Permanent Primary Deficits, Idiosyncratic Long-Run Risk, and Growth

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today's talk

- an endogenously growing economy
  - perpetual youth, inelastic labor supply, Cobb-Douglas
  - linear knowledge capital accumulation,
  - which is subject to uninsurable idiosyncratic risk
  - could add physical capital, as in Lucas [1988]

1. in this economy, if markets were *complete instead*, then

  - must have $r - g > 0$ even in OLG setting, unlike Diamond [1965]

2. with incomplete markets,

  - if feasible, permanent primary deficits *cause* $r - g < 0$

3. a well-defined bound on deficits when IES = 1,

  - unfunded baby bonds good for growth, bad for old consumers

4. there may be *no bound* on feasible deficits when IES > 1 (!)

  - can use large deficits to fund baby bonds, not roads
  - large enough deficits will be a Pareto improvement

but, deficits may imply multiple equilibria

- nominal debt without any money
  - as if the Fed pays interest on its money, just like the Treasury
  - unique equilibrium in the surplus case

when IES = 1:

1. two steady states in the deficit case

- one of these is continuous around a balanced budget policy

2. can make this the only equilibrium

- using off-equilibrium plans for fiscal stringency
- a one-time transfer causes an upward jump in the price level

3. deficits cause inflation via low r

- raising nominal interest rates further increases inflation

4. assuming adaptive expectations (following Lucas [1986])

- a unique equilibrium
- converges to the "continuous" perfect-foresight steady state

the Uzawa-AK technology with idiosyncratic risk

- many households, indexed by j
  - perpetual youth, death rate δ
- the aggregate technology for consumption goods is

$$Y_t \;\le\; X_t^{1-\alpha} L^{\alpha}, \qquad X_t \le \int_0^1 X_{j,t} \mathrm{d}t$$

- household j accumulates capital according to

$$\mathrm{d}K_{j,t} \;=\; \left(\mu K_{j,t} - X_{j,t}\right) \mathrm{d}t + \varsigma K_{j,t} \mathrm{d}Z_{j,t} + \mathrm{d}I_{j,t}$$

  - the Brownian motions $Z_{j,t}$ are independent
  - cumulative purchases of capital are $I_{j,t}$
- could include physical capital
  - Lucas [1988], but with labor as an additional fixed factor

stationary allocations

1. the level and growth rate of aggregate consumption

$$X_t = xK_t, \quad g = (1-\alpha)(\mu - x)$$

– a choice between level or growth

2. the growth rate of individual consumption

$$g_y = g + \delta \left(1 - \frac{C_y}{C}\right) < g + \delta$$

– here, C_y/C is newborn/aggregate consumption

– another choice between level or growth

3. risky individual consumption trajectories

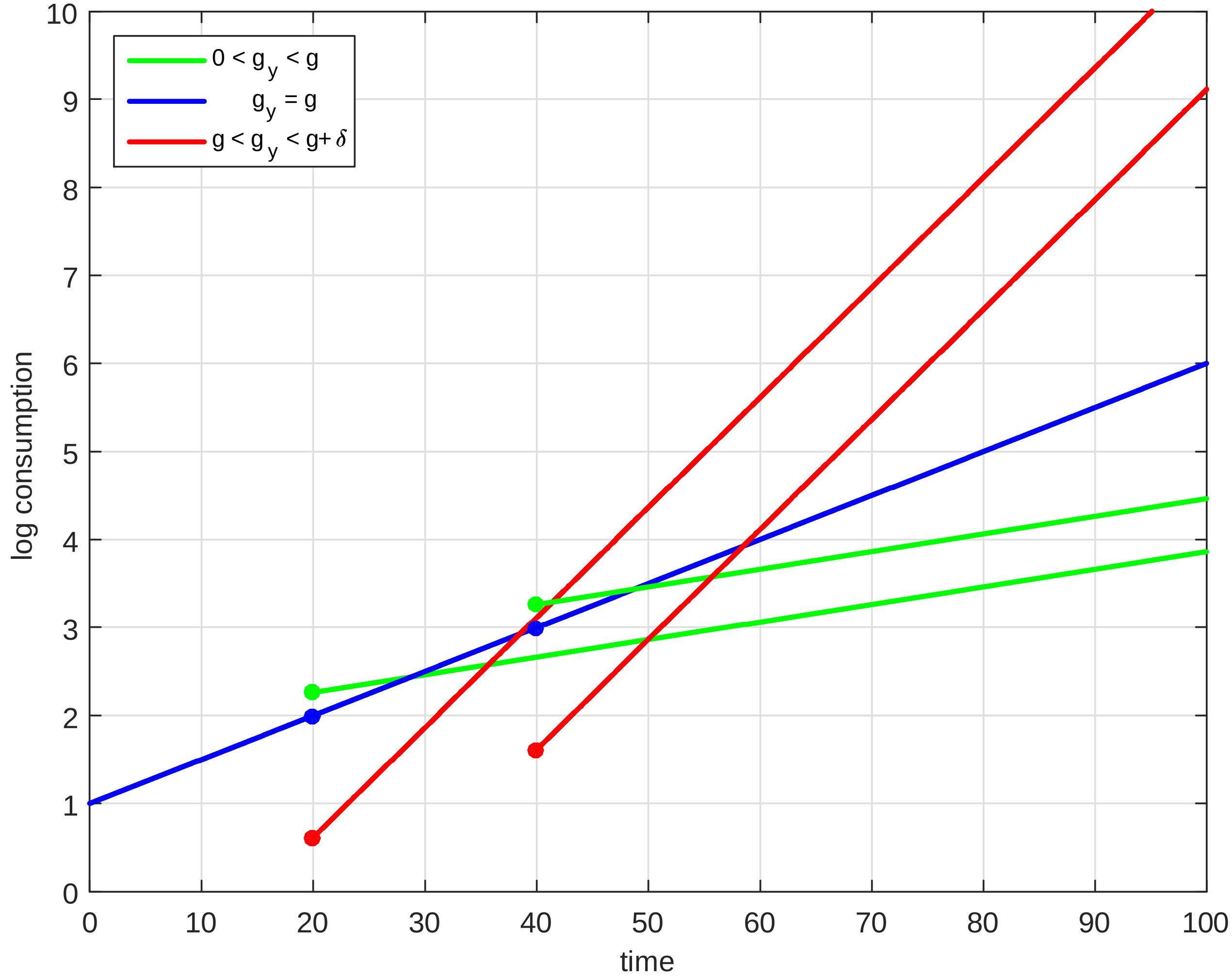
$$\mathrm{d}C_{j,t} = C_{j,t} \left(g_y \mathrm{d}t + \sigma_y \mathrm{d}Z_{j,t}\right),$$

conditional on survival

– an omniscient central planner would set $\sigma_y = 0$

– a large supply of government debt can push in that direction

some feasible trajectories for $\ln(C_t)$ and $\ln(C_{j,t})$



finite utility—risk-free consumption

- consider an infinitely lived consumer
  - suppose $C_t = Ce^{gt}$
  - and utility is

$$\mathcal{U}(C) = \left(\int_0^\infty \rho e^{-\rho t} C_t^{1-1/\varepsilon} \mathrm{d}t\right)^{1/(1-1/\varepsilon)}$$

