INTERNATIONAL TRADE AND PAYMENTS THEORY **T. J. KEHOE**
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PROBLEM SET #1

1. Consider a world with two countries. The representative consumer in each country has the utility function

$$u(c_1^i,\ c_2^i) = \log c_1^i + \log c_2^i,\ i = 1,2 .$$

This consumer is endowed with capital and labor in the amounts $(\bar{k}^i, \bar{\ell}^i)$. The production technologies in the two countries are identical.

$$y_j^i = \min[k_j^i / a_{Kj}, \ell_j^i / a_{Lj}],\ i, j = 1,2 .$$

To make things simple assume that $a_{K1} = a_{L2} = 1$ and that $a_{K2} = a_{L1} = 2$.

a) Define an autarky equilibrium for country *i*. Under what conditions on $(\bar{k}^i, \bar{\ell}^i)$ are both of the equilibrium factor prices positive? Under what conditions on $(\bar{k}^i, \bar{\ell}^i)$ are all of the factors fully employed in the sense that $k_j^i / a_{Kj} = \ell_j^i / a_{Lj}$, $j = 1,2$, $k_1^i + k_2^i = \bar{k}^i$, and $\ell_1^i + \ell_2^i = \bar{\ell}^i$?

b) Define a free trade equilibrium. Under what conditions on $(\bar{k}^1, \bar{\ell}^1)$ and $(\bar{k}^2, \bar{\ell}^2)$ do both countries produce both goods?

c) State and prove a version of the factor price equalization theorem for this particular world economy. [Hint: The proofs of this theorem and those in parts d-f are very simple. You should provide careful statements of the theorems. In particular you should be careful about the restrictions on parameters needed for the theorem to hold.]

d) State and prove a version of the Stolper-Samuelson theorem.

e) State and prove a version of the Rybczynski theorem.

f) State and prove a version of the Heckscher-Ohlin theorem.

2. Consider an economy in which there are two types of goods, agriculture and manufactured goods. Agricultural goods are homogeneous and are produced using labor according to the constant returns to scale production function

$$y_0 = \ell_0.$$

Manufactured goods are differentiated by firm. The production function for firm *j* is

$$y_j = (1/b)\max[\ell_j - f, 0].$$

Here f is the fixed cost, in terms of labor, necessary to operate the firm and b is the unit labor requirement. Suppose that there is a representative consumer with preferences

$$\log c_0 + (1/\rho)\log \sum\nolimits_{j=1}^{n} c_j^{\rho} ,$$

where $1 \geq \rho > 0$. There is an endowment of $\bar{\ell}$ units of labor

a) Define a monopolistically competitive equilibrium for this economy in which firms follow Cournot pricing rules and there is free entry and exit.

b) Suppose that $b = 1$, $f = 3$, $\rho = 1/2$, and $\bar{\ell} = 50$. Calculate the autarky equilibrium.

c) Suppose now that $\bar{\ell} = 200$. Calculate the equilibrium.

d) Interpret the equilibrium in part c as a trading equilibrium among two countries, one with $\bar{\ell}^1 = 50$ and the second with $\bar{\ell}^2 = 150$. Assume that production of the homogeneous good is distributed proportionally across the two countries. What impact does trade have on the number of manufacturing firms in each country? The average output of firms? The total number of products available? Consumer utility and real income? Illustrate the efficiency gains using an average cost curve diagram.

3. Repeat the analysis of question 2 for two variants of the model. Make sure to compare the gains from trade in these two alternative models with those in the model in question 2.

a) Suppose that consumers have the utility function

$$\log c_0 + (1/\rho)\log \int_0^n c(j)^{\rho}\, dj.$$

Here there is a continuum $[0, n]$ of differentiated goods. (Hint: You need to be very careful in taking derivatives when solving the firms' profit maximization problems. In particular, the answers change drastically.)

b) Suppose that there are again a finite number of differentiated goods but that firms are now Bertrand competitors, rather than Cournot competitors.

4. Consider an economy similar to that in question 2. Suppose that production of the agricultural good is governed by the function

$$y_0 = \ell_0^{\alpha} t_0^{1-\alpha}.$$

Here t_0 denotes inputs of land. Suppose now that there are two such countries, one with endowments $(\bar{\ell}^1, \bar{t}^1)$ and the other with endowments $(\bar{\ell}^2, \bar{t}^2)$, but otherwise identical.

a) Define a trade equilibrium.

b) Suppose that $\bar{\ell}^1 / \bar{t}^1 \neq \bar{\ell}^2 / \bar{t}^2$. Explain what changes you would expect to see in prices, number of firms, average output levels, and utility levels as these two countries, initially in autarky, open to trade. Explain carefully what patterns of specialization are possible and what pattern of trade you would expect to see.

5. Consider an economy in which there are two countries and a continuum of goods in indexed $z \in [0,1]$. Goods are produced using labor:

$$y_j(z) = \ell_j(z) / a_j(z).$$

where

$$a_1(z) = e^{\alpha z}$$

$$a_2(z) = e^{\alpha(1-z)}.$$

Here $y_j(z)$ is the production of good z in country j and $\ell_j(z)$ is the input of labor. The stand-in consumer in each country has the utility function

$$\int_0^1 \log c_j(z)\, dz.$$

This consumer is endowed with $\bar{\ell}_j$ units of labor where $\bar{\ell}_1 = \bar{\ell}_2 = \bar{\ell}$.

a) Define an equilibrium of the economy. Calculate expressions for all of the equilibrium prices and quantities. Draw a graph that illustrates the pattern of specialization in production and trade.

b) Suppose now the each country faces iceberg transportation costs of τ to import the goods from the other country. Repeat the analysis of part a.

c) Suppose finally that the two countries engage in a tariff war in which each country imposes an *ad valorem* tariff τ on imports from the other country. Repeat the analysis of part a.

d) For the model in part c, calculate gross domestic product, exports, and the real income index

$$v_j = \exp \int_0^1 \log c_j(z)\, dz$$

as functions of τ. Suppose that in the base period $\tau = 0$ and calculate real GDP — that is, GDP in base period prices — as well as GDP in current prices.