \geq 1$,

$$(22) \qquad (1+\lambda_1)U_{mt}+\lambda_1 m_t U_{mmt}=0.$$

We first show that if the Friedman rule holds, then the allocation can be made time consistent. (By doing so, we highlight some of the key ideas in Proposition 3, below.) We set $M_0+B_{0,t}=0$ and $B_{0,t}=0$ for all $t \geq 1$ and construct b_{0t} and λ_1 in a way related to that of Lucas and Stokey (1983). Since c_t, m_t, and l_t are constant in (21), the level of real government debt b_{0t} does not vary with t; we denote it by b'. We use (21) to find b' as a function of λ_1:

$$b' = -\frac{(U_{ct}+U_{lt})}{U_{llt}}\left(\frac{1}{\lambda_1}\right) - \frac{U_{ct}+U_{cct}c_t+U_{lt}+U_{llt}l_t}{U_{llt}}.$$

We then substitute b' into the period $t=1$ implementability constraint to solve for λ_1. Such a λ_1 can be found if $-(U_{ct}+U_{lt})/U_{llt} \neq 0$, a condition that we will assume in Proposition 3. Finally, we need to verify that the first-order condition for m_t (22) holds for the constructed multiplier λ_1. Notice that, if the Friedman rule holds, then by (18), this equation is satisfied for any λ_1. In this sense, when agents are satiated with money, money disappears from the Ramsey problem.

Second, we show that if the Friedman rule does not hold, then the period 0 Ramsey problem is not time consistent. (By doing so, we highlight some of the key ideas in Proposition 4, below.) Our lemma implies that if interest rates are always positive, then the nominal government debt has to be zero in each period. Motivated by this implication, we suppose here that the new nominal debt satisfies $M_0+B_{0,t}=0$ and $B_{0,t}=0$ for all $t \geq 1$. We argue that with these restrictions on nominal debt, the period 0 allocations cannot be supported. For $t \geq 1$, taxes are levied, and the allocations are constant; hence, $b_{0,t}$ is some positive constant b' for all t. Comparing the first-order conditions for m_t in periods 0 and 1, we conclude that since $U_{mt}>0$ for some $t \geq 1$, $\lambda_1=\lambda_0$. Using this multiplier, we evaluate the first-order condition of the period 1 problem (21) at the continuation of the period 0 allocations, which solve (19), to conclude that for all $t \geq 1$,

$$(1+\lambda_0)(U_{ct}+U_{lt})+\lambda_0 c_t(U_{cct}+U_{llt})+\lambda_0 U_{llt} b' = \lambda_0 U_{llt} b' < 0,$$

since U_{llt} is negative and both λ_0 and b' are positive and the rest of the terms are zero by (19). This inequality means that at the period 0 allocations, the government has an incentive to deviate from the allocations chosen by the period 0 government. Hence, the period 0 Ramsey problem is not time consistent.

Consider this result at a more abstract level than that simple example. When the restrictions imposed by the lemma are satisfied, the government in period 0 does not have enough instruments of influence to induce the government in period 1 to follow the continuation of the Ramsey policy. The government in period 0 must find a vector of real bonds and a multiplier for the implementability constraint that satisfy the first-order conditions for consumption and leisure (21), the first-order conditions for real balances (22), and the implementability constraints of the period 1 government at the period 0 Ramsey allocation. When the Friedman rule does not hold, in general, no combination of b' and λ satisfies all these equations.

We can now use the logic of the example to show that if the Friedman rule holds, then the Ramsey problem is time consistent. To cover the general case, we assume that two regularity conditions hold in each period t: $U_{ct} + U_{lt} \geq 0$, so that taxes are nonnegative, and $U_{clt} + U_{llt} < 0$, which is essentially normality of consumption. (In our working paper, Alvarez, Kehoe, and Neumeyer (2002), we give sufficient conditions for taxes to be nonnegative.)

PROPOSITION 3: *Assume that the initial nominal government liabilities are all zero and our regularity conditions hold. If the Friedman rule holds for each period, then the Ramsey problem in period* 0 *is time consistent.*

The proof is a generalization of that used in the example and is in the Appendix. Strictly speaking, in the proof of this proposition, we show that if the maturity structure of the government debt is adequately managed, then the continuation of the Ramsey allocation in period 0 satisfies the first-order conditions of the Ramsey problem faced by the successor governments. An allocation that satisfies the first-order conditions may not solve the Ramsey problem; it could be a local maximum, a minimum, or a saddle point. In our working paper, Alvarez, Kehoe, and Neumeyer (2002), we give conditions under which the first-order conditions of the Ramsey problem are sufficient for a maximum.

The proof of Proposition 3 makes clear that to ensure time consistency, there is a unique way to restructure the real government debt. In general, there are many ways to restructure the nominal debt. (See Alvarez, Kehoe, and Neumeyer (2002) for details.)

We now consider the converse of Proposition 3, that if the Ramsey problem is time consistent, then the Friedman rule is optimal. We assume the following regularity condition. At a Ramsey allocation, if the Friedman rule does not hold for some period t, then

$$(23) \qquad (R_{ct} + R_{lt})U_{lmt} - R_{mt}(U_{lct} + U_{llt}) \neq 0$$

holds, where $R(c, m, l) = cU_c + mU_m + lU_l$. (This regularity condition ensures that the first-order conditions for the Ramsey problem are not collinear.) This regularity condition is satisfied, for example, when the period utility function U is additively separable in leisure. For such preferences, the left-hand side of (23) reduces to $-R_{mt}U_{llt}$, and the first-order condition (11) implies that $R_{mt} = -U_{mt}/\lambda \neq 0$.