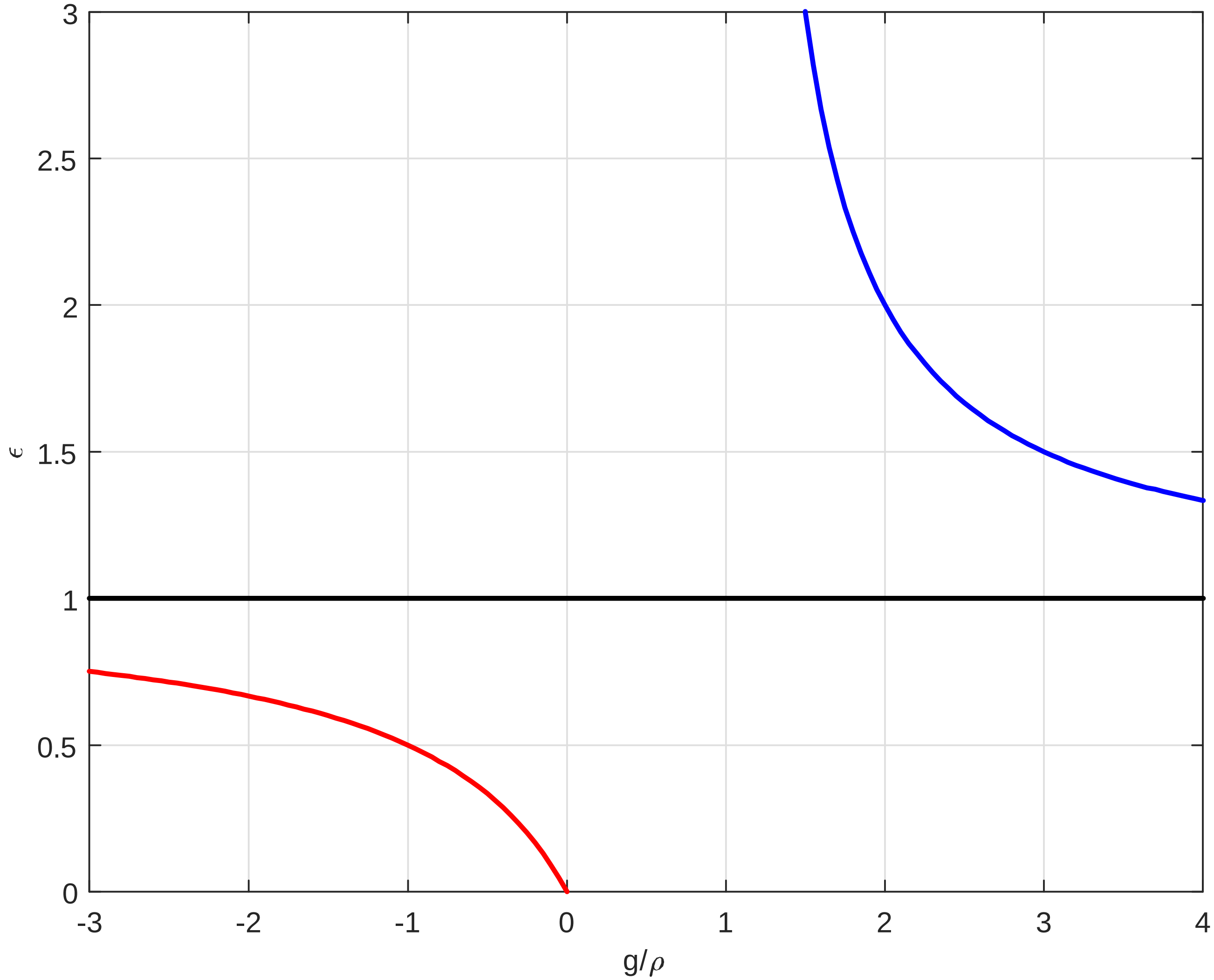
- then $\mathcal{U}(C) \in (0, \infty)$ if and only if $\rho > (1 - 1/\varepsilon)g$
  - requires lower bound on g if $\varepsilon \in (0, 1)$,

$$\frac{1}{1 - 1/\varepsilon} < \frac{g}{\rho}$$

  - requires upper bound on g if $\varepsilon \in (1, \infty)$,

$$\frac{g}{\rho} < \frac{1}{1 - 1/\varepsilon}$$

the boundaries implied by $\left(1 - \frac{1}{\varepsilon}\right) g < \rho$



finite utility—risky consumption

- consider an infinitely-lived consumer j
  - suppose $\mathrm{d}C_t = C_t\left(g\mathrm{d}t + \varsigma \mathrm{d}Z_{j,t}\right)$
  - and utility satisfies

$$\mathrm{d}U_t = U_t \times \left(\mathcal{A}_t U \mathrm{d}t + \mathcal{S}_t U \mathrm{d}Z_{j,t}\right)$$

    where

$$\rho U_t^{1-1/\varepsilon} \;=\; \rho C_t^{1-1/\varepsilon} + \left(1 - \frac{1}{\varepsilon}\right) U_t^{1-1/\varepsilon}\left(\mathcal{A}_t U - \frac{1}{2}\xi\left(\mathcal{S}_t U\right)^2\right)$$

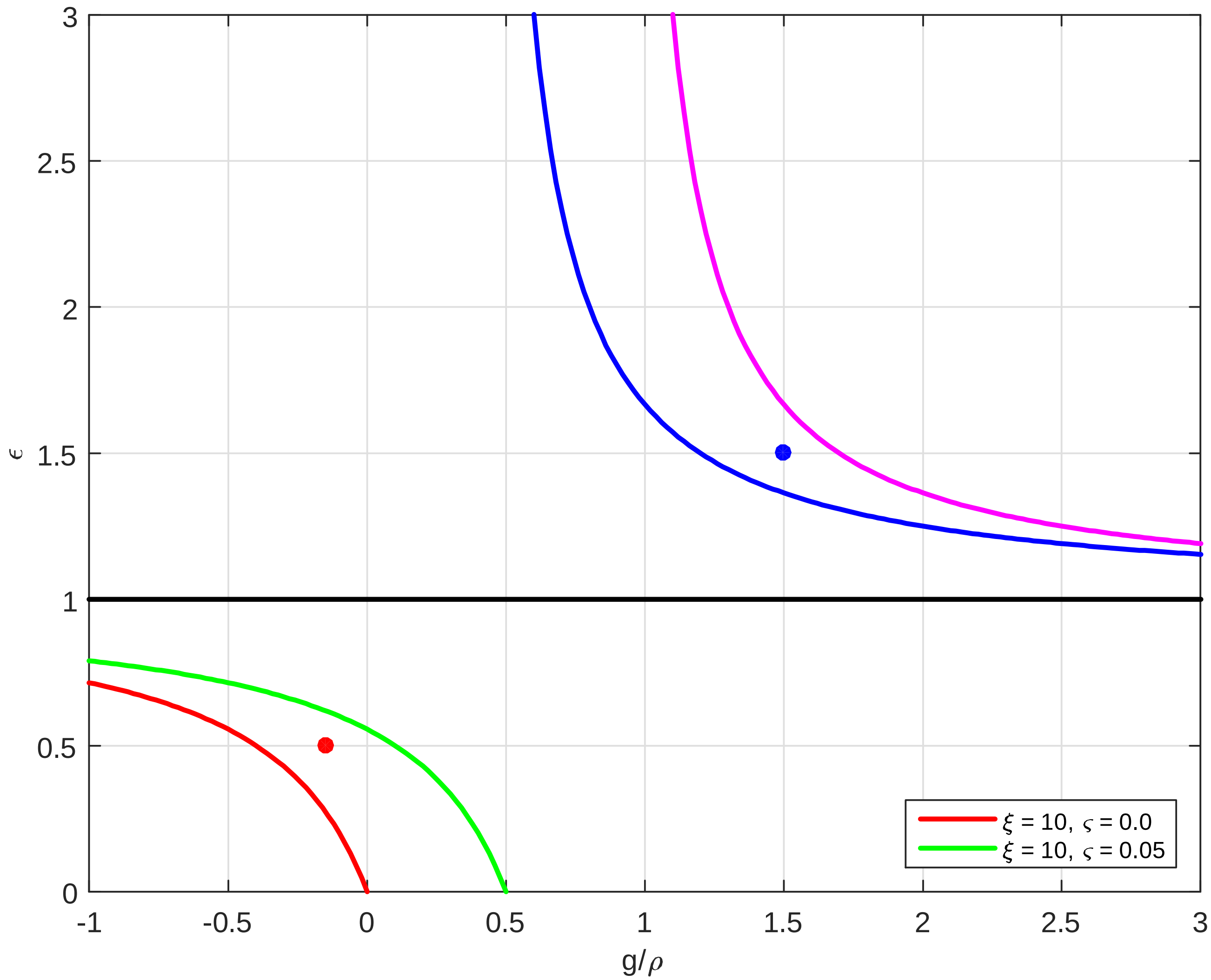
- then $\mathcal{U}\left(C\right) \in \left(0, \infty\right)$ if and only if $\rho > \left(1 - 1/\varepsilon\right)\left(g - \frac{1}{2}\xi\varsigma^2\right)$
  - requires lower bound on g if $\varepsilon \in \left(0, 1\right)$,

$$\frac{1}{1 - 1/\varepsilon} < \frac{1}{\rho}\left(g - \frac{1}{2}\xi\varsigma^2\right)$$

