MACROECONOMIC THEORY **T. J. KEHOE**
ECON 8105 **FALL 2014**

MIDTERM EXAMINATION

Answer ***two*** of the following three questions.

1. Consider an economy with two infinitely lived consumers. There is one good in each period. Consumer i, $i = 1, 2$, has the utility function

$$\sum_{t=0}^{\infty} \beta^t \log c_t^i .$$

Here β, $0 < \beta < 1$, is the common discount factor. Each of the consumers is endowed with a sequence of goods:

$$(w_0^1, w_1^1, w_2^1, w_3^1, ...) = (2, 2, 2, 2, ...)$$

$$(w_0^2, w_1^2, w_2^2, w_3^2, ...) = (1, 4, 1, 4, ...) .$$

There is no production or storage.

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium for this economy.

(b) Describe a sequential market structures for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium for this economy.

(c) Carefully state a proposition or propositions that establish the essential equivalence of the equilibrium concept in part a with that in part b. Be sure to specify the relationships between the objects in the Arrow-Debreu equilibrium and those in the sequential markets equilibrium. (You do not have to prove these propositions.)

(d) Calculate the Arrow-Debreu equilibrium for this economy. (This equilibrium is unique up to a normalization of prices, but you do not have to prove this fact.) Use this answer and the answer to part c to calculate the sequential markets equilibrium.

(e) Suppose now that there is a third consumer. Consumer 3 has the same utility function

$$\sum_{t=0}^{\infty} \beta^t \log c_t^3 ,$$

and has the endowment sequence

$$(w_0^3, w_1^3, w_2^3, w_3^3, ...) = (4, 1, 4, 1, ...) .$$

Define a sequential markets equilibrium.

2. Consider an overlapping generations economy in which the representative consumer born in period t, $t = 1, 2, \ldots$, has the utility function over consumption of the single good in periods t and $t+1$

$$u(c_t^t, c_{t+1}^t) = c_t^t + \log c_{t+1}^t$$

and endowments $(w_t^t, w_{t+1}^t) = (w_1, w_2)$. (Notice that the utility function is not $\log c_t^t + \log c_{t+1}^t$.) Suppose that the representative consumer in the initial old generation has the utility function

$$u^0(c_1^0) = \log c_1^0$$

and endowment $w_1^0 = w_2$ of the good in period 1 and endowment m of fiat money.

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium for this economy.

(b) Describe a sequential market structures for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium for this economy.

(c) Suppose that $m = 0$. Calculate both the Arrow-Debreu equilibrium and the sequential markets equilibrium.

(d) Define a Pareto efficient allocation. Suppose that $w_2 > 1$. Is the equilibrium allocation in part c Pareto efficient? Explain carefully why or why not.

(e) Suppose now that there are two types of consumers of equal measure in each generation. The representative consumer of type 1 born in period t, $t = 1, 2, \ldots$, has the utility function over consumption of the single good in periods t and $t+1$

$$u_1(c_t^{1t}, c_{t+1}^{1t}) = c_t^{1t} + \log c_{t+1}^{1t},$$

while the representative consumer of type 2 has the utility function

$$u_2(c_t^{2t}, c_{t+1}^{2t}) = \log c_t^{2t} + c_{t+1}^{2t}.$$

The endowments of these consumers are $(w_t^{it}, w_{t+1}^{it}) = (w_1^i, w_2^i)$, $i = 1, 2$. The representative consumers of type 1 and 2 who live only in period 1 have utility functions $\log c_1^{10}$ and c_1^{20}, endowments $w_1^{10} = w_2^1$ and $w_1^{20} = w_2^2$ of the good in period 1, and endowments m^1 and m^2 of fiat money. Define an Arrow-Debreu equilibrium for this economy.

3. Consider an overlapping generations economy in which the representative consumer in generation t, $t = 1, 2, \ldots,$ has preferences over the consumption of the single good in each of the two periods of her life given by the utility function

$$u(c_t^t, c_{t+1}^t) = \log\ c_t^t + \log\ c_{t+1}^t.$$

This consumer is endowed with quantities of labor $(\ell_t^t, \ell_{t+1}^t) = (\bar{\ell}_1, \bar{\ell}_2)$. In addition, there is a generation 0 who representative consumer lives only in period 1 and has the utility function

$$u^0(c_1^0) = \log\ c_1^0,$$

and the endowment of $\bar{\ell}_2$ units of labor and $\bar{k}_1^0$ units of capital in period 1. In addition, this consumer has an endowment of fiat money m, which can be positive, negative or zero.

The production function is

$$f(k_t, \ell_t) = \theta k_t^{\alpha} \ell_t^{1-\alpha},$$

where $\theta > 1$ and $1 > \alpha > 0$. Capital depreciates at the rate δ per period, $0 \leq \delta \leq 1$.

(a) Define a sequential market equilibrium for this economy.

(b) Assume that consumers own capital and rent it to the firms. Define an Arrow-Debreu equilibrium for this economy. State and prove two theorems that establish the equivalence between a sequential market equilibrium and an Arrow-Debreu equilibrium.

(c) Suppose that the generation of consumers born in period t, $t = 1, 2, \ldots$, live for three periods and have the utility function

$$u(c_t^t, c_{t+1}^t, c_{t+2}^t) = \log\ c_t^t + \log\ c_{t+1}^t + \log\ c_{t+2}^t$$

and labor endowments $(\ell_t^t, \ell_{t+1}^t, \ell_{t+2}^t) = (\bar{\ell}_1, \bar{\ell}_2, \bar{\ell}_3)$. In addition there is an old generation, generation -1, that lives only in period 0 and a middle aged generation, generation 0, that lives in period 1 and 2. The consumers in these generations have utility functions

$$u(c_1^{-1}) = \log\ c_1^{-1}$$

$$u(c_1^0, c_2^0) = \log\ c_1^0 + \log\ c_2^0,$$

endowments of labor $\ell_1^{-1} = \bar{\ell}_3$ and $(\ell_1^0, \ell_2^0) = (\bar{\ell}_2, \bar{\ell}_3)$, endowments of capital $\bar{k}_1^{-1}$ and $\bar{k}_1^0$, and endowments of fiat money m^{-1} and m^0. Define a sequential market equilibrium for this economy.

(d) Assume that consumers own capital and rent it to the firms. Define an Arrow-Debreu equilibrium for the economy in part c.