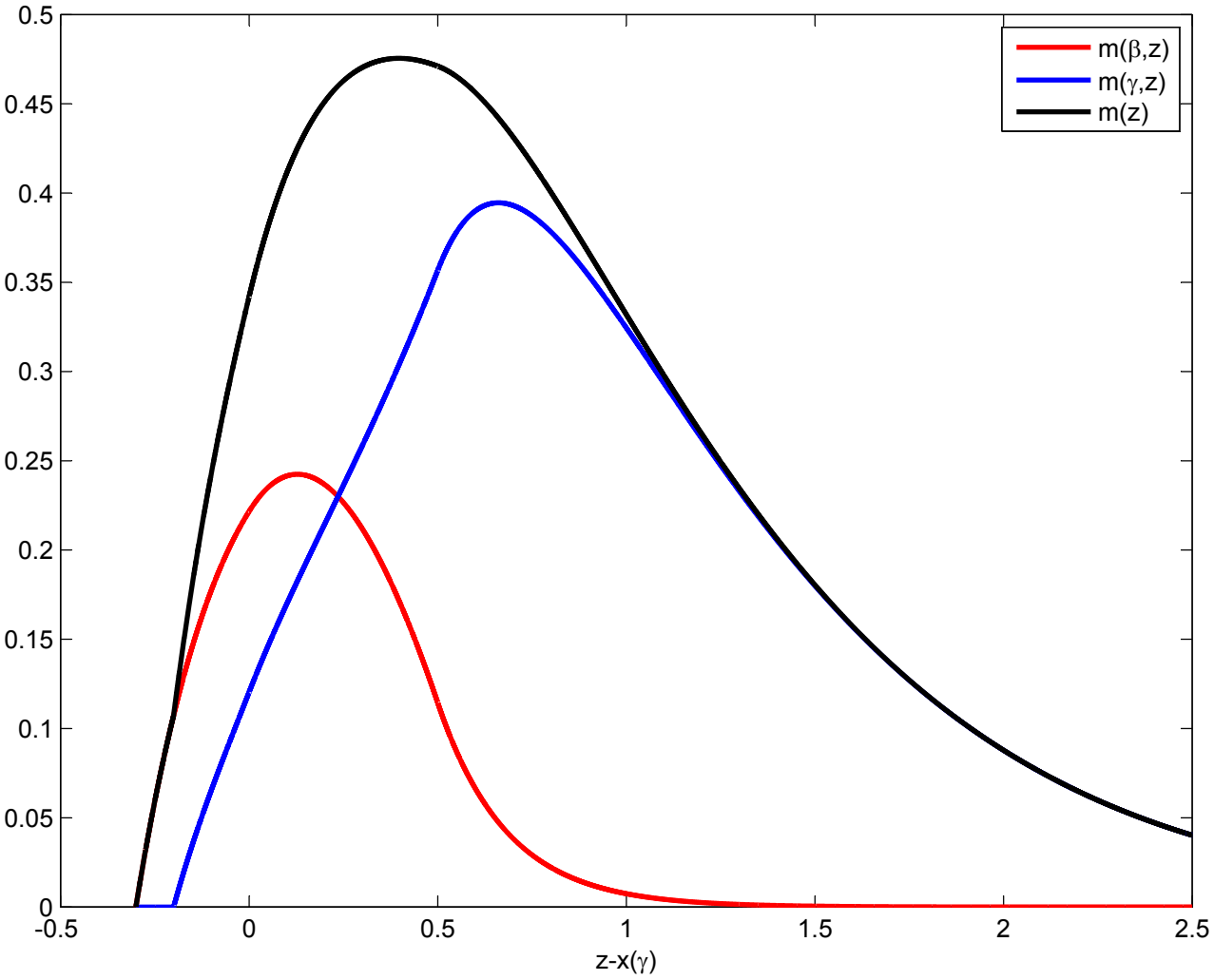
```
%-------------------------------------------------------------------------%
%  SD.m                                                                   %
%-------------------------------------------------------------------------%
% "An Assignment Model of Knowledge Diffusion and Income Inequality"  %
%  Minneapolis Federal Reserve Bank Staff Report 509, April 2015      %
%  Erzo G.J. Luttmer                                                  %
%  University of Minnesota                                            %
%  See: Appendix II at www.luttmer.org                                %
%-------------------------------------------------------------------------%

clear;

%----------------%
% threshold gaps %
%----------------%

TH = 0.30; % x(ga) - b(be) %
OM = 0.20; % x(ga) - b(ga) %
DE = 0.50; %     y - x(ga) %

%------------%
% parameters %
%------------%
de = 0.0400; %
be = 0.0500; %
ga = 0.0600; %
mu = 0.0000; %
si = 0.1000; %
%------------%

ss = si*si;

ze_g =        sqrt((ga-de)/(ss/2));
ze_b = ze_g + sqrt(    ga /(ss/2));

ka = mu + ss*ze_g;

%--------------------------------%
% roots characteristic equations %
%--------------------------------%

ze_p = ze_g+sqrt((ze_g^2)-(ga-de)/(ss/2));
ze_n = ze_g-sqrt((ze_g^2)-(ga-de)/(ss/2));
th_p = ze_g+sqrt((ze_g^2)-(be-de)/(ss/2));
th_n = ze_g-sqrt((ze_g^2)-(be-de)/(ss/2));
xi_p = ze_g+sqrt((ze_g^2)+    de /(ss/2));
xi_n = ze_g-sqrt((ze_g^2)+    de /(ss/2));
om_p = ze_g+sqrt((ze_g^2)+(ga+de)/(ss/2));
om_n = ze_g-sqrt((ze_g^2)+(ga+de)/(ss/2));

%---------%
% A and B %
%---------%

 T = [1 1; th_n th_p];
 X = [1 1; xi_n xi_p];
```

```
 O = [1 1; om_n om_p];

DT = diag([exp(-th_n*TH),exp(-th_p*TH)]);
DD = diag([exp(-th_n*DE),exp(-th_p*DE)]);
DX = diag([exp(-xi_n*DE),exp(-xi_p*DE)]);
DO = diag([exp(-om_n*OM),exp(-om_p*OM)]);

MA = T*DD*inv(T)-X*DX*inv(X);
MA = [T*DT*DD*[1;-1] MA*O*DO*[1;-1]];

A  = inv(MA)*[1;xi_p];

Bb = DT*[1;-1]*A(1)+inv(T)*O*DO*[1;-1]*A(2);
Bg =               inv(X)*O*DO*[1;-1]*A(2);

mDm = T*DD*Bb; % [m(y),-Dm(y)] %

%--------------%
% plot domains %
%--------------%

zb   =  -max(TH,OM):.0001:0   ;
zx   =            0:.0001:DE  ;
zy   =           DE:.0001:DE+2;

%-----------%
% densities %
%-----------%

mb_b = max(A(1)*(exp(-th_n*(zb+TH))-exp(-th_p*(zb+TH))),0);
mb_g = max(A(2)*(exp(-om_n*(zb+OM))-exp(-om_p*(zb+OM))),0);

mx_b = Bb(1)*exp(-th_n*zx) + Bb(2)*exp(-th_p*zx);
mx_g = Bg(1)*exp(-xi_n*zx) + Bg(2)*exp(-xi_p*zx);
mx_b = mx_b - mx_g;

my_b =                                          exp(-xi_p*(zy-DE))       ;
my_g =  mDm(1)*(1 + (ze_g-mDm(2)/mDm(1))*(zy-DE)).*exp(-ze_g*(zy-DE)) - my_b;

%---------------------%
% numbers of managers %
%---------------------%