  - requires upper bound on g if $\varepsilon \in \left(1, \infty\right)$,

$$\frac{1}{\rho}\left(g - \frac{1}{2}\xi\varsigma^2\right) < \frac{1}{1 - 1/\varepsilon}$$

incomplete versus complete markets



- red dot: no equilibrium if consumption subject to idiosyncratic risk
- blue dot: large welfare gains from eliminating idiosyncratic risk

TreasuryDirect deposits

- nominal interest rate $i \in \mathbb{R}_+$
- aggregate consumption is C_t
  - government purchases $G_t = \gamma C_t$
  - consumption tax revenues $T_t = \tau C_t$
  - consumption expenditures $E_t = (1+\tau)C_t$
- the "surplus ratio" is defined as

$$\frac{T_t - G_t}{E_t} = 1 - \frac{1+\gamma}{1+\tau} = \mathcal{S}$$

- the producer price of consumption in units of treasury deposits is P_t
- the supply of treasury deposits follows,

$$\mathrm{d}D_t = iD_t\mathrm{d}t - \mathcal{S}P_tE_t\mathrm{d}t$$

- given $\mathrm{d}E_t = g_tE_t\mathrm{d}t$, this implies

$$\mathrm{d}\left(\frac{D_t}{P_tE_t}\right) = (r_t - g_t) \times \left(\frac{D_t}{P_tE_t}\right)\mathrm{d}t - \mathcal{S}\mathrm{d}t \qquad (!)$$

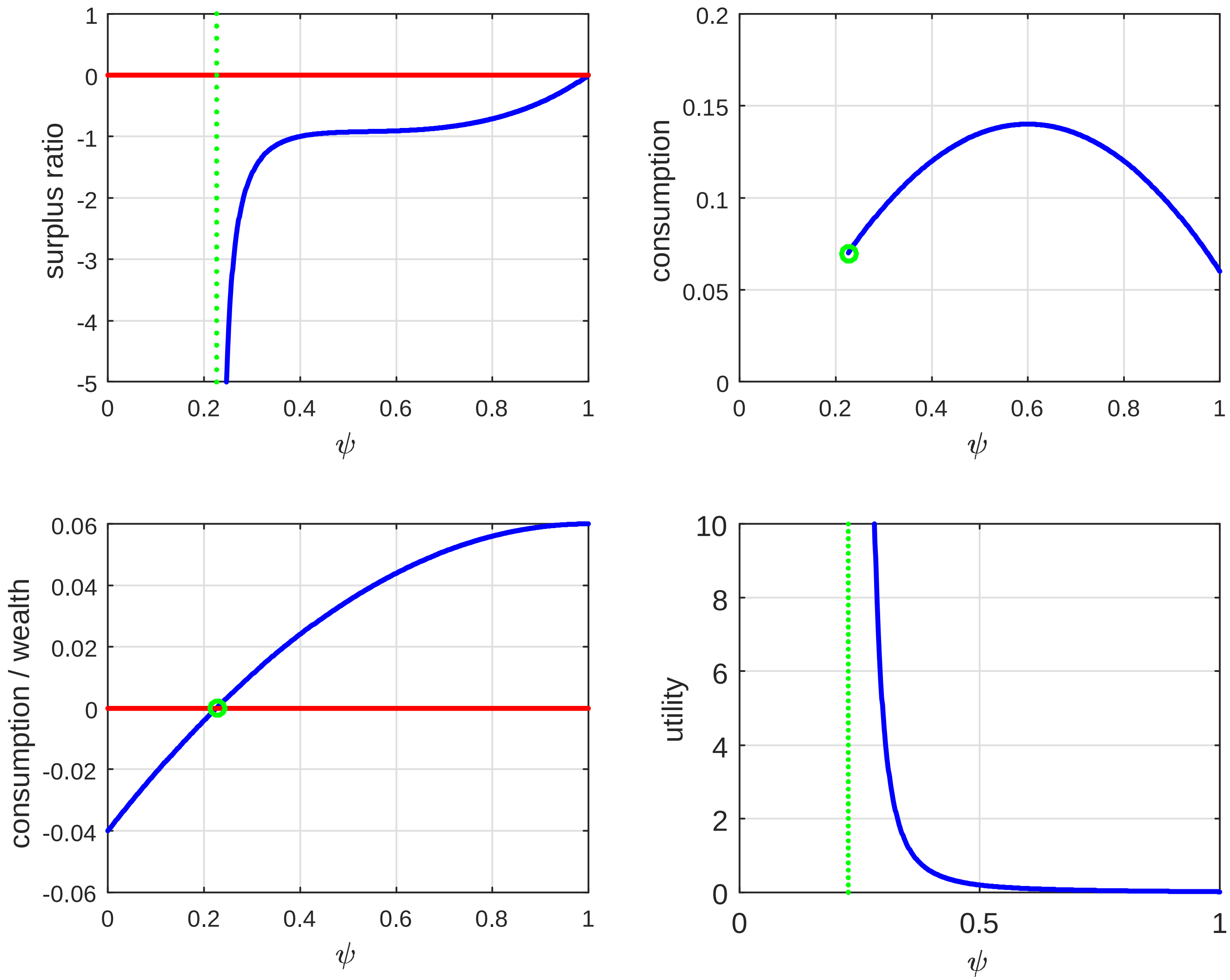
if the economy has a complete markets equilibrium...

- fix γ and take τ large
  - this gives a surplus ratio

$$\mathcal{S} = \frac{T_t - G_t}{E_t} = 1 - \frac{1+\gamma}{1+\tau} \uparrow 1$$

- government securities become a large and safe Lucas tree
- we show that
  - as the consumption tax rate becomes large,
  - the incomplete markets equilibrium converges,
  - to a complete markets equilibrium
- but what if there is no complete markets equilibrium?

backsolving for $\mathcal{S}$ when $\left(1 - \frac{1}{\varepsilon}\right)\mu > \rho$



ψ = portfolio share of risky capital

the economy with $\delta = 0$, $\alpha = 0$, and $\varepsilon = 1$

- household-j real consumption expenditures $E_{j,t}$
- capital accumulation

$$\mathrm{d}K_{j,t} = (\mu K_{j,t} - Y_{j,t})\,\mathrm{d}t + \varsigma K_{j,t}\mathrm{d}Z_{j,t} + \mathrm{d}I_{j,t}$$

  - consumption goods produced $Y_{j,t}$
  - cumulative purchases of capital $I_{j,t}$

- holdings of TreasuryDirect deposits $D_{j,t}$,
  - earn a nominal interest rate $i \in \mathbb{R}_+$
  - the producer price of consumption is P_t
- wealth is

$$W_{j,t} = K_{j,t} + \frac{D_{j,t}}{P_t}$$

- Merton says

$$E_{j,t} = \rho W_{j,t}, \qquad \frac{K_{j,t}}{W_{j,t}} = \psi_{j,t} = \frac{\mu - r_t}{\xi\varsigma^2}$$

  where $r_t = i - (\mathrm{d}P_t/\mathrm{d}t)\,/P_t$

perfect foresight equilibria

- the treasury deposit dynamics, the definition $\psi_t = 1 - (D_t/P_t)/W_t$, and the decision rule $E_t = \rho W_t$, immediately imply

$$\mathrm{d}\psi_t = (\rho\mathcal{S} - (r_t - g_t)(1 - \psi_t))\,\mathrm{d}t \tag{1}$$