We also assume that in period 0 the government's budget is not balanced, in that

$$\tau_0 c_0 + (1 - Q_{0,1})m_0 \neq g_0 + b_{-1,0}, \tag{24}$$

where $Q_{0,1} = 1 + (U_{m0}/U_{l0})$ and $1 + \tau_0 = -U_{c0}/U_{l0}$.

The proof is a generalization of that used in the example and is in the Appendix.

PROPOSITION 4: *Assume that the government's initial nominal liabilities are all zero. Assume also that in some period r, consumption taxes are levied and the conditions* (16) *and the regularity condition* (23) *hold; in all periods, the normality condition $U_{lct} + U_{llt} < 0$ holds; and in period* 0, *the government's budget is not balanced. If the Ramsey problem is time consistent, then the Friedman rule holds for each period $s \geq 1$.*

Note that if, in the period 0 Ramsey problem, the government's budget is balanced in every period, then the Ramsey problem can be time consistent even if interest rates are strictly positive in all periods. For example, consider an economy in which g_t is constant and $b_{-1,t} = B_{-1,t} = 0$ for all t, so that there is no initial government debt. The period 0 Ramsey allocation is constant and prescribes a balanced budget. The Ramsey problem at any future period is the same as the period 0 Ramsey problem; thus, its solution is the continuation of the period 0 solution. Therefore, the period 0 Ramsey problem is time consistent. Nevertheless, the solution of the period 0 Ramsey problem may have strictly positive interest rates, depending on the preferences. For an example of such preferences, see the work of Chari, Christiano, and Kehoe (1996, p. 209).

Clearly, our results, especially Proposition 4, are at odds with the results of Persson, Persson, and Svensson (1987). They construct a nominal debt vector to be inherited by the period 1 government and suppose that with this vector as an initial condition, the period 1 government chooses an interior point for p_1, so that $0 < p_1 < \infty$. As our lemma shows, unless the Friedman rule is satisfied, the Ramsey problem in period 1 does not have a solution (with an interior point for p_1). Thus, the construction by Persson, Persson, and Svensson (1987) is invalidated by endogenous restrictions on the nominal debt vector that they do not take into account.

3. INTUITION: COUNTING EQUATIONS AND UNKNOWNS

Whether or not a policy is time consistent depends critically on how many instruments a government in any period has to influence its successor relative to how many choices the successor makes. For time consistency, the number of instruments must be greater than or equal to the number of choices.

We simplify the counting of instruments and choices by considering a finite horizon version of our economy with T periods. To mimic the opportunity cost of holding money in period T that would arise in an infinite horizon economy from the nominal interest rate between period T and period $T+1$, we add a direct tax on real balances for period T.

We focus on how the government in period 0 can induce the government in period 1 to carry out the continuation of its plans. We first count the number of instruments that the period 0 government can use to influence the period 1 government. In doing so, we count neither the nominal bonds nor the money supply. We exclude the nominal bonds because the proofs of Propositions 3 and 4 indicate that the Ramsey problem is time consistent if and only if the Ramsey allocations can be supported by the period 0 government giving the period 1 government zero nominal liabilities in each period by setting $M_1 + B_{0,1} = 0$ and $B_{0,t} = 0$ for all $t > 1$. (In each period, either $U_{mt} > 0$ and the lemma applies, or $U_{mt} = 0$ and (18) holds. Either way, the terms in the first-order conditions involving nominal debt drop out.) We do not count the money supply because the period 0 plan includes M_1, so that there is no freedom in picking this variable. Thus, the only other instruments that the government in period 0 can use to influence the government in period 1 are the T real bonds $b_{0,t}$ for $t = 1, \dots, T$.

Now we consider the conditions that define the period 1 government choices, the first-order conditions and the implementability constraint. For $t = 1, \dots, T$ the first-order conditions for c_t and l_t combined give

$$R_{ct} + R_{lt} + (U_{lct} + U_{llt})b_{0t} = -\frac{(U_{ct} + U_{lt})}{\lambda_1},$$

while the first-order condition for real balances m_t is

$$R_{mt} + U_{lmt}b_{0t} = \frac{U_{mt}}{\lambda_1}. \tag{25}$$

The implementability constraint is

$$\sum_{t=1}^{T} \beta^{t-1} R(c_t, m_t, l_t) = -\sum_{t=1}^{T} \beta^{t-1} U_{lt} b_{0t}.$$

These equations form a linear system in $2T+1$ equations and $T+1$ unknowns, namely, the T real bonds and the multiplier λ_1. This system has (many) more equations than unknowns.

When the Friedman rule is optimal, $U_{mt} = 0$, and using either weak separability or (18), we know that $U_{lmt} = 0$, and neither b_{0t} nor λ_1 enters (25). Thus, the linear system reduces to $T+1$ equations in $T+1$ unknowns and, under our regularity conditions, has a unique solution. Thus, here the period 0 government has enough instruments to induce the government in period 1 to carry out the continuation of its plan. Conversely, if the Ramsey problem is time consistent, then T equations must be redundant. Given our regularity conditions, the T redundant equations are the first-order conditions for money, which implies that $U_{mt} = 0$. Thus, the Friedman rule is optimal.

We have shown that determining whether or not the Ramsey problem is time consistent can be reduced to counting the number of instruments that the government of period 0 has to influence the government in period 1 and the number of conditions that define the choices of the period 1 government and comparing the two numbers. For time consistency, government 0's instruments of influence must be at least as many as government 1's choices.

