

PROBLEM SET #3

1. Consider an economy in which the equilibrium solves the optimal growth problem

$$\begin{aligned} & \max \sum_{t=0}^{\infty} \beta^t \log C_t / N_t \\ \text{s.t. } & C_t + K_{t+1} - (1 - \delta)K_t \leq (\gamma^{1-\alpha})^t \theta K_t^\alpha N_t^{1-\alpha} \\ & C_t, K_t \geq 0 \\ & K_0 = \bar{K}_0 \\ & N_t = \eta^t N_0. \end{aligned}$$

a) Define a balanced growth path for this economy. Write down conditions that characterize this balanced growth path. Verify that the balanced growth path exhibits characteristics consistent with Kaldor's stylized facts on economic growth.

b) Calibrate the parameters of this economy — β , δ , γ , θ , α , and η — so that the balanced growth path matches as closely as possible the national income and product accounts data of some country not the United States.

2. Find annual time series data on real output, real investment, hours worked, and working age population for some country. If you have sufficient data for other variables, calibrate an annual depreciation rate δ and a capital share α . Otherwise, use the values $\delta = 0.05$ and $\alpha = 0.30$ in what follows.

a) Use the data for real investment to construct a series for the capital stock following the rule

$$\begin{aligned} K_{t+1} &= (1 - \delta)K_t + I_t \\ K_{T_0} &= \bar{K}_{T_0}. \end{aligned}$$

where T_0 is the first year for which you have data on output and investment. Choose \bar{K}_{T_0} so that

$$K_{T_0+1} / K_{T_0} = (K_{T_0+10} / K_{T_0})^{1/10}.$$

b) Repeat part a, but choose \bar{K}_{T_0} so that

$$K_{T_0} / Y_{T_0} = \left(\sum_{t=T_0}^{T_0+9} K_t / Y_t \right) / 10.$$

c) Compare the two series constructed in parts a and b.

d) Perform a growth accounting exercise for this economy. That is, decompose the growth and fluctuation in real GDP per working-age person into three factors, one of which depends on total factor productivity, one of which depends on the capital/output ratio, and the third of which depends on hours worked per working-age person. Discuss what happens during different time periods.

3. Consider the optimal growth problem

$$\begin{aligned} & \max \sum_{t=0}^{\infty} (0.6)^t \log c_t \\ \text{s.t. } & c_t + k_{t+1} \leq 20k_t^{0.3} \\ & c_t, k_t \geq 0 \\ & k_0 = \bar{k}_0. \end{aligned}$$

a) Write down the Euler conditions and the transversality condition for this problem. Calculate the steady state values of c and k .

b) Write down the functional equation that defines the value function for this problem. Guess that the value function has the form $a_0 + a_1 \log k$. Calculate the value function and the policy function. Verify that the policy function generates a path for capital that satisfies the Euler conditions and transversality condition in part a.

c) Let capital take values for the discrete grid (2, 4, 6, 8, 10). Make the original guess $V_0(k) = 0$ for all k , and perform the first three steps of the value function iteration

$$V_{i+1}(k) = \max \log(20k^{0.3} - k') + 0.6V_i(k').$$

d) Perform the value function iterations until

$$\max_k |V_{i+1}(k) - V_i(k)| < 10^{-5}.$$

Report the value function and the policy function that you obtain. Compare these results with what you obtained in part b. (Hint: you probably want to use a computer.)

e) Repeat part d for the grid of capital stocks (0.05, 0.10, ..., 9.95, 10). Compare your answer with those of parts b and d. (Hint: you need to use a computer).

f) Repeat part e for the problem

$$\begin{aligned} & \max \sum_{t=0}^{\infty} (0.6)^t \log c_t \\ \text{s.t. } & c_t + k_{t+1} - 0.5k_t \leq 20k_t^{0.3} \\ & c_t, k_t \geq 0 \\ & k_0 = \bar{k}_0. \end{aligned}$$

(There is now no comparison with an analytical answer to be made, however.)