- aggregate output and consumption grow at the rate

$$g_t = r_t + \psi_t(\mu - r_t) - \rho$$

  - the decision rule $\psi_t = (\mu - r_t)/(\xi\varsigma^2)$ then implies

$$r_t - g_t = \rho - \xi\varsigma^2 \times \psi_t^2 \tag{2}$$

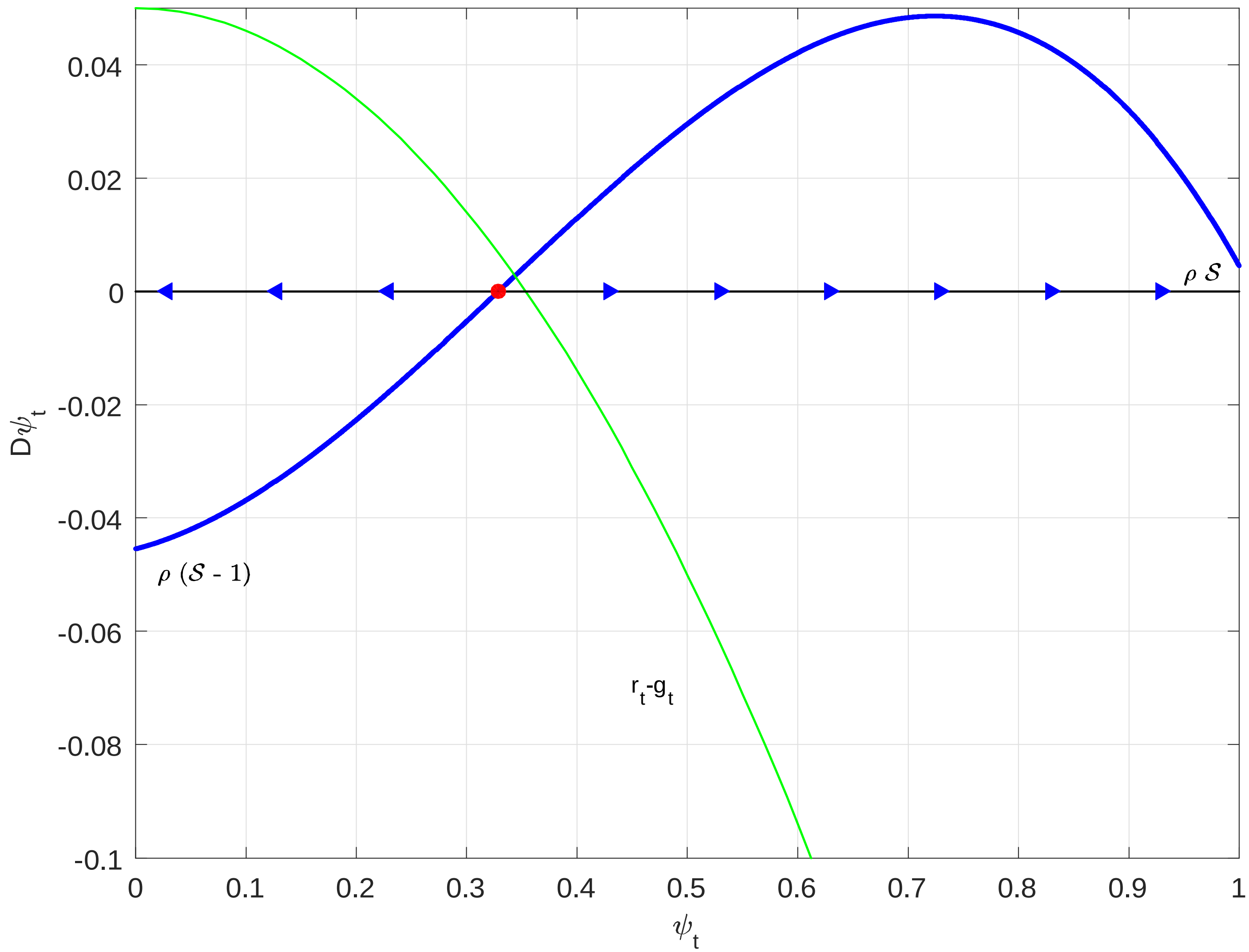
  - so the household Euler conditions aggregate

- combining (1) and (2) gives

$$\mathrm{d}\psi_t = \rho\left(\mathcal{S} - \left(1 - \frac{\xi\varsigma^2}{\rho} \times \psi_t^2\right)(1 - \psi_t)\right)\mathrm{d}t \tag{ODE}$$

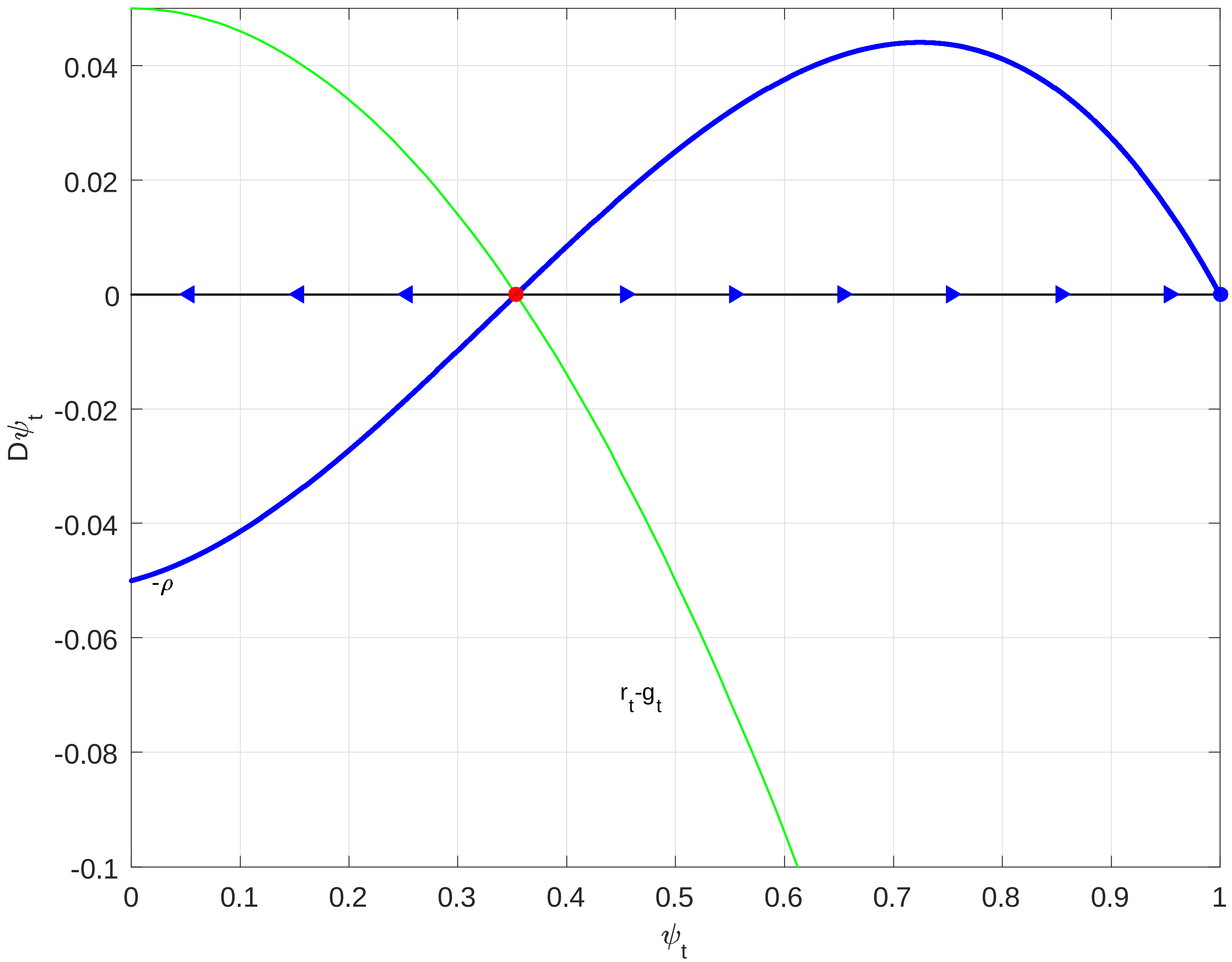
  - the price level follows from $\psi_t = P_t K_t/(P_t K_t + D_t)$
  - there is no initial condition for ψ_t

