EXAMINATION

Please answer **three** of the four questions:

1. Consider an economy in which there are two types of goods, primary goods and manufactured goods. Primary goods are homogeneous and are produced using capital and labor services subject to the production function

$$y_0 = k_0^{\alpha_p} \ell_0^{1-\alpha_p}.$$

Manufactured goods are differentiated by firm. There are n firms and the production function for firm j is

$$y_j = \max[\beta k_j^{\alpha_m} \ell_j^{1-\alpha_m} - f, \ 0].$$

where f is the fixed cost. Suppose that $1 > \alpha_p > \alpha_m > 0$.

   Suppose that there is a representative consumer with preferences given by the utility function

$$\log x_0 + (1/\rho)\log \sum_{j=1}^{n} x_j^{\rho}$$

where $1 \geq \rho > 0$. There is an endowment of $\bar{k}$ units of capital and $\bar{\ell}$ units of labor.

a) Suppose that the number of manufacturing firms is variable, that these firms are Cournot competitors, and that there is free entry and exit in manufacturing. Define an (autarkic) equilibrium. Explain carefully how you would calculate this equilibrium (You do not need to calculate it.)

b) Suppose now that there are two such countries, one with endowments $(\bar{k}^1, \ \bar{\ell}^1)$ and and the other with endowments $(\bar{k}^2, \ \bar{\ell}^2)$, but otherwise identical. Define a trade equilibrium.

c) Suppose that $\bar{k}^1 / \bar{\ell}^1 > \bar{k}^2 / \bar{\ell}^2$. Explain what changes you would expect to see in prices, average output levels, and utility levels as these two countries, initially in autarky, open to trade. Explain carefully what patterns of specialization are possible and what pattern of trade you would expect to see.

2. Consider a world with two countries. There is a representative consumer in each country who has preferences over the interval of goods $X = [0, 1]$ given by the utility function

$$\int_X \log c(x)\,dx.$$

In each country there is a single factor, labor. Endowments are $\bar{\ell}_1 = \bar{\ell}_2 = \bar{\ell}$. Production functions are linear but differ across countries:

$$y_j(x) = \ell_j(x)/a_j(x).$$

Here $y_j(x)$ is the amount of good x produced in country j; $\ell_j(x)$ is the amount of labor used in this production; and $a_j(x)$ is the unit labor required, given by

$$a_1(x) = e^x$$

$$a_2(x) = e^{1-x}.$$

a) Define a competitive equilibrium of the world economy.

b) Characterize as much as possible the patterns of specialization and trade in the unique competitive equilibrium

c) Suppose now that there are tariffs, so that the price paid by a consumer in county i for a good x imported from country j, $i \neq j$, is

$$p^i(x) = (1+\tau)a_j(x)w_j$$

where τ is the *ad valorem* tariff rate and w_j is the wage rate in country j. Tariff revenues are redistributed to consumers in a lump-sum form. Explain how your definition of equilibrium is altered and characterize as much as possible how the equilibrium in the world with tariffs differs from the equilibrium in the world without tariffs.

3. Consider a two-sector growth model in which the representative consumer has the utility function

$$\int_0^\infty e^{-\rho t} \log(c_1^b + c_2^b)^{1/b} dt.$$

and in which investment is produced according to

$$\dot{k} + \delta k = (x_1^b + x_2^b)^{1/b}.$$

Feasible consumption/investment plans satisfy the feasibility constraints

$$c_1 + x_1 = f_1(k_1, \ell_1) = k_1$$
$$c_2 + x_2 = f_2(k_2, \ell_2) = \ell_2 .$$

The initial value of $k(t)$ is $k(0)$; $\ell(t)$ is fixed at 1.

a) Define a competitive equilibrium.

b) Reduce the equilibrium conditions to two differential equations in k and $z = c/k$ and a transversality condition. Here $c = (c_1^b + c_2^b)^{1/b}$ is aggregate consumption.

c) Draw a phase diagram in (k, z) space illustrating the equilibrium path for any given $k(0)$. (You will have to consider different cases depending on the value of b; you can do one carefully for a case where $b < 0$ and just sketch out the other(s).)

d) Consider the case where $b < 0$ and suppose that there is a world of many such countries, each with a different population $\overline{\ell}_j$ and a different initial capital-labor ratio $k_j(0)$. Define a competitive equilibrium for the world economy.

e) Consider a country whose initial capital-labor ratio $k_j(0)$ is much lower than the initial worldwide capital-labor ratio $k(0)$, which is in turn less than the steady state capital-labor ratio. Explain how the development path for this economy depends crucially on whether it is open or closed to trade. (You can assume that the relation

$$\frac{k_j(t)}{k(t)} - 1 = \frac{z(t)}{z(0)} \left(\frac{k_j(0)}{k(0)} - 1 \right)$$

holds in the trade equilibrium without proving that it does.)

4. [You need a calculator to answer this question.] Consider two countries, one with a higher output per worker than the other. In the more developed country, the level of output per worker in 1990 is 40,000 (measured in Summers-Heston 1985 international dollars). The level of capital per worker is 90,000. Over the five-year period 1985-1990 the average *ex post* real interest rate on bank loans to prime level business borrowers was 5 per cent per year. In this country the share of GDP earned by capital is 0.3 and capital depreciates at a rate of 5 percent per year.

In the less developed country output per worker is 20,000. The level of capital per worker is 30,000. Over the period 1985-1990 the real interest rate averaged 10 percent per year.

a) Consider a simple, one sector model in which output in each country is produced using a Cobb-Douglas production function. Argue that differences in output per worker cannot be explained by differences in capital per worker alone. To do this, use two different sets of calculations, one in which you use the information on capital per worker directly, and the other in which you use differences in real interest rates to infer differences in capital per worker.

b) Calibrate production Cobb-Douglas functions for each country that have different total factor productivity parameters but are otherwise identical and that are consistent with the observed data on output per worker and capital per worker.

c) Suppose that the two countries, which have been closed to investment flows, now open themselves. Both the developed and the less developed country have 50 million workers. Calculate the impact of this opening on both countries.

d) Discuss some of the limitations of the analysis in parts a, b, and c. In your discussion pay some attention to the differences in the calculations that utilize capital stock data and those that use differences in real interest rates to infer differences in capital stocks. Indicate how you could alter the analysis to eliminate or lessen these limitations.