

# Optional Exercises for 8102

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These questions are for your own education only, not for credit. Solving them is not necessary nor sufficient for passing exams/prelims. They are intended to make you think at a deeper level about some of the main conceptual (as opposed to technical) issues raised in this course. Many of them are decidedly open-ended.

1. In this course we have been studying how to analyze models of market behavior in which people behave purposefully. A potential alternative to this approach would be take things like demand curves and supply curves as primitives of the model without explicitly deriving them from the principles of individual optimization. What do you think might be the benefits from *not* taking this “easier” approach?
2. Consider a pure exchange economy with only one commodity.
  - (a) What is the competitive equilibrium allocation for this economy?
  - (b) Does your answer in (a) create a conflict with partial equilibrium analyses in which people focus on a market of a single good? Why or why not?
3. Construct an example of an exchange economy to which you can apply the Very Easy Existence Theorem. Can you find an “interesting” example? Why or why not?
4. Using an Edgeworth box, illustrate how equilibrium existence can be jeopardized if the hypotheses of the Proposition covered in the recitation fail.
5. Consider an exchange economy for which the condition  $\sum_{i=1}^n e_i \gg 0$  fails. As you demonstrated in the previous question, this may create an equilibrium non-existence problem. Can you go back and reformulate the model in a reasonable way to get rid of this problem?
6. More generally, suppose you have constructed an interesting model to study a question that you are interested in, but after some work you have figured out that an equilibrium does not exist. What do you think you can do in response?
7. Now suppose you have constructed a model and it has turned out that there are more than one equilibria. Do you think this is a deficiency of your model? Or is it a virtue? What should be done in response? Explain.

8. Suppose you have constructed a model and have found a successful numerical algorithm for computing equilibria, but you have not been able to rigorously establish neither the existence nor the uniqueness of equilibria. Can you justify using your algorithm to study questions you are interested in? Why or why not?
9. Consider the production economy model discussed in the recitation. Suppose that (by some miracle, say) you have managed to figure out what the equilibrium production profile  $\{y_j^*\}_{j=1}^m$  is. Show that you can now solve for the equilibrium consumption allocation and equilibrium price vector *as if* you were dealing with an exchange economy with suitably re-defined endowment levels.
10. Take the production economy model discussed in the recitation and introduce a new market (a “stock market”) in which people are allowed to trade their entitlements to firm profits. Give a sensible definition of an equilibrium for this model. Does this change the set of equilibria (in any reasonable sense)? Would it be interesting to compare the equilibrium predictions of what goes on in this market with real-world stock market data?
11. The definition of competitive equilibrium for production economies includes a condition which states that firms maximize profits. Consider replacing this condition with one that says “a firm’s production plan is determined by voting among the firm’s owners.” (This may be more appealing given that firms are owned by consumers rather than some profit-hungry manager.) Formalize this idea. Does it change the set of competitive equilibria? Why or why not?
12. Consider again the production economy model discussed in the recitation. It is sometimes useful to classify the elements comprising the model as either “physical” or “institutional.” Loosely speaking, the former refers to things like demographics, preferences, technology, etc that “cannot be changed,” whereas the latter refers to a description of how people are allowed to interact and how their interactions determine social outcomes, which presumably can be changed given sufficient political power.
  - (a) Change the “institutional” part of the model to something that resembles socialism (or some other regime, if you prefer) rather than capitalism, and define a sensible notion of equilibrium for your model.
  - (b) Does an equilibrium exist? Is an equilibrium allocation Pareto optimal? Can it be blocked by a coalition?
  - (c) Why/how do you think society might end up being organized as in your model rather than as in the competitive equilibrium model of free markets? What about the converse?
13. The first welfare theorem is sometimes taken to be a “proof” of the superiority of capitalism over other institutional arrangements. Do you agree or not? Explain.

14. We have noted in class that robbery is “welfare neutral” so long as we take Pareto optimality to be our welfare criterion. This might seem to suggest that there is nothing to be said about the following policy question: Should the U.S. government continue to punish thieves or not? Do you agree? Why or why not?
15. Consider a finite-horizon version of the growth model you studied in macro, and assume that: (i) firms own the capital and make investment decisions, and that (ii) the market structure is Arrow-Debreu (i.e., time-0).
  - (a) Map this model into the production economy setup described in the recitation. That is, specify all of the elements that comprise a production economy (the commodity space, set of consumers, their consumption sets, preferences, and so forth) in terms of the model.
  - (b) Demonstrate that under this mapping, the definition of equilibrium for the growth model given in the macro class is equivalent to the definition of competitive equilibria for production economies given in the recitation.
16. Now consider the infinite-horizon version of the growth model. How do you think the production economy model described in the recitation needs to be extended for you to be able to map this version of the growth model into it?
17. Show that in a two-consumer exchange economy, an allocation is in the core if and only if it is both individually rational and Pareto optimal. Can you construct an example of an allocation in a three-consumer economy that is both individually rational and Pareto optimal but is not in the core?
18. Suppose you have constructed a model that looks interesting but it has turned out that the equilibrium allocation is not in the core. Does this change the amount of confidence you have in your model? Explain why or why not. Would you want to condition your answer on the minimum size of the coalitions that can possibly block the equilibrium allocation? If so, describe why.