MACROECONOMIC THEORY **T. J. KEHOE**
ECON 8105 **FALL 2022**

MIDTERM EXAMINATION

Answer ***two*** of the following three questions..

1. Consider an economy with two types of infinitely lived consumers, each a continuum of measure one. There is one good in each period. Consumers of type i, $i=1,2$, have the utility function

$$\sum_{t=0}^{\infty} \beta^t \log c_t^i$$

Here β, $0<\beta<1$, is the common discount factor. Each of type of consumer is endowed with a sequence of goods:

$$(w_0^1, w_1^1, w_2^1, w_3^1, ...) = (3,1,3,1,...)$$
$$(w_0^2, w_1^2, w_2^2, w_3^2, ...) = (1,3,1,3,...)\ .$$

There is no production or storage.

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium for this economy.

(b) Describe a sequential market structures for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium for this economy.

(c) Carefully state a proposition or propositions that establish the essential equivalence of the equilibrium concept in part a with that in part b. Be sure to specify the relationships between the objects in the Arrow-Debreu equilibrium and those in the sequential markets equilibrium.

(d) Calculate the Arrow-Debreu equilibrium for this economy. Use your answer to part c to calculate the sequential markets equilibrium.

(e) Consider a version of this model with production. The consumers have endowments of labor

$$(\bar{\ell}_0^1, \bar{\ell}_1^1, \bar{\ell}_2^1, \bar{\ell}_3^1, ...) = (3,1,3,1,...)$$
$$(\bar{\ell}_0^2, \bar{\ell}_1^2, \bar{\ell}_2^2, \bar{\ell}_3^2, ...) = (1,3,1,3,...)\ .$$

In addition, consumers of type 1 have an endowment of $\bar{k}_0^1$ units of capital in period 0 and consumers of type 2 have an endowment of $\bar{k}_0^2$. Aggregate allocations satisfy the feasibility conditions

$$c_t + k_{t+1} - (1-\delta)k_t \le \theta k_t^{\alpha} \ell_t^{1-\alpha}\ ,$$

where $1 > \alpha, \delta > 0$ and $\theta > 0$. Define a sequential markets equilibrium for this economy.

2. Consider an overlapping generations economy in which the representative consumer born in period $t, t = 1, 2, \ldots,$ has the utility function over consumption of the single good in periods t and $t+1$

$$u(c_t^t, c_{t+1}^t) = \log c_t^t + \log c_{t+1}^t$$

and endowments $(w_t^t, w_{t+1}^t) = (w_1, w_2)$. Suppose that the representative consumer in the initial old generation has the utility function

$$u^0(c_1^0) = \log c_1^0$$

and endowment $w_1^0 = w_2$ of the good in period 1 and endowment m of fiat money.

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium for this economy.

(b) Describe a sequential market structures for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium for this economy.

(c) Suppose that $m = 0$. Calculate both the Arrow-Debreu equilibrium and the sequential markets equilibrium.

(d) Define a Pareto efficient allocation. Suppose that $w_1 > w_2$. Is the equilibrium allocation in part c Pareto efficient? Explain carefully why or why not.

(e) Relax now the assumption that the good is not storable. Suppose instead that 1 unit of the good in period t, $t = 1, 2, \ldots$, can be transformed into $\theta > 0$ units of the good in period $t+1$. Define a sequential markets equilibrium for this economy. Under what conditions will the storage technology be used in equilibrium?

3. Consider an economy with a representative consumer with the utility function

$$\sum_{t=0}^{\infty} \beta^t \log c_t$$

where $0 < \beta < 1$. This consumer has an endowment of $\bar{\ell}_t = 1$ unit of labor in each period and $\bar{k}_0$ units of capital in period 0. Feasible allocation/production plans satisfy

$$c_t + k_{t+1} \leq \theta k_t^{\alpha} \ell_t^{1-\alpha} .$$

(a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium.

(b) Describe a sequential markets structure for this economy, explaining when markets are open, who trades with whom, and so on. Define a sequential markets equilibrium.

(c) Carefully state a proposition or propositions that establish the essential equivalence of the equilibrium concept in part a with that in part b. Be sure to specify the relationships between the objects in the Arrow-Debreu equilibrium and those in the sequential markets equilibrium.

(d) Define a Pareto efficient allocation/production plan. Prove either that an Arrow-Debreu allocation/production plan is Pareto efficient or that a sequential markets allocation/production plan is Pareto efficient.

(e) Write down Bellman's equation that defines the value function for the dynamic programming problem that a Pareto efficient allocation/production plan solves. Explain how you would derive the policy function $k' = g(k)$ from this value function. Guess that the value function has the form $V(k) = a_0 + a_1 \log k$ for some yet-to-be-determined constants a_0 and a_1. Solve for the policy function $k' = g(k)$. Use this value function to calculate the sequential markets equilibrium of this economy.