

PROBLEM SET #1

1. Consider an economy two infinitely lived consumers, each of whom has the same utility function,

$$u(c_0^i, c_1^i, \dots) = \sum_{t=0}^{\infty} \beta^t \log c_t^i,$$

where  $0 < \beta < 1$ . Suppose that consumer 1 has the endowments

$$(w_0^1, w_1^1, w_2^1, w_3^1, \dots) = (1, 3, 1, 3, \dots)$$

and consumer 2 has the endowments

$$(w_0^2, w_1^2, w_2^2, w_3^2, \dots) = (3, 1, 3, 1, \dots)$$

- a) Describe an Arrow-Debreu market structure for this economy, explaining when markets are open, who trades with whom, and so on. Define an Arrow-Debreu equilibrium.
- b) Define a Pareto efficient allocation for this economy. Calculate a Pareto efficient allocation by maximizing a weighted sum of utilities,  $\alpha_1 u_1 + \alpha_2 u_2$ .
- c) Define an Arrow-Debreu equilibrium with transfers. Find the transfer payments necessary to implement the Pareto efficient allocation in part b as equilibrium with transfers. Demonstrate that the transfer payments are homogeneous of degree one in  $(\alpha_1, \alpha_2)$  and sum to 0.
- d) Find the transfer payments necessary to implement the allocation  $(c_t^1, c_t^2) = (2, 2)$  as an equilibrium with transfers.
- e) Calculate the (unique) Arrow-Debreu equilibrium of this economy.
- f) Define a sequential markets equilibrium. Calculate the unique sequential markets equilibrium of the economy.
- g) Redo the calculations in part e for the economy in which

$$(w_0^1, w_1^1, w_2^1, w_3^1, \dots) = (1, 5, 1, 5, \dots)$$

$$(w_0^2, w_1^2, w_2^2, w_3^2, \dots) = (3, 1, 3, 1, \dots).$$

2. Consider a simple overlapping generations economy in which the representative consumer born in period  $t$ ,  $t = 1, 2, \dots$ , has the utility function

$$u(c_t^t, c_{t+1}^t) = c_t^t + \left[ (c_{t+1}^t)^b - 1 \right] / b,$$

where  $b < 1$ . Suppose that his endowment is  $(w_t^t, w_{t+1}^t) = (w_1, w_2)$ .

- a) What is the utility function in the case where  $b = 0$ ? [Hint: use l'Hôpital's rule.]
- b) Write down the utility maximization problem in an environment with Arrow-Debreu markets. Derive the excess demand functions  $y(p_t, p_{t+1})$  and  $z(p_t, p_{t+1})$ . Demonstrate that they are homogeneous of degree zero and that they satisfy Walras's law.
- c) Suppose that the representative consumer in the first generation has the utility function  $u^0(c_1^0) = \left[ (c_1^0)^b - 1 \right] / b$ . This consumer is endowed with  $w_1^0 = w_2$  of the good in period 1 as well as  $m$  units of fiat money, where  $m$  can be positive, negative, or 0. Explain the role of  $m$ . Define an Arrow-Debreu equilibrium of this model. Write down the equilibrium conditions using the excess demand functions.
- d) Find an expression for the offer curve for this model. (Hint: you have to solve for  $y$  as a function of  $z$ .)
- e) Suppose that  $w_1 = 2$  and  $w_2 = 0.75$ . Draw the offer curve for the three cases  $b = 0.5$ ,  $b = 0$ , and  $b = -1$ .
- f) Define a sequential market equilibrium for this economy. Suppose that you have calculated the Arrow-Debreu equilibrium in part d. Explain how you can use the Arrow-Debreu equilibrium to calculate the sequential market equilibrium.