Throughout we use the primal approach to analyze the Ramsey problem in which all the tax rates and interest rates are substituted out. If instead we had used the dual approach, our counting procedure would be more reminiscent of that of Tinbergen (1956). We could then compare the number of policies chosen by the government of period 1 to the number of instruments the period 0 government has to influence the period 1 government. The period 1 government must choose $2T$ policies, the T taxes on consumption and T taxes on real balances ($T-1$ nominal interest rates and one direct tax) while the period 0 government has effectively only the T real bonds as instruments. When the Friedman rule is optimal, money effectively disappears, and the period 1 government has only T policies and the period 0 government has sufficiently many instruments, the T real bonds, to induce its successor to carry out its plan. When the Friedman rule is not optimal, the period 0 government does not have enough instruments to do this.

4. EXTENSIONS TO OTHER MONETARY ECONOMIES

Our main result is that optimal monetary and fiscal policies are time consistent for an interesting class of economies discussed in Propositions 1–3. The benchmark economy we have focused on is an economy with money in the utility function, in which end-of-period real money balances enter that function. Now we show that optimal monetary and fiscal policies are time consistent for other interesting economies as well, including a version of our economy with beginning-of-period money balances in the utility function, a cash-credit economy, and a shopping time economy. For completeness, in Proposition 4, we have also given necessary conditions for the time consistency of optimal policies for our benchmark model. Here, we briefly discuss how these necessary conditions may change in other models.

In our model with end-of-period money balances, increases or decreases in the price level have no costs other than their effect on the real value of nominal debt. This makes the time consistency problem severe: if private agents owe the government even a tiny nominal amount, then the government has an incentive to engineer a sharp decrease in the price level to greatly increase the real value of this nominal debt. Even with this severe problem, however, policy is time consistent in an interesting class of economies.

In general, as changing the price level ex post becomes either harder or costlier, the time consistency problem becomes less severe, and the set of circumstances for which optimal policy is time consistent becomes larger. As an extreme, consider the example of Lucas and Stokey (1983) in which the government (or "central bank") can commit to a path for the nominal price level, so that ex post prices cannot be changed from this path. Then, as Lucas and Stokey show, optimal policy will always be time consistent, regardless of the form of preferences. The principle is that, as changing prices ex post becomes harder, the set of sufficient conditions for time consistency expands and the set of necessary conditions for time consistency shrinks.

We turn now to showing how that optimal policy will be time consistent for other commonly used monetary economies.

Beginning-of-Period Real Balances

We start with economies in which the money balances entering a period's utility function are those for the beginning of the period rather than the end. Explicitly, consider a utility function $U(c_t, m_t, l_t)$, in which now $m_t = M_{t-1}/p_t$ denotes the beginning-of-period real money balances.

The implementability constraint for this economy is

$$(26)\qquad c_0 U_{c0} + U_{l0}(l_0 + b_{-10}) + \sum_{t=1}^{\infty} \beta^t [c_t U_{ct} + m_t U_{mt} + U_{lt}(l_t + b_{-1t})]$$
$$= -\frac{U_{l0}}{p_0}\left(M_{-1} + B_{-10} + \sum_{t=1}^{\infty} Q_{0,t} B_{-1t}\right), \quad \text{where}$$
$$Q_{0,t} = \prod_{s=1}^{t}\left(1 - \frac{U_{ms}}{U_{ls}}\right)^{-1} \quad \text{and} \quad p_0 = M_{-1}/m_0.$$

Notice that here M_{-1} is a predetermined variable, so that changing p_0 necessarily changes m_0, while in the end-of-period balances economy $p_0 = M_0/m_0$, and both M_0 and m_0 are choice variables.

Note that the time consistency problem is less severe here than in the benchmark economy. The reason is that an increase in the initial price level decreases the initial real money balances as well as the real value of the nominal debt. To see this, consider the first-order conditions for p_0:

$$(27)\qquad -\frac{\lambda_0 U_{l0}}{p_0^2}\left(M_{-1} + B_{-10} + \sum_{t=1}^{\infty} Q_{0,t} B_{-1t}\right) = \frac{U_{m0}}{p_0} m_0,$$

where, for notational simplicity, we let U be additively separable in m. The left side of (27) is the benefit from increasing the price level. Comparing (27) with (12) demonstrates that here increasing the price level has an extra cost coming from the decrease in real money balances captured by the term $U_{m0}m_0/p_0$.

The exact analogs of Propositions 1, 2, and 3 hold here. To see this, note that when the initial nominal government liabilities are all zero, the only difference between the Ramsey problems of our benchmark economy and this economy is that the implementability constraint for the benchmark economy, (7), has the term $m_0 U_{m0}$ while the implementability constraint for this economy, (26), does not. In the economy with beginning-of-period balances and zero nominal debt in all periods, using (18), we see that setting $U_{m0} = 0$ is optimal, regardless of preferences. The first-order conditions for all other periods are the same in the two economies.

For this economy, the proof of our lemma does not hold; hence, our proof of Proposition 4 does not either. Here, in a Ramsey problem, the initial price level may be interior even if the value of the nominal debt is not zero, as the first-order condition (27) shows. We conjecture that necessary conditions for time consistency are much weaker here than for the benchmark economy.

Cash-Credit and Shopping Time Economies

Consider extending our results to two other commonly used economies—a cash and credit goods economy and a shopping time economy. These economies can be represented as economies with money in the utility function, but the assumption that $U_m \geq 0$ everywhere may well be violated. Note that Propositions 1 and 2 apply to these economies because in them we do not require that the utility function be such that $U_m \geq 0$. Propositions 3 and 4, however, require a slight modification. Here we discuss the modification to Proposition 3, and in Alvarez, Kehoe, and Neumeyer (2002), we discuss the one corresponding to Proposition 4.