a unique equilibrium when $\mathcal{S} > 0$



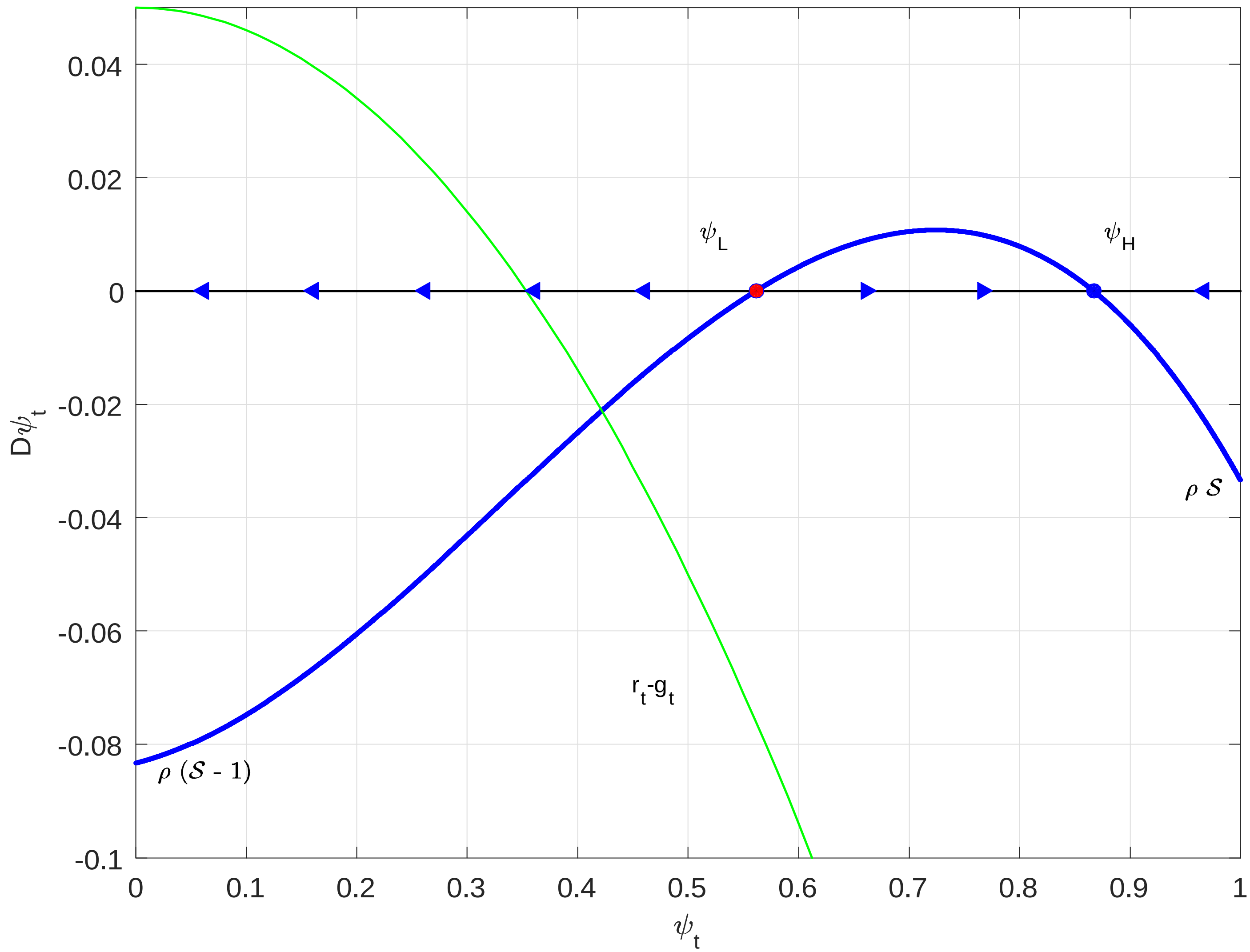
- inflation π_t follows from $\psi_t = (\mu + \pi_t - i)/(\xi\varsigma^2)$.

a continuum of equilibria when $\mathcal{S} = 0$



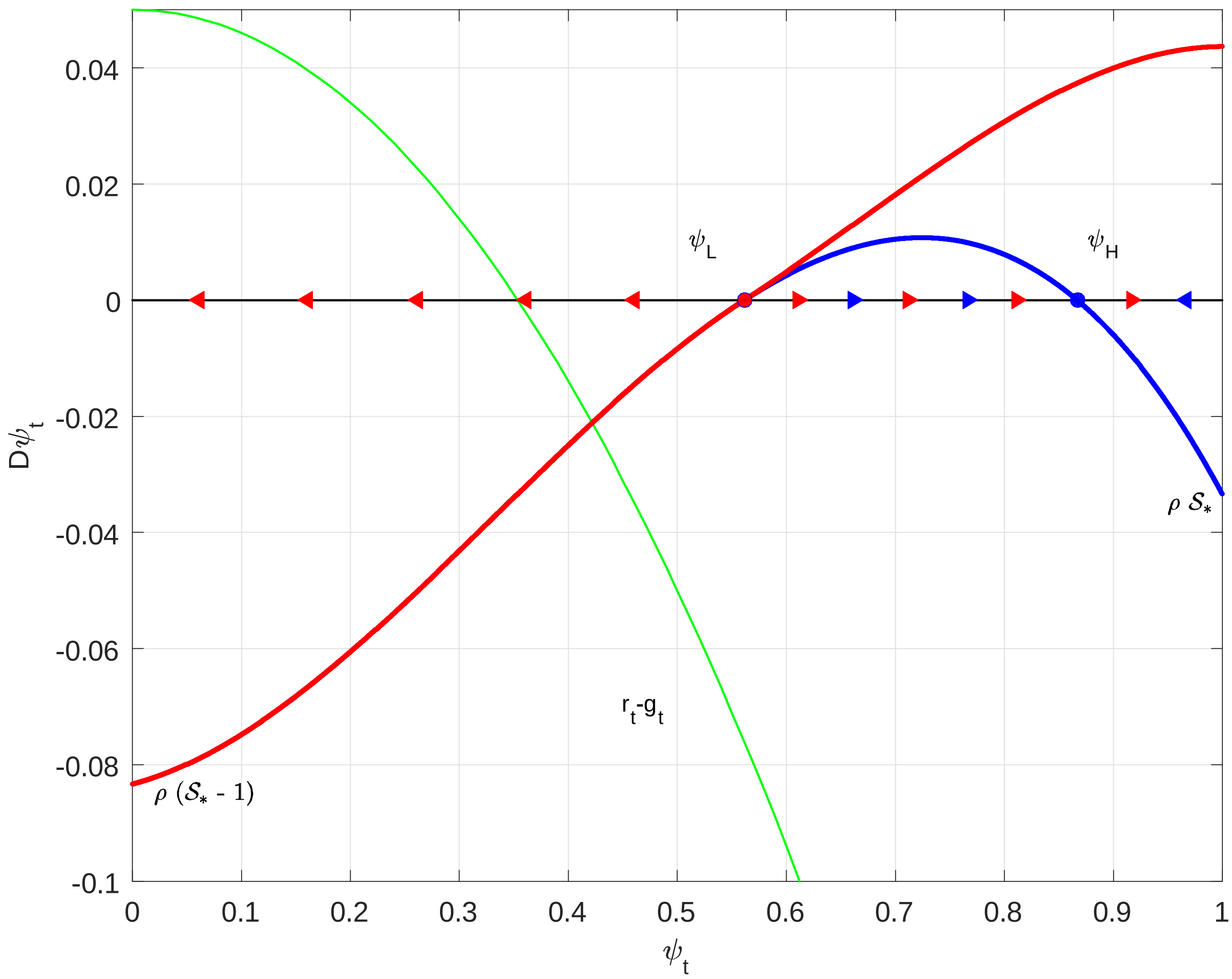
- remember: no initial condition for ψ_t

a continuum of equilibria when $\mathcal{S} < 0$



note that in a steady state: $$\frac{D}{PE} = \frac{\mathcal{S}}{r-g} = \frac{1-\psi}{\rho}$$

a smooth fiscal policy $\mathcal{S}(\cdot)$ that implies a unique equilibrium



$$\mathcal{S}(\psi) = \mathcal{S}_* + B \times (\max\{0, \psi - \psi_L\})^2$$

Adaptive Behavior and Economic Theory

Robert E. Lucas, Jr.

