

PROBLEM SET #1

1. Consider an economy in which there are two types of goods, agriculture and manufactured goods. Agricultural goods are homogeneous and are produced using labor according to the constant returns to scale production function

$$y_0 = \ell_0.$$

Manufactured goods are differentiated by firm. The production function for firm  $j$  is

$$y_j = (1/b) \max[\ell_j - f, 0].$$

Here  $f$  is the fixed cost, in terms of labor, necessary to operate the firm and  $b$  is the unit labor requirement. Suppose that there is a representative consumer with preferences

$$\log x_0 + (1/\rho) \log \sum_{j=1}^n x_j^\rho$$

where  $\rho > 0$ . There is an endowment of  $\bar{\ell}$  units of labor

a) Define a monopolistically competitive equilibrium for this economy in which firms follow Cournot pricing rules and there is free entry and exit.

b) Suppose that  $b = 1$ ,  $f = 3$ ,  $\rho = 1/2$ , and  $\bar{\ell} = 50$ . Calculate the autarky equilibrium.

c) Suppose now that  $\bar{\ell} = 200$ . Calculate the equilibrium.

d) Interpret the equilibrium in part c as a trading equilibrium among two countries, one with  $\bar{\ell} = 50$  and the second with  $\bar{\ell} = 150$ . Assume that production of the homogeneous good is distributed proportionally across the two countries. What impact does trade have on the number of manufacturing firms in each country? The average output of firms? The total number of products available? Consumer utility? Illustrate the efficiency gains using an average cost curve diagram.

e) Repeat the analysis of parts a-d in a model in which consumers have the utility function

$$\log x_0 + (1/\rho) \log \int_0^n x(j)^\rho dj.$$

Here there is a continuum  $[0, n]$  of differentiated goods. (Hint: You need to be very careful in taking derivatives when solving the firms' profit maximization problems. In particular, the answers change drastically.)

f) Now suppose that production of the agricultural good is governed by the function

$$y_0 = \ell_0^\alpha t_0^{1-\alpha}.$$

Here  $t_0$  denotes inputs of land. Suppose now that there are two such countries, one with endowments  $(\bar{\ell}^1, \bar{t}^1)$  and the other with endowments  $(\bar{\ell}^2, \bar{t}^2)$ , but otherwise identical. Define a trade equilibrium.

g) Suppose that in the model of part f  $\bar{\ell}^1 / \bar{t}^1 \neq \bar{\ell}^2 / \bar{t}^2$ . Explain what changes you would expect to see in prices, average output levels, and utility levels as these two countries, initially in autarky, open to trade. Explain carefully what patterns of specialization are possible and what pattern of trade you would expect to see.

2. Consider a world with two countries. The representative consumer in each country has the utility function

$$u(x_1^i, x_2^i) = \log x_1^i + \log x_2^i, \quad i = 1, 2.$$

This consumer is endowed with capital and labor in the amounts  $(\bar{k}^i, \bar{\ell}^i)$ . The production technologies in the two countries are identical.

$$y_j^i = \min[k_j^i / a_{Kj}, \ell_j^i / a_{Lj}], \quad i, j = 1, 2.$$

To make things simple assume that  $a_{K1} = a_{L2} = 1$  and that  $a_{K2} = a_{L1} = 2$ .

- a) Define an autarky equilibrium for country  $i$ . Under what conditions on  $(\bar{k}^i, \bar{\ell}^i)$  are both of the equilibrium factor prices positive?
- b) Define a free trade equilibrium. Under what conditions on  $(\bar{k}^1, \bar{\ell}^1)$  and  $(\bar{k}^2, \bar{\ell}^2)$  do both countries produce both goods?
- c) State and prove a version of the factor price equalization theorem for this particular world economy.
- d) State and prove a version of the Stolper-Samuelson theorem.
- e) State and prove a version of the Rybczynski theorem.

f) State and prove a version of the Heckscher-Ohlin theorem.

3. Consider a two sector growth model in which the representative consumer has the utility function

$$\int_0^{\infty} e^{-\rho t} \log(c_1^b + c_2^b)^{1/b} dt.$$

and in which investment is produced according to

$$\dot{k} + \delta k = (x_1^b + x_2^b)^{1/b}.$$

Feasible consumption/investment plans satisfy the feasibility constraints

$$\begin{aligned} c_1 + x_1 &= f_1(k_1, \ell_1) = k_1 \\ c_2 + x_2 &= f_2(k_2, \ell_2) = \ell_2. \end{aligned}$$

where

$$\begin{aligned} k_1 + k_2 &= k \\ \ell_1 + \ell_2 &= \ell. \end{aligned}$$

The initial value of  $k(t)$  is  $\bar{k}_0$ .  $\ell(t)$  is fixed at  $\bar{\ell}$ .

a) Carefully define a competitive equilibrium for this economy.

b) Reduce the equilibrium conditions to two differential equations in  $k$  and  $z = c/k$  and a transversality condition. Here  $c = (c_1^b + c_2^b)^{1/b}$  is aggregate consumption. Draw phase diagrams illustrating the different possible equilibrium paths for any  $\bar{k}_0 > 0$ .

c) Suppose now that there is a world made up of  $m$  different countries all with the same technologies and preferences, but different endowments,  $\bar{\ell}^j$  and  $\bar{k}_0^j$ . Suppose that there is no international borrowing or lending and there are no international capital flows. Define an equilibrium for the world economy. Prove that in this equilibrium the variables

$$c_i(t) = \sum_{j=1}^m c_i^j(t), \quad k(t) = \sum_{j=1}^m k^j(t), \quad p_i(t), \quad r(t), \quad \text{and} \quad w(t)$$

satisfy the equilibrium conditions for the equilibrium in part a where  $\bar{k}_0 = \sum_{j=1}^m \bar{k}_0^j$ ,  $\bar{\ell} = \sum_{j=1}^m \bar{\ell}^j$ .

d) State and prove versions of the factor price equalization theorem, the Stolper-Samuelson theorem, the Rybczynski theorem, and the Heckscher-Ohlin theorem for this particular world economy.

e) Derive equations that govern the relationship between  $k^j(t)$  and  $k(t)$  and that between  $y^j(t) = p_1(0)k^j(t) + p_2(0)\bar{\ell}^j$  and  $y(t)$ . Explain why it is more difficult to derive the relationship between  $p_1(t)k^j(t) + p_2(t)\bar{\ell}^j$  and  $p_1(t)k(t) + p_2(t)\bar{\ell}$ .

f) What effects do the assumptions of no international borrowing and lending and no international capital flows have on your analysis? Try to be precise about which variables are uniquely determined in the equilibrium with borrowing and lending and/or capital flows are which would not be uniquely determined.

4. Repeat parts a and b of question 3 for an economy where the production technologies take the form

$$f_1(k_1, \ell_1) = \theta_1 \ell_1^{1-\alpha_1} k_1^{\alpha_1}$$

$$f_2(k_2, \ell_2) = \theta_2 \ell_2^{1-\alpha_2} k_2^{\alpha_2} .$$

Where does the integrated equilibrium approach of part c break down? What impact would assuming international borrowing and lending or capital flows have on your analysis?