Consider an economy with cash and credit goods, similar to that of Lucas and Stokey (1983). In particular, let the period utility function be $h(c_1, c_2, l)$, where c_1 and c_2 are cash and credit goods and l is labor. Assuming that end-of-period real balances are exclusively used to purchase cash goods, we can map the cash-credit economy into our notation by defining $U(c, m, l) = h(m, c - m, l)$, where $c = c_1 + c_2$. Notice that even if h is strictly increasing in c_1 and c_2, the associated utility function U is typically not weakly increasing in m everywhere, as we have assumed in Proposition 3. The monotonicity assumption $U_m \geq 0$, together with the assumptions of concavity and differentiability, implies that $U_{mm} = 0$ whenever $U_m = 0$. For the cash-credit economy, $U_m = 0$ when $h_1 = h_2$. It can be shown that our monotonicity and concavity assumptions require that at the point where $h_1 = h_2$, $h_{11} = h_{22} = h_{12}$, which in turns implies an infinite elasticity of substitution between cash and credit goods, a very special condition.

Proposition 3 holds for the cash-credit economy if we add an extra regularity condition:

$$\begin{aligned} &U_{mt} = 0 \quad \text{and} \quad U_{mmt} < 0 \quad \text{imply that} \\ &(U_{lct} + U_{llt})U_{mmt} > U_{lmt}(U_{mct} + U_{mlt}). \end{aligned} \tag{28}$$

To understand this condition, define the satiation level of money $m^*(c, l)$ to be the smallest level of real balances for which its marginal utility is zero; that is, $m^*(c, l) = \min\{m : U_m(c, m, l) = 0\}$. This regularity condition is met if U is such that $m^*(c, l)$ is independent of l and leisure is a normal good. A sufficient condition for this to be true is that U is weakly separable in l, as in $U(c, m, l) = u(w(c, m), l)$.

Proposition 3 also holds for a shopping time economy. In this economy, the regularity condition also holds when U is derived from a shopping time technology, as in $U(c, m, l) = u(c, l + s(c, m))$, where s is the time an agent that consumes c and has m real balances needs to devote to shopping and where the utility u is defined over c and total labor $l + s$. We then have the following proposition:

PROPOSITION 3′: *Assume that the initial nominal claims are zero period by period, so that* (8) *holds and our strengthened regularity conditions hold. If the Friedman rule holds for each period, then the Ramsey problem is time consistent.*

The proof uses a construction similar to that of Proposition 3. The only difference is that now we have to construct η_t, the multiplier on the interest rate constraint $U_{mt} \geq 0$, as well as b_0 and λ_1. Our extra regularity condition (28) ensures that there is a unique solution for b_{0t} and η_t as a function of λ_1. We then substitute the resulting expression for $b_{0,t}$ into the implementability constraint, and under our regularity conditions, there is a unique λ_1.

5. RELATING TIME CONSISTENCY TO SUSTAINABLE PLANS

Now we relate Lucas and Stokey's (1983) notion of time consistency to the literature on sustainable plans (Chari and Kehoe (1990, 1993)) and thus to the closely related literature on credible policies (Stokey (1991)). We show that if the Ramsey problem is time consistent, then its solution is sustainable. More precisely, we show that the Ramsey allocations and policies are sustainable outcomes generated by a Markov sustainable equilibrium. Note that the converse is clearly not true; sustainable outcomes are not typically time consistent.

In the Lucas–Stokey definition of time consistency, which we have used here, the government in period 0 solves a problem under the presumption that it has the ability to commit to all its future policies, and consumers act under this presumption as well. What the government in period 0 actually gets to set, however, is the period 0 policies, including the new initial conditions for the government to face in period 1. The problem in period 0 is time consistent for the problem in period 1 if initial conditions exist such that the government in period 1, under a similar presumption about commitment, chooses to continue with the allocations and policies chosen by the government in period 0. Under this definition, the government in period 0 does not explicitly think through how altering the initial conditions for the government of period 1 affects the future government's choices, since the government in period 0 simply presumes it can commit to all future policies.

The sustainable plan literature takes a lack of commitment as given and explicitly builds it into the definition of an equilibrium. In this definition, the government in period 0 realizes both that it cannot commit to all its future policies and that consumers understand this lack of commitment. This government also explicitly thinks through how altering the initial conditions for the period 1 problem affects the choices of the period 1 government.

In the sustainable equilibrium, the lack of commitment is modeled by having the government choose policy sequentially. Consumer allocations, prices, and government policy are specified as functions of the history of past policies of the government. These functions specify behavior for any possible history, even for histories in which the government deviates from prescribed behavior.

The time consistent equilibrium, in contrast, simply specifies a sequence of allocations, prices, and policies and thus is not directly comparable to a sustainable equilibrium. Along the equilibrium path, however, a sustainable equilibrium generates a particular sequence of allocations, prices, and policies, called a *sustainable outcome*, which is comparable to the sequences specified by a time consistent equilibrium.