Journal of Business, 1986

Abstract

This essay uses a series of examples to illustrate the use of rationality and adaptation in economic theory. It is argued that these hypotheses are complementary and that stability theories based on adaptive behavior may help to narrow the class of empirically interesting equilibria in certain economic models. An experiment is proposed to test this idea.

adaptive expectations

- decision rules

$$E_{j,t} = \rho W_{j,t}, \qquad \frac{P_t K_{j,t}}{P_t K_{j,t} + D_{j,t}} = \psi_t = \frac{\mu - (i - \pi_t)}{\xi \varsigma^2},$$

  where

$$\mathrm{d}\pi_t = -\lambda \left(\pi_t - \frac{1}{P_t} \frac{\mathrm{d}P_t}{\mathrm{d}t} \right) \mathrm{d}t$$

  starting from some initial beliefs π_0

- the aggregate state variables evolve according to

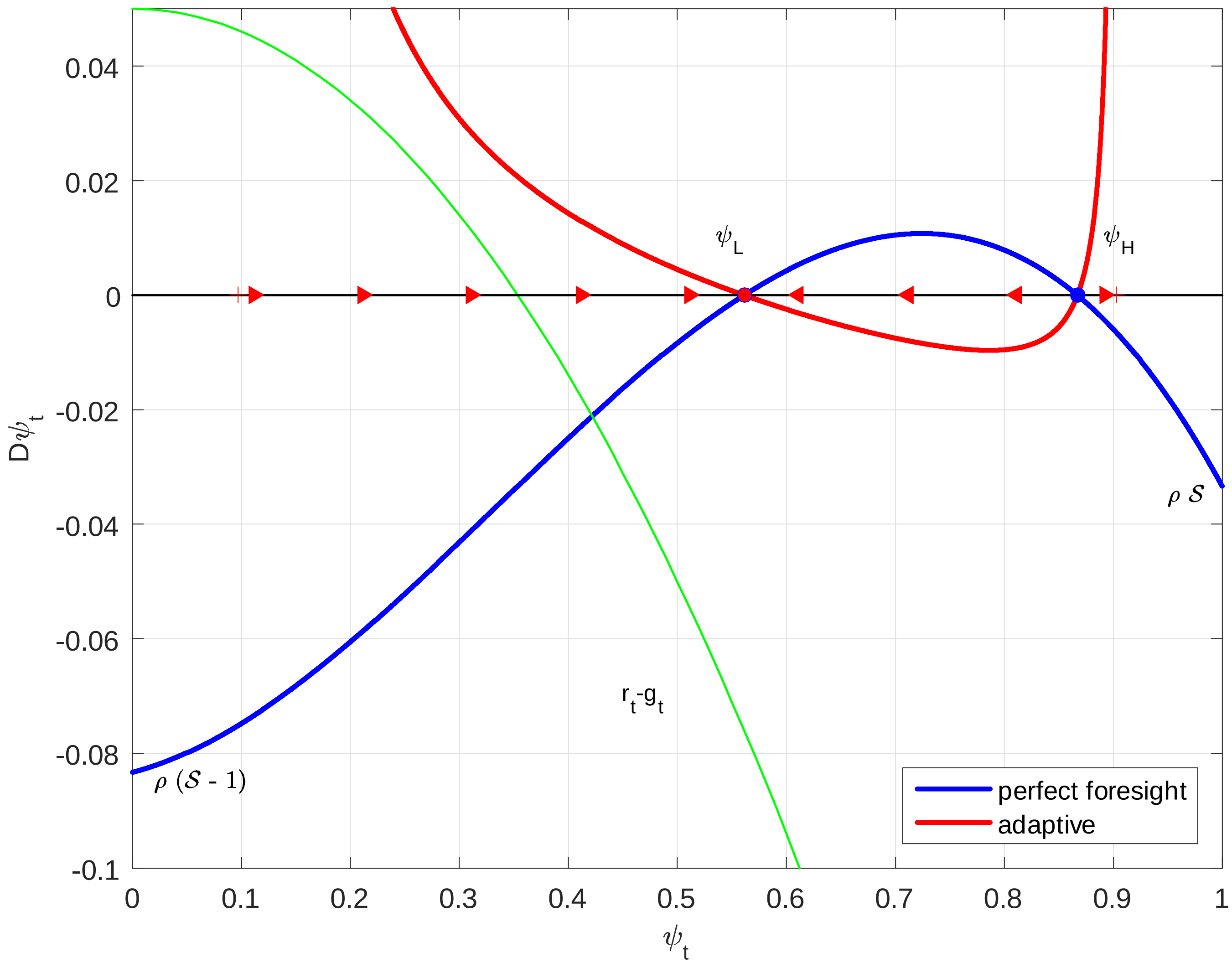
$$\begin{aligned} \mathrm{d}K_t &= (\mu K_t - (1 - \mathcal{S}) E_t) \, \mathrm{d}t \\ \mathrm{d}D_t &= (i D_t - \mathcal{S} P_t E_t) \, \mathrm{d}t \end{aligned}$$

- this implies

$$\frac{\mathrm{d}\psi_t}{\mathrm{d}t} = \rho \times \frac{\mathcal{S} - \left(1 - \frac{\xi\varsigma^2}{\rho} \times \psi_t^2\right)(1 - \psi_t)}{1 - (1 - \psi_t)\psi_t \times \frac{\xi\varsigma^2}{\lambda}}$$

  – when $\lambda > 0$ is small, the sign switches for all intermediate ψ_t

dynamics under adaptive expectations when $(1-\psi_t)\psi_t\xi\varsigma^2 > \lambda$



- the initial beliefs π_0 pin down $\psi_0 = (\mu + \pi_0 - i)/(\xi\varsigma^2)$

adding inelastic labor

- consumption produced using capital and labor

$$Y_t = X_t^{1-\alpha} L^\alpha, \qquad X_t = \int_0^1 X_{j,t} \mathrm{d}t$$

where

$$\mathrm{d}K_{j,t} = (\mu K_{j,t} - X_{j,t})\,\mathrm{d}t + \varsigma K_{j,t} \mathrm{d}Z_{j,t} + \mathrm{d}I_{j,t}$$

- the real price of capital is

$$q_t = (1-\alpha)\left(\frac{X_t}{L}\right)^{-\alpha}$$

  - more growth implies faster economic depreciation

- real wages are $w_t = \alpha Y_t / L$, and so $w_t = we^{gt}$
  - the natural borrowing limit forces $r - g > 0$
  - hence

$$(r-g) \times \frac{D}{PE} = \mathcal{S}$$

    must be positive

▶ market incompleteness is no longer enough to allow $\mathcal{S} < 0$

adding perpetual youth

- die at rate $\delta > 0$
  - perfect annuity markets, natural borrowing limit
- growth rate

$$g = (1-\alpha)(\mu - x), \qquad x = X_t/K_t$$

- Merton says

$$\psi = \frac{1}{\xi\varsigma^2}\left[\left(\mu + \frac{1}{q_t}\frac{\mathrm{d}q_t}{\mathrm{d}t} - g\right) - (r-g)\right]$$

- the "Gordon growth" discount rate for risky capital is

$$\mu + \frac{1}{q_t}\frac{\mathrm{d}q_t}{\mathrm{d}t} - g = \mu - \frac{\alpha g}{1-\alpha} - g = x > 0 \qquad (!)$$

