INTERNATIONAL TRADE AND PAYMENTS THEORY **T. J. KEHOE**
ECON 8401 **FALL 2009**

EXAMINATION

Please answer **two** of the three questions.

You can consult class notes, working papers, and articles while you are working on the exam, but you are asked not to discuss the exam with anyone until the exam period is finished for everyone.

1. Consider a two-sector growth model in which the representative consumer has the utility function

$$\sum_{t=0}^{\infty} \beta^t \log(c_{1t}^{a_1} c_{2t}^{a_2}).$$

The investment good is produced according to

$$k_{t+1} = d x_{1t}^{a_1} x_{2t}^{a_2}.$$

Feasible consumption/investment plans satisfy the feasibility constraints

$$c_{1t} + x_{1t} = \phi_1(k_{1t}, \ell_{1t}) = k_{1t}$$
$$c_{2t} + x_{2t} = \phi_2(k_{2t}, \ell_{2t}) = \ell_{2t},$$

where

$$k_{1t} + k_{2t} = k_t$$
$$\ell_{1t} + \ell_{2t} = 1.$$

The initial value of k_t is $\bar{k}_0$. All of the variables specified above are in per capita terms. There is a measure L of consumer/workers.

a) Define an equilibrium for this economy.

b) Write out a social planner's problem for this economy. Explain how solution to this social planner's problem is related to that of the one-sector social planner's problem

$$\sum_{t=0}^{\infty} \beta^t \log c_t$$
$$\text{s.t. } c_t + k_{t+1} = d k_t^{a_1}$$
$$c_t, k_t \geq 0$$
$$k_0 = \bar{k}_0.$$

[You can write done a proposition or propositions without providing a proof or proofs, but be sure to carefully relate the variables in the two-sector model to the variables in the one-sector model.]

c) Solve the one-sector social planner's problem in part b. [Recall that the policy function for investment has the form $k_{t+1}(k_t) = Adk^{a_1}$ where A is a constant that you remember or can determine with a bit of algebra and calculus.]

d) Suppose now that there is a world made up of n different countries, all with the same technologies and preferences, but with different constant populations, L^i, and with different initial capital-labor ratios $\bar{k}_0^i$. Suppose that goods 1 and 2 can be freely traded across countries, but that the investment good cannot be traded. Suppose too that there is no international borrowing. Define an equilibrium for the world economy.

e) State and prove versions of the factor price equalization theorem, the Stolper-Samuelson theorem, the Rybczynski theorem, and the Heckscher-Ohlin theorem for this particular world economy.

f) Let $s_t = c_t / y_t$ where $y_t = p_{1t}k_t + p_{2t} = dk_t^{a_1}$ is world GDP per capita. Transform the first-order conditions for the one-sector social planner's problem in part b into two difference equations in k_t and s_t. Use the first-order conditions for the consumer's problem of the equilibrium in part d to show that

$$\frac{y_t^i - y_t}{y_t} = \frac{s_t}{s_{t-1}}\left(\frac{y_{t-1}^i - y_{t-1}}{y_{t-1}}\right) = \frac{s_t}{s_0}\left(\frac{y_0^i - y_0}{y_0}\right).$$

g) Use the solution to the one-sector social planner's problem in part c to solve for s_t. Discuss the economic significance of the result that you obtain.

2. Consider an economy in which the consumption space is the set of functions $c: R_+ \times R_+ \to R_+$. In $c(x,t)$ the index x denotes the type of good and the index t denotes the date at which it is consumed. An individual consumer has preferences given by the functional

$$u(c) = \int_0^\infty e^{-\rho t}\left[\int_0^\infty \log(c(x,t)+1)dx\right]dt.$$

Goods are produced using a single factor of production, labor:

$$y(x,t) = \ell(x,t) / a(x,t).$$

Each consumer has an endowment of labor equal to 1, and the total number of consumers is fixed at $\bar{\ell}$. The unit labor requirement $a(x,t)$ is bounded from below, $a(x,t) > \bar{a}(x)$, where

$$\bar{a}(x) = e^{-x}.$$

At $t = 0$ there is a $z(0) > 0$ such that $a(x,0) = e^{-x}$ for all $x < z(0)$ and that $a(x,0) = e^{x-2z(0)}$ for all $x \geq z(0)$. There is learning by doing of the form

$$\frac{\dot{a}(x,t)}{a(x,t)} = \begin{cases} -\int_0^\infty b(v,t)\ell(v,t)dv & \text{if } a(x,t) > \bar{a}(x) \\ 0 & \text{if } a(x,t) = \bar{a}(x) \end{cases}.$$