Nb_A =  A(1)*(((1-exp(-th_n*TH))/th_n) -       ((1-exp(-th_p*TH))/th_p));
Ng_A =  A(2)*(((1-exp(-om_n*OM))/om_n) -       ((1-exp(-om_p*OM))/om_p));

Nb_B = Bb(1)* ((1-exp(-th_n*DE))/th_n) + Bb(2)*((1-exp(-th_p*DE))/th_p) ;
Ng_B = Bg(1)* ((1-exp(-xi_n*DE))/xi_n) + Bg(2)*((1-exp(-xi_p*DE))/xi_p) ;
Nb_B = Nb_B - Ng_B;

Nb_C = 1/xi_p;
Ng_C = (2*mDm(1)-mDm(2)/ze_g)/ze_g;
Ng_C = Ng_C - Nb_C;

Nb = Nb_A + Nb_B + Nb_C;
Ng = Ng_A + Ng_B + Ng_C;
```

```
%--------------------------%
% exit rates at thresholds %
%--------------------------%

er_b = (ss/2)*(th_p-th_n)*A(1)/Nb;
er_g = (ss/2)*(om_p-om_n)*A(2)/Ng;

%--------------------------%
% implied type populations %
%--------------------------%

Mb = Nb*(1+(de+er_b)/be); % lower bound %
Mg = Ng*(1+(de+er_g)/ga); % actual      %

%---------------------------------%
% normalize gamma population to 1 %
%---------------------------------%

mb_b = mb_b/Mg;
mb_g = mb_g/Mg;
mx_b = mx_b/Mg;
mx_g = mx_g/Mg;
my_b = my_b/Mg;
my_g = my_g/Mg;

%----------------%
% plot densities %
%----------------%

figure(1);clf;hold;box on;

plot(zb,mb_b     ,'r','LineWidth',2);
plot(zb,mb_g     ,'b','LineWidth',2);
plot(zb,mb_b+mb_g,'k','LineWidth',2);
legend('m(\beta,z)','m(\gamma,z)','m(z)');

plot(zx,mx_b     ,'r','LineWidth',2);
plot(zx,     mx_g,'b','LineWidth',2);
plot(zx,mx_b+mx_g,'k','LineWidth',2);

plot(zy,my_b     ,'r','LineWidth',2);
plot(zy,     my_g,'b','LineWidth',2);
plot(zy,my_b+my_g,'k','LineWidth',2);

xlabel('z-x(\gamma)');

%---------------%
% print results %
%---------------%

clc;

fprintf('kappa - mu..................... %2.3f%%',     ka-mu); disp([' ']);
fprintf('tail index beta................ %2.3f%%',      ze_b); disp([' ']);
fprintf('tail index gamma............... %2.3f%%',      ze_g); disp([' ']);
```

```
fprintf('exit rate beta.................. %2.3f%',   de+er_b); disp([' ']);
fprintf('exit rate gamma................. %2.3f%',   de+er_g); disp([' ']);
fprintf('beta worker-students............ %2.3f%',(Mb-Nb)/Mg); disp([' ']);
fprintf('beta managers................... %2.3f%',     Nb/Mg); disp([' ']);
fprintf('gamma workers................... %2.3f%',   1-Ng/Mg); disp([' ']);
fprintf('gamma managers.................. %2.3f%',     Ng/Mg); disp([' ']);
```

```
kappa - mu..................... 0.020
tail index beta................ 5.464
tail index gamma............... 2.000
exit rate beta................. 0.080
exit rate gamma................ 0.047
beta worker-students........... 0.263
beta managers.................. 0.165
gamma workers.................. 0.438
gamma managers................. 0.562
>>
```


m(β,z)
m(γ,z)
m(z)
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.5
0
0.5
1
1.5
2
2.5
z-x(γ)