- the definition of ψ implies

$$\psi = \frac{q_t K_t}{W_t} = \frac{qX}{Y} \times \frac{Y}{E} \times \frac{K}{X} \times \frac{E}{W} = (1-\alpha) \times \frac{1+\gamma}{1+\tau} \times \frac{1}{x} \times \frac{E}{W}$$

balanced growth conditions for $\varepsilon = 1$

- government transfers in the form of baby bonds, equal to σY_t
  - the surplus ratio is now

$$\mathcal{S} = 1 - \frac{(1+\gamma)(1+\sigma)}{1+\tau}$$

- the $\varepsilon = 1$ assumption implies $E/W = \rho + \delta$

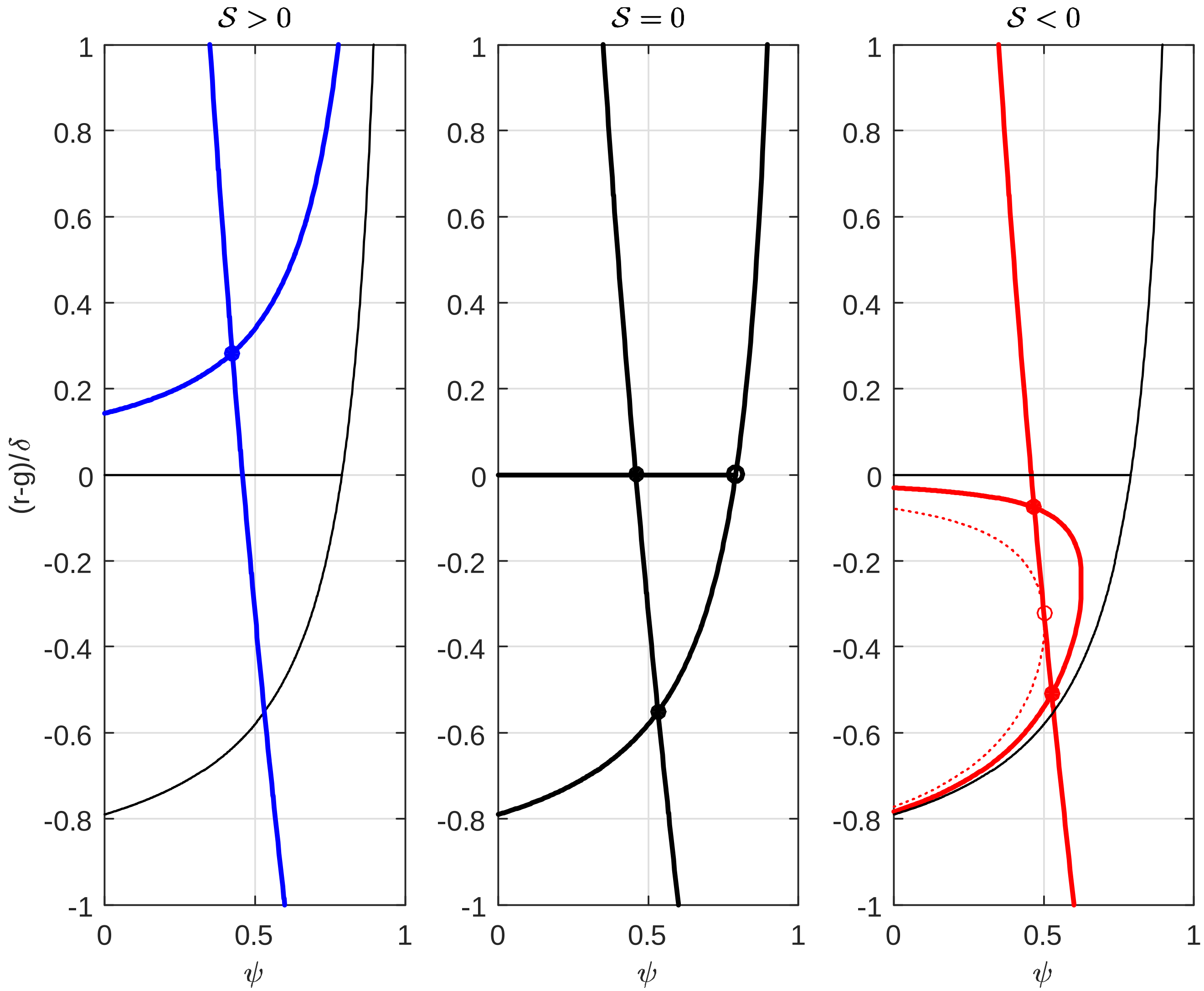
▶ the risky market clearing condition is

$$\frac{\psi}{\rho+\delta} = \frac{1-\alpha}{r-g+\xi\varsigma^2\psi}\frac{1+\gamma}{1+\tau} \tag{1}$$

▶ the risk-free market clearing condition is

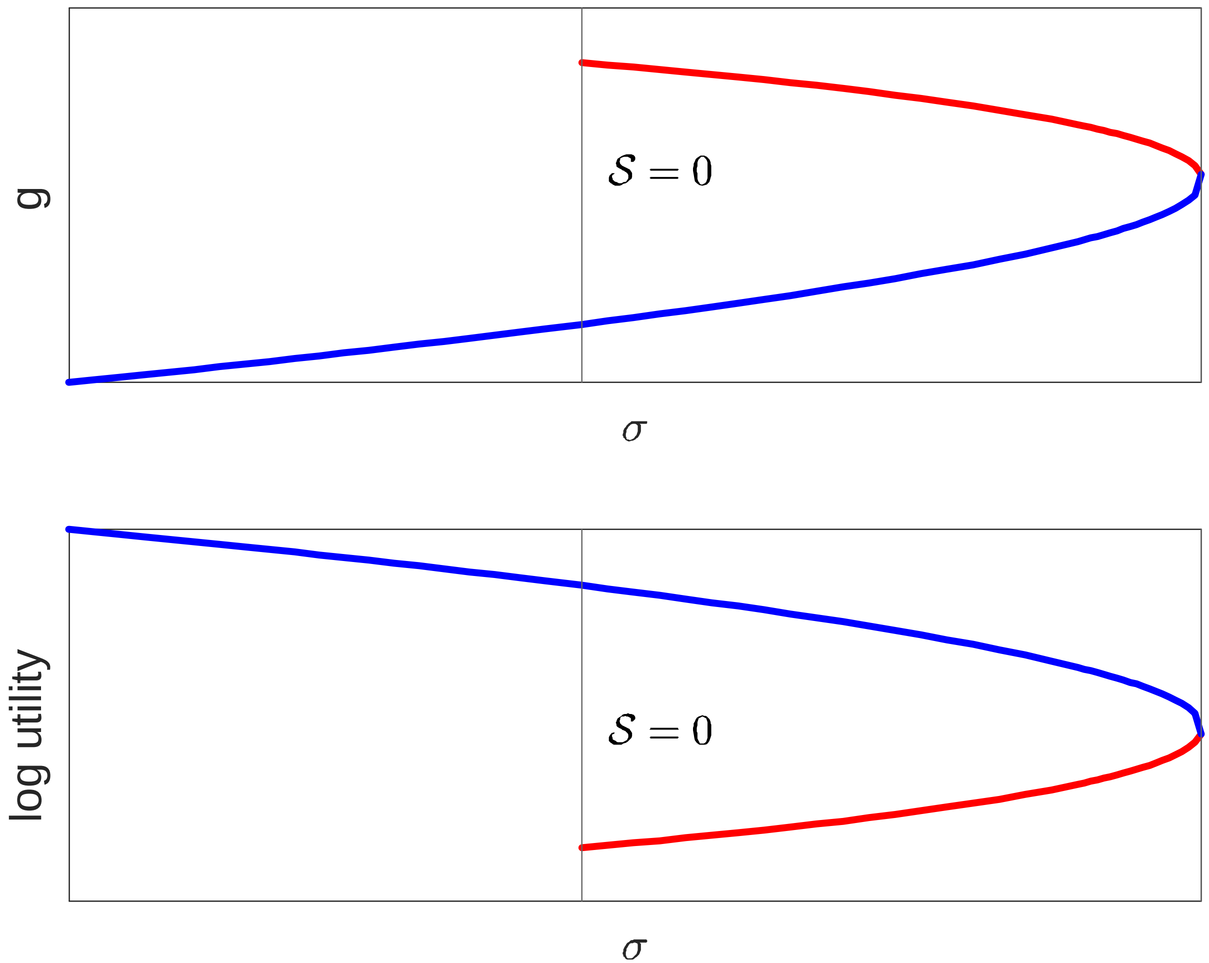
$$\frac{1-\psi}{\rho+\delta} = \frac{\alpha}{r-g+\delta}\frac{1+\gamma}{1+\tau} + \frac{1}{r-g}\left(1 - \frac{(1+\gamma)(1+\sigma)}{1+\tau}\right) \tag{2}$$