Here $\dot{a}(x,t)$ denotes the partial derivative of $a(x,t)$ with respect to t and $b(v,t) = b > 0$ if $a(v,t) > \bar{a}(v)$ and $b(v,t) = 0$ if $a(v,t) = \bar{a}(v)$. There is no borrowing or lending, and there is no storage.

a) Provide a motivation for the both the utility function and the production technology described above.

b) Define an equilibrium for this economy. Characterize the equilibrium as much as possible.

c) Consider now a two country world in which the two countries are identical except in their endowments of labor and their initial technology levels. In particular, $z^1(0) > z^2(0)$. There is no borrowing or lending across countries. Define an equilibrium for this economy.

d) Suppose that $z^1(t) > z^2(t)$. Explain carefully and illustrate two of the five qualitatively different possible equilibrium configurations for production, consumption, and trade at time t. (To make things easy, assume that $z^1(t)$ and $z^2(t)$ are sufficiently large so that good $x = 0$ is not produced in equilibrium.)

e) Briefly describe the dynamics of this model, explaining the crucial role played by the sizes of the two countries, $\bar{\ell}^1$ and $\bar{\ell}^2$.

3. Consider an economy where the consumers have Dixit-Stiglitz utility functions and solve the problem

$$\max \; (1-\alpha)\log c_0 + \frac{\alpha}{\rho}\log\int_0^m c(z)^\rho dz$$
$$\text{s.t. } p_0 c_0 + \int_0^m p(z)c(z)dz = w\bar{\ell} + \pi$$
$$c(z) \geq 0.$$

Here $1 > \alpha > 0$ and $1 > \rho > 0$. Suppose that good 0 is produced with the constant-returns production function $y_0 = \ell_0$.

a) Suppose that the producer of good z takes the prices $p(z')$, for $z' \neq z$, as given. Suppose too that this producer has the production function

$$y(z) = \max\left[x(z)\left(\ell(z) - f\right), 0\right].$$

where $x(z) > 0$ is the firm's productivity level and $f > 0$. Solve the firm's profit maximization problem to derive an optimal pricing rule.

b) Suppose that good 0 is produced with the constant-returns production function $y_0 = \ell_0$ Suppose that $\mu > 0$ is the fixed measure of potential firms and that firm productivities are distributed on the interval $x \geq 1$ according to the Pareto distribution with distribution function

$$F(x) = 1 - x^{-\gamma},$$

where $\gamma > 2$ and $\gamma > \rho/(1-\rho)$. Define an equilibrium for this economy.

c) Suppose that the parameters are such that, in equilibrium, not all firms produce. Find an expression for the productivity of the least productive firm that produces. That is, find an $\bar{x} > 1$ such that no firm with $x(z) < \bar{x}$ produces and all firms with $x(z) \geq \bar{x}$ produce.

d) Suppose now that there are two symmetric countries that engage in free trade. Each country i, $i = 1, 2$, has a population of $\bar{\ell}$ and a measure of potential firms of μ. Firms' productivities are again distributed according to the Pareto distribution, $F(x) = 1 - x^{-\gamma}$. A firm in country i faces a fixed cost of exporting to country j, $j \neq i$, of f_e where $f_e > f_d = f$ and an iceberg transportation cost of $\tau - 1 \geq 0$. Define an equilibrium for this world economy.

e) Suppose again that, in equilibrium, not all firms produce. Explain how to characterize the equilibrium production patterns with a cutoff value, or values, as in part c. [You should explain carefully how to calculate any cutoff values, but you to not actually need to calculate it.] Compare this value, or these values, with that in part c.

f) Briefly discuss the strengths and limitations of this sort of model for accounting for firm-level data on exports.