For a version of the Lucas and Stokey (1983) economy without money, Chari and Kehoe (1993) show that the sustainable outcome generated by a Markov sustainable equilibrium solves a simple programming problem. With a little work, their results can be extended to our economy, and it can be shown that for some given initial conditions M_{-1}, B_{-1}, and b_{-1}, the allocations $({}_0c, {}_0m, {}_0l)$ are *sustainable Markov allocations* if and only if they are part of the solution to the following programming problem: Choose allocations $({}_0c, {}_0m, {}_0l)$, a nominal money supply M_0, and nominal and real government debt B_0 and b_0 to solve the *sustainable Markov problem*

$$V_0(M_{-1}, B_{-1}, b_{-1}) = \max \sum_{t=0}^{\infty} \beta^t U(c_t, m_t, l_t) \tag{29}$$

subject to the resource constraint and the nonnegative interest rate constraint for $t \geq 0$, the implementability constraint for $t = 0$

$$\sum_{t=0}^{\infty} \beta^t R_t = -\frac{U_{l0}}{p_0}\left(M_{-1} + \sum_{t=0}^{\infty} Q_{0,t} B_{-1,t}\right) - \sum_{t=0}^{\infty} \beta^t U_{lt} b_{-1,t},$$

and for $t = 1$

$$\sum_{t=1}^{\infty} \beta^{t-1} R_t = -\frac{U_{l1}}{p_1}\left(M_0 + \sum_{t=1}^{\infty} Q_{1,t} B_{0,t}\right) - \sum_{t=1}^{\infty} \beta^{t-1} U_{lt} b_{0,t}, \tag{30}$$

and the *sustainability constraint* for all $t > 0$

$$\sum_{s=t}^{\infty} \beta^{s-t} U(c_t, m_t, l_t) \geq V_t(M_{t-1}, B_{t-1}, b_{t-1}), \tag{31}$$

where $Q_{0,t} = \Pi_{s=0}^{t-1}[1 + (U_{ms}/U_{ls})]$ and $p_1 = M_1/m_1$ and the value functions $V_t(M_{t-1}, B_{t-1}, b_{t-1})$ are defined recursively by (29). The sustainability constraint (31) captures the restriction that whatever sequence of allocations from period 0 to infinity is contemplated by the government in period 0, given the state variables (M_0, B_1, b_1) that this government passes to the government in period 1, the government in period 1 has an incentive to implement the continuation of these allocations from period 1 onward. The government in period 1 faces a similar constraint with respect to the government in period 2, and so on, for the governments in all future periods.

Notice that the *sustainable Markov problem* is essentially the Ramsey problem in period 0 with two extra constraints: the implementability constraint in period 1 and the sustainability constraint with extra choice variables (M_0, B_0, b_0) and p_1. From the definition of time consistency and the sustainable Markov problem, we have this proposition:

PROPOSITION 5: *If the Ramsey problem is time consistent, then the Ramsey allocations are Markov sustainable allocations.*

PROOF: Our previous propositions contain the essence of the proof. Let $V_0^R(M_{-1}, B_{-1}, b_{-1})$ denote the value of the Ramsey problem in period 0 with state variables (M_{-1}, B_{-1}, b_{-1}). Since the Ramsey problem is a less constrained version of the sustainable Markov problem, its value is necessarily higher, so that

$$V_0^R(M_{-1}, B_{-1}, b_{-1}) \geq V_0(M_{-1}, B_{-1}, b_{-1}).$$

Thus, if the Ramsey allocations are feasible for the sustainable Markov problem, then they necessarily solve it. So, consider the Ramsey allocations given the state variables (M_{-1}, B_{-1}, b_{-1}). These allocations clearly satisfy the resource constraint and the implementability constraint in period 0 in the sustainable Markov problem. Given the values for the new state variables (M_0, B_0, b_0) constructed as in the definition of time consistency, these state variables plus the continuation of the period 0 allocations clearly satisfy the remaining constraints of the sustainable Markov problem—the implementability constraint in period 1, by construction, and the sustainability constraint, since by the same logic as that of period 0, it follows that

$$V_1^R(M_0, B_0, b_0) \geq V_1(M_0, B_0, b_0). \qquad Q.E.D.$$

Here we have shown the connection between the concepts of time consistency and sustainable plans for a class of economies. A similar connection should hold for other economies as well.

6. CONCLUSION

In a simple economy similar to that of Lucas and Stokey (1983), we have found that optimal monetary and fiscal policies are time consistent if and only if the Friedman rule is optimal in this economy. The key ideas behind this result are three: (i) A government has little freedom in using nominal debt to influence successor governments; the primary influence comes, instead, from the appropriate setting of real debt. (ii) When the Friedman rule is not optimal, no government has enough free debt instruments to adequately control the incentives of its successor to carry out its plan. When the Friedman rule is optimal, the government has no desire to use one of its taxes, the nominal interest rate, but the free debt instrument, real debt, is rich enough to control the incentives of the successor government in setting the remaining tax, the consumption tax. (iii) The Friedman rule is optimal for preferences that are widely used in applied work. Hence, this study suggests that, in practice, the type of time consistency problem considered here can be adequately solved by a careful management of government debt.

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APPENDIX: PROOFS OF PROPOSITIONS 1, 2, 3, AND 4 AND THE LEMMA

PROOF OF PROPOSITION 1: Assume, by way of contradiction, that $U_{mt} > 0$. We can arrange the first-order conditions for consumption c_t and real money balances m_t to be

$$(1+\lambda) + \lambda\left(\frac{U_{cct}c_t + U_{cmt}m_t}{U_{ct}}\right) + \lambda(l_t + b_{-1t})\frac{U_{lct}}{U_{ct}} = \frac{\gamma_t}{U_{ct}}, \tag{A.1}$$

$$(1+\lambda) + \lambda\left(\frac{U_{cmt}c_t + U_{mmt}m_t}{U_{mt}}\right) + \lambda(l_t + b_{-1t})\frac{U_{lmt}}{U_{mt}} = 0. \tag{A.2}$$

Differentiating (13) with respect to α and evaluating it at $\alpha = 1$ gives that

$$\text{(A.3)} \qquad \frac{c_t U_{cmt} + m_t U_{mmt}}{U_{mt}} = \frac{c_t U_{cct} + m_t U_{cmt}}{U_{ct}}.$$

By weak separability, $U_{lct}/U_{ct} = U_{lmt}/U_{mt}$. Subtracting (A.1) from (A.2) using (A.3) and weak separability gives that $\gamma_t/U_{ct} = 0$, which is a contradiction since γ_t and U_{ct} are strictly positive. *Q.E.D.*

PROOF OF PROPOSITION 2: We prove here that the Friedman rule solves the Ramsey problem under the conditions of Proposition 2 by showing that under our hypotheses, we can construct a solution to the first-order conditions and the implementability constraint that satisfies the Friedman rule.