alternative deficit scenarios



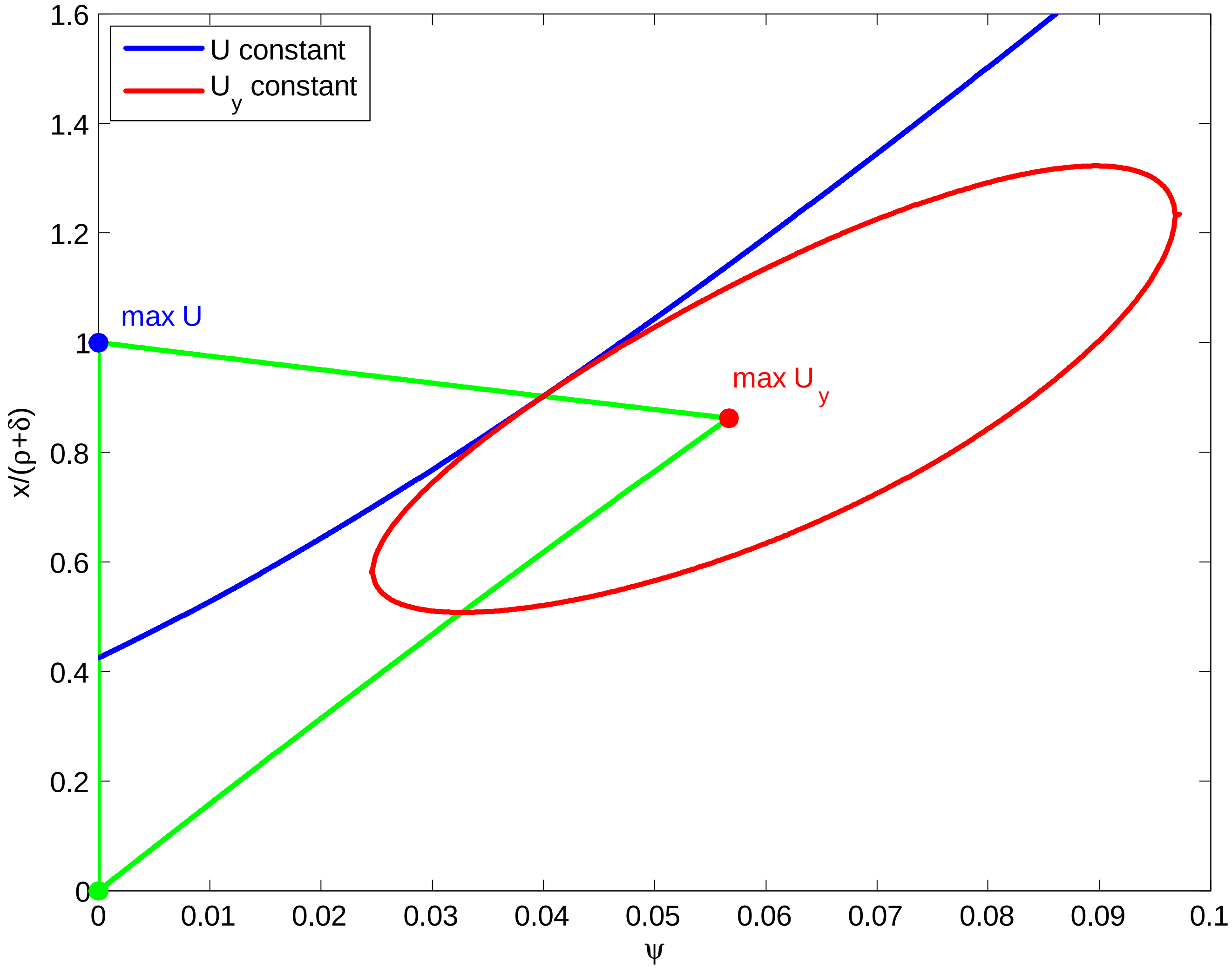
(thin lines for reference only)

varying transfers σY_t of baby bonds



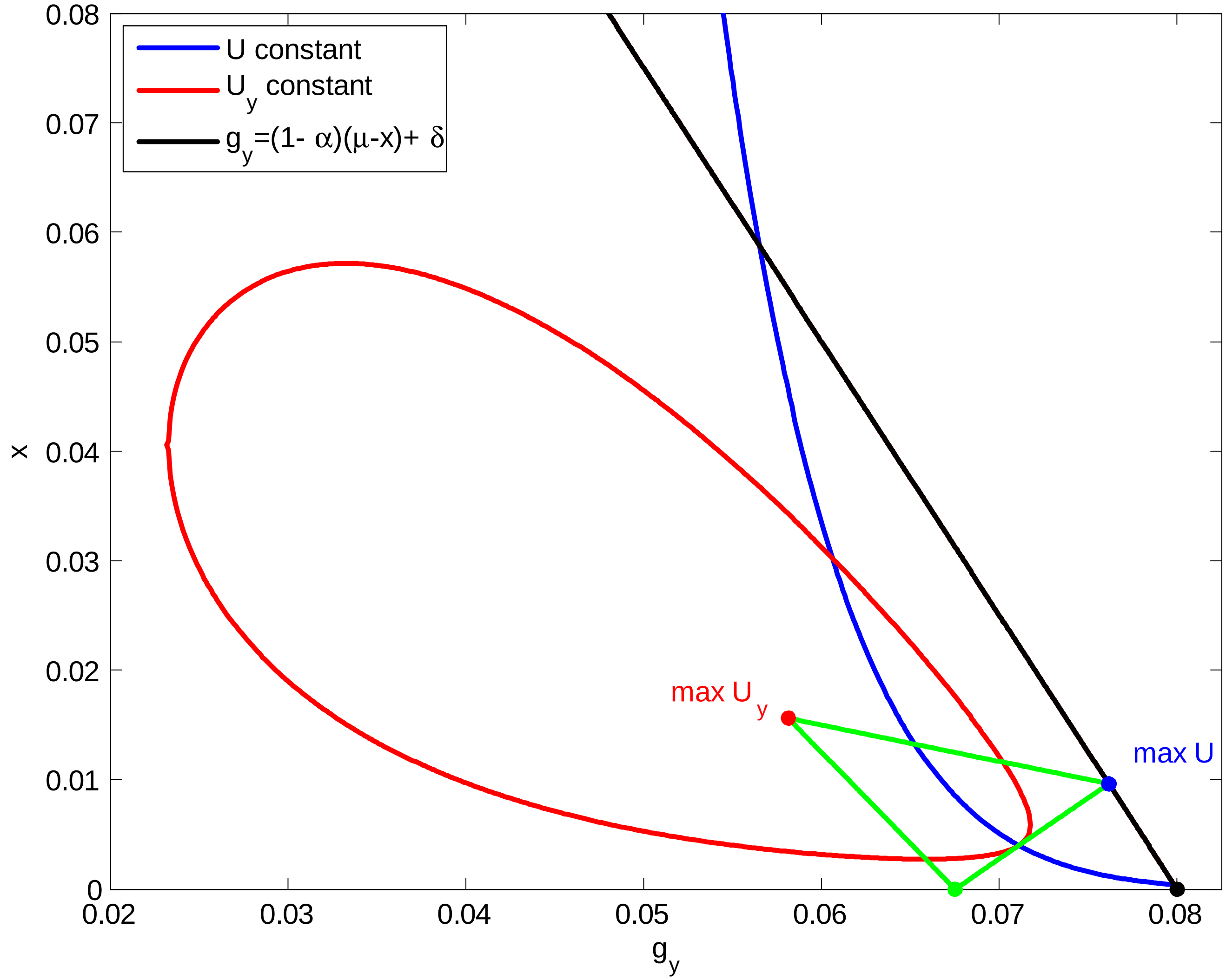
- that is, *our* log utility; and Gen 3023 only really cares about g

varying consumption taxes and baby bonds



- but $\psi > 0$ allocations are Pareto-dominated by $\psi = 0$ allocations
- can add wealth tax to generate Pareto improvements and bring $\psi \downarrow 0$

stationary allocations with perfect risk sharing



main results

1. the government can use large consumption taxes to create a large Lucas tree
   - approximate complete markets equilibrium, and $r - g > 0$
   - but only if the complete markets equilibrium is well defined…
2. if IES $>$ 1 and there is no complete markets equilibrium, then
   - well-defined incomplete markets equilibrium if the economy is not too productive
   - large enough baby bonds will then be a Pareto improvement
   - bounded welfare gains when age capped at some $T < \infty$
3. if IES = 1, then there is a conflict of interest between consumers already alive and future newborn generations
   - deficit-financed baby bonds are good for growth
   - because they lower current consumption
   - consumers already alive do care about current consumption
   - growth is all that matters for Gen 3023