Problem Set 2

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due November 14

1 Real wage rate and labor supply

a). Consider one period representative consumer problem. Consumer utility function is $U(c,n) = \frac{c^{1-\sigma}}{1-\sigma} + \frac{(1-n)^{1-\sigma}}{1-\sigma}$. Wage rate is w. How does change in w will affect consumer's labor supply decision?

b). Consider economy with identical infinitly lived consumers. They have the same preferences as in 1a. Discount factor is $\beta \in (0,1)$. Production in the economy is given by $y_t = A_t n_t$. How change in A_t will affect consumer's labor supply decision? Relate your answer to the problem 1a.

c). Consider standard growth model with consumer preferences as specified above. What is the relationship between the wage rate and labor supply of representative consumer on the balanced growth path with exogenous labor augmented technological change?

2 Consumption externalities

a). Let $u_1 = ln(c_1) + ln(c_2)$. Let the second consumer have utility function $u_2 = ln(c_2)$. Is the competetive equilibrium Pareto optimal? Now allow gifts. Is the competetive equilibrium with gifts Pareto optimal?

b). Let

$$u_1 = ln(c_1) + ln(c_3)$$

$$u_2 = ln(c_2) + ln(c_3)$$

$$u_3 = ln(c_3)$$

Allow gifts. Is the equilibrium allocation Pareto optimal?

c). Say that consumers have envious preferences if they have utility function of the form

$$U_i(c_i, c_{-i})$$
$$\frac{\partial U_i}{\partial c_i} > 0$$
$$\frac{\partial U_i}{\partial c_{-i}} < 0$$

where c_{-i} denotes consumption of consumers other then i.

Show that if consumers have envious preferences then any equilibrium allocation is Pareto optimal.

d).Consider the following generation of agents:

$$u_t = ln(c_t) + \beta u_{t+1}, \qquad t = 0, 1, \ldots$$

Each agent lives for one period, has endowment e_t in that period and cares about next generation.

Allow agents to give gifts. Then show that the solution to the above problem corresponds to the solution to a certain representative agent problem.

3 Equilibrium with $\beta \geq 1$

Consider social planner problem

$$max \sum_{t=0}^{\infty} \beta^t u(c_t)$$
$$c_t + k_{t+1} \leq f(k_t) + (1-\delta)k_t$$
$$u' > 0, f' > 0$$

.

Find u and f s.t. there is a solution with $\beta \geq 1$.