Define the *satiation level of money* $m^*(c, l)$ to be the smallest level of real balances for which its marginal utility is zero; that is, $m^*(c, l) = \min\{m : U_m(c, m, l) = 0\}$. As a preliminary result, we show that this satiation level $m^*(c, l) = cg(l)$ for some positive and increasing function $g(l)$ and that at that satiation level,

$$\text{(A.4)} \qquad U_{mc} + U_{mm} g(l) = 0 \quad \text{and} \quad U_{mm} c g'(l) + U_{ml} = 0.$$

To see this, note that if $U_m(\alpha c, \alpha m, l) = 0$ for all α, we can set $\alpha = 1/c$ and define m^* implicitly as $U_m(1, m^*(c, l)/c, l) = 0$. Clearly, m^* is of the form $cg(l)$. Differentiating $U_m(c, cg(l), l) = 0$ with respect to c and with respect to m gives (A.4). To see that g is increasing, notice that if $U_{mm} < 0$, then $U_{ml} \geq 0$ if and only if $g'(l) \geq 0$. If $U_{mm} = 0$ at the point where $U_m = 0$, as in (18), then $U_{mc} = U_{ml} = 0$.

We establish our result by constructing an allocation that solves the first-order conditions of the Ramsey problem in the monetary economy using the first-order conditions for a corresponding real economy. To do so, define $\bar{U}(c, l) = U(c, m^*(c, l), l)$, and define the Ramsey problem for the corresponding real economy as choosing $\{c_t, l_t\}$ to maximize $\sum \beta^t \bar{U}(c_t, l_t)$ subject to $c_t + g_t = l_t$ and the implementability constraint

$$\text{(A.5)} \qquad \sum_{t=0}^{\infty} \beta^t [\bar{U}_{ct} c_t + \bar{U}_{lt}(l_t + b_{-1,t})] = 0.$$

Denote the solution as $\{c_t^*, l_t^*\}$ and the multipliers for the resource constraints and the implementability constraints as $\{\gamma_t^*\}$ and λ^*.

For the corresponding monetary economy, we need to define the multiplier on the constraint $U_{mt} \geq 0$. Let this multiplier be given by

$$\text{(A.6)} \qquad \eta_t^* = \lambda^* m_l(c_t^*, l^*)(l_t^* + b_{-1t}).$$

Notice that $\eta_t^* \geq 0$ since $l_t^* + b_{-1t} \geq 0$, $m_l(c_t^*, l^*) = c_t^* g'(l_t^*) \geq 0$, and $\lambda^* \geq 0$. Let $m_t^* = m^*(c_t^*, l_t^*)$. We claim that $\{c_t^*, m_t^*, l_t^*\}$ and the multipliers λ^* and $\{\gamma_t^*, \eta_t^*\}$ solve the first-order conditions and the implementability constraint of the Ramsey problem in the monetary economy. Clearly, the allocation is resource feasible. It remains to be shown that the first-order conditions with respect to m_t, c_t, and l_t as well as the implementability constraint are satisfied.

We now show that the first-order condition for money

$$\text{(A.7)} \qquad (1 + \lambda^*) U_{mt} + \lambda^* [U_{mct} c_t^* + U_{mmt} m_t^* + U_{mlt}(l_t^* + b_{-1,t})] + \eta_t^* U_{mmt} = 0$$

holds at the constructed allocations and multipliers. At our constructed allocation, $U_{mt} = 0$. Multiplying the first equality in (A.4) by $\lambda^* c_t^*$ and using the definition of m^*, we get that $\lambda^*(U_{mct} c_t^* + U_{mmt} m_t^*) = 0$. Using (A.4) to solve for U_{mlt} and the definition of η_t^*, we get that

$$\lambda(l_t^* + b_{-1t}) U_{mlt} + \eta_t^* U_{mmt} = U_{mmt}[-\lambda^*(l_t^* + b_{-1t}) c_t^* g'(l_t^*) + \eta_t^*].$$

Hence, the first-order condition for m_t holds.

The first-order condition with respect to c_t in the real economy is that

$$(A.8) \qquad (1+\lambda^*)\bar{U}_{ct} + \lambda^*[\bar{U}_{cct}c_t^* + \bar{U}_{clt}(l_t^* + b_{-1,t})] = \gamma_t^*.$$

Using the definition of $\bar{U}$ and the form of m^* and evaluating the derivatives at the candidate allocation, we have that $\bar{U}_c = U_c$, $\bar{U}_{cc} = U_{cc} + U_{cm}m_c$, and $\bar{U}_{cl} = U_{cl} + U_{ml}m_c$. From (A.4), we get that $U_{ml}m_c = U_{mc}m_l$. Using all of these expressions and the definition of η_t^*, we get this:

$$(1+\lambda^*)U_{ct} + \lambda^*(U_{cct}c_t^* + U_{cmt}m_t^*) + \lambda^*U_{clt}(l_t^* + b_{-1t}) + \lambda^*\eta_t^* = \gamma_t^*,$$

which is the first-order condition for c_t in the monetary economy. An analogous argument holds for the first-order conditions with respect to l_t. Finally, from $m_t^* = m(c_t^*, l_t^*)$, $\bar{U}_{ct} = U_{ct}$, $\bar{U}_{lt} = U_{lt}$, and $U_{mt} = 0$, it is immediate that the implementability constraint in the real economy implies the implementability constraint in the monetary economy. *Q.E.D.*

PROOF OF LEMMA: We prove the lemma by showing that if the assumptions of the lemma hold and (15) does not hold, then we can perturb the allocations and increase utility.

We construct the perturbation in two steps. In the first step, we perturb the allocation in order to make the present value of the government's nominal liabilities negative and then lower the initial price level. If $Q_{s,s+1} < 1$ and (14) holds but (15) does not, then we can make the present value of the government's nominal liabilities negative by a small change in $Q_{s,s+1}$. This change, which may entail either raising or lowering $Q_{s,s+1}$, is feasible since the Friedman rule does not hold at s. We change c_s, m_s, and l_s in a way that satisfies the resource constraint and produces the desired change in $Q_{s,s+1}$. Then, by lowering the initial price level p_0, we can generate any desired level of real assets for the government.

In the second step of the perturbation, we lower the taxes in period r, the period that we have hypothesized has positive taxes, in a way that raises utility in that period, satisfies the resource constraint, and holds fixed $Q_{r,r+1}$, so that we know that the perturbed allocation still implies a strictly negative value for nominal government liabilities and the first step of the perturbation still works. To that end, note that positive taxes in period r imply that $-U_{lr} < U_{cr}$. Since $U_{mr} \geq 0$, we can increase c_r and m_r and decrease l_r in a way that keeps $Q_{r,r+1}$ constant, satisfies the resource constraint, and increases utility in period r. Clearly, by the implicit function theorem, for a fixed $Q_{r,r+1}$ and g_r, functions $m(c)$ and $l(c)$ exist such that c, $m(c)$, and $l(c)$ satisfy

$$(A.9) \qquad U_m(c, m(c), l(c)) + (1 - Q_{r,r+1})U_l(c, m(c), l(c)) = 0$$

and $c + g_r = l(c)$. These functions satisfy $l'(c) = 1$, and if $U_m > 0$, then

$$(A.10) \qquad m'(c) = -\left[\frac{U_{mc} + U_{ml} + (1 - Q_{r,r+1})(U_{ll} + U_{lc})}{U_{mm} + (1 - Q_{r,r+1})U_{lm}}\right],$$

which is nonnegative under our assumptions on second derivatives. (Note that since $U_{mm} \leq 0$, the denominator in (A.10) is nonpositive even if $U_{lm} > 0$, because $U_{mm} + U_{lm} < 0$ and $1 - Q_{r,r+1} \leq 1$.)

If $U_m > 0$, then increasing c, and thus changing m and l by $m'(c)$ and $l'(c)$, leads utility in period r to change by $U_c + U_l + U_m m'(c)$, which is strictly positive since by assumption at r, $-U_l < U_c$, $U_m \geq 0$, and $m'(c) \geq 0$.

If $U_m = 0$, it must be that $Q_{r,r+1} = 1$. Consider the case where $U_{mm} < 0$. To ensure that (A.9) holds, it is enough to let $m'(c) = -(U_{mc} + U_{ml})/U_{mm} > 0$. In this case, the resulting change in utility in period r is $U_c + U_l > 0$. If at some point $U_m = 0$ and $U_{mm} = 0$, by concavity it must be that $U_{ml} = U_{cm} = 0$ at this point as well. (To see this, note that if $U_m = 0$ at some point m, then since U is weakly increasing in m, $U_m = 0$ at all points $m' \geq m$. Thus, since U is twice continuously differentiable, $U_{mm} = 0$. To see that $U_{mc} = 0$, note that by concavity $U_{cc}U_{mm} - U_{mc}^2 \geq 0$, so $U_{mc} = 0$. A similar argument applies for U_{ml}.) Hence, a small change in c and l does not change the value of U_m, and in particular, this change keeps $U_m = 0$. Thus, increasing c and l by the same small amount changes the period r utility by $U_c + U_l > 0$. This establishes the contradiction. *Q.E.D.*

PROOF OF PROPOSITION 3: Now we prove that if the Friedman rule holds under the conditions of Proposition 3, then the Ramsey problem is time consistent.

We begin this proof by showing that the Ramsey problem for period 0 is time consistent for period 1 by constructing the appropriate initial conditions for the period 1 Ramsey problem, namely, M_0, B_0, and b_0 together with a period 1 multiplier λ_1, so that the first-order conditions and the implementability constraint for the period 1 Ramsey problem hold when evaluated at the continuation of the period 0 Ramsey allocations. For the nominal assets, we set $M_0 + B_{0,0} = 0$ and $B_{0,t} = 0$ for $t \geq 1$. (The breakdown of M_0 and $B_{0,0}$ is irrelevant as long as $M_0 > 0$.)

We construct the values for b_0 and λ_1 in a way similar to that of Lucas and Stokey (1983). Consider the combined first-order conditions for c_t and l_t from (9) and (10), which can be rewritten as

$$b_{0,t} = -\left(\frac{1}{\lambda_1}\right)\frac{(U_{ct} + U_{lt})}{(U_{lct} + U_{llt})} - \frac{(R_{ct} + R_{lt})}{(U_{lct} + U_{llt})},$$

which gives our expression for $b_{0,t}$ for an arbitrary λ_1. To construct λ_1, we substitute this expression for $b_{0,t}$ into the period 1 implementability constraint. Given our assumptions, the fraction $(U_{ct} + U_{lt})/(U_{lct} + U_{llt})$ is negative, and there is a unique solution for λ_1.

It remains to be shown that the first-order conditions for m_t in the period 1 problem

$$U_{mt} + \lambda_1 R_{mt} = -\lambda_1 U_{lmt} b_{0,t} \tag{A.11}$$

hold. Since $U_{mt} = 0$, it follows from (18) that $R_{mt} = U_{mt} + c_t U_{cmt} + m_t U_{mmt} + l_t U_{mlt} = 0$. Since both sides of (A.11) are identically zero regardless of the multiplier, these first-order conditions trivially hold. *Q.E.D.*

PROOF OF PROPOSITION 4: We show here that under the stated conditions, if the Ramsey problem is time consistent, then the Friedman rule is optimal. We prove this proposition by showing that if the Friedman rule does not hold in some period $s \geq 1$, then the Ramsey problem is not time consistent. By way of contradiction, suppose that the Friedman rule does not hold in $s \geq 1$, but the Ramsey problem is time consistent. We show that this implies that in period 0, the Ramsey allocation's first-order conditions for the period 1 problem cannot hold, thus establishing a contradiction.

We first show that all the terms involving the nominal government liabilities in the first-order conditions for the period 1 problem are zero. Consider the first-order conditions with respect to c_t and l_t in the period 1 Ramsey problem. We claim that in any period $t \geq 1$, terms of the form

$$Q_{0t}\frac{\partial Q_{t,t+1}}{\partial c_t}\sum_{v=t+1}^{\infty} Q_{t+1,v}B_{-1,v} = 0. \tag{A.12}$$

Suppose initially that t is some period in which the Friedman rule holds. Then $U_{mt} = 0$, and from (18) we know that $\partial Q_{t,t+1}/\partial c_t = (U_{mct}U_{lt} - U_{lct}U_{mt})/U_{lt}^2 = 0$, where we have used $Q_{t,t+1} = 1 + (U_{mt}/U_{lt})$. A similar argument implies that $\partial Q_{t,t+1}/\partial l_t = 0$; hence, the corresponding terms are also zero under the first-order conditions for l_t. Suppose next that t is a period, like period s, in which the Friedman rule does not hold; then (15) does hold, and these terms are zero as well. Moreover, for the period 1 problem, the first-order condition with respect to p_1 implies that terms of the form $(M_0 + \sum_{t=0}^{\infty} Q_{1,t}B_{0,t})/p_1 = 0$.

We now show that the multipliers on the implementability constraints for the Ramsey problems in periods 0 and 1, λ_0 and λ_1, satisfy $\lambda_0 = \lambda_1$. To see this, consider the first-order conditions to these problems for period s. From (8) we know that $B_{-1,t}$ is zero for all $t \geq 1$. The first-order condition for m_s has the form of (11), which can be written as

$$R_{ms}\lambda_0 + U_{lms}\lambda_0 b_{-1,s} = -U_{ms}. \tag{A.13}$$

Combining the first-order conditions for c_s and l_s gives that

$$(R_{cs} + R_{ls})\lambda_0 + (U_{lcs} + U_{lls})\lambda_0 b_{-1,s} = -U_{cs} - U_{ls}. \tag{A.14}$$

We can regard (A.13) and (A.14) as a system of two linear equations in two unknowns, λ_0 and $\lambda_0 b_{-1,s}$. For the period 1 Ramsey problem, $U_{ms} > 0$ for some $s \geq 1$. Our lemma implies that $\sum_{t=s}^{\infty} Q_{1,t} B_{0,t}$ is zero, so that the first-order conditions for the period 1 problem can be written as

$$R_{ms}\lambda_1 + U_{lms}\lambda_1 b_{0,s} = -U_{ms}, \tag{A.15}$$

$$(R_{cs} + R_{ls})\lambda_1 + (U_{lcs} + U_{lls})\lambda_1 b_{0,s} = -U_{cs} - U_{ls}, \tag{A.16}$$

which is a system of linear equations in the two unknowns λ_1 and $\lambda_1 b_{0,s}$. By hypothesis, the Ramsey problem is time consistent; hence, the allocations in the two systems of equations (A.13)–(A.14) and (A.15)–(A.16) are the same. Our regularity condition (23) implies that there is a unique solution to both and, hence, that $\lambda_0 = \lambda_1$.

Now we will show that there is some period T for which $b_{0,T} \neq b_{-1,T}$. By way of contradiction, suppose not. Since the solution to the Ramsey problem in period 1 is interior, the first-order condition for p_1 implies that

$$\frac{\lambda_1}{p_1}\left(M_0 + \sum_{t=1}^{\infty} Q_{1,t} B_{0,t}\right) = 0. \tag{A.17}$$

Subtracting the product of β and the period 1 implementability constraint from the period 0 implementability constraint gives $U_c c_0 + U_{m0} m_0 + U_{l0} l_0 = -U_{l0} b_{-1,0}$, which implies that the budget must be balanced in period 0. This is a contradiction. Hence, there must be some period T for which $b_{0,T} \neq b_{-1,T}$. Using $\lambda_1 = \lambda_0$ and $b_{0,T} \neq b_{-1,T}$, evaluate the first-order condition for the period 1 problem in T for the period 0 allocation. That gives

$$(R_{cT} + R_{lT})\lambda_1 + (U_{lcT} + U_{llT})\lambda_1 b_{0,T} \neq -U_{cT} - U_{lT}, \tag{A.18}$$

where we have used the assumption that $U_{lcT} + U_{llT} < 0$. Thus, the continuation of the period 0 allocation cannot solve the period 1 problem. Hence, the Ramsey problem is not time consistent. *Q.E.D.*

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