## INTERNATIONAL TRADE AND PAYMENTS THEORY T. J. KEHOE ECON 8401 FALL 2009 PROBLEM SET #1

1. Consider an economy in which the representative consumer has the utility function

$$u(c_1, c_2) = a_1 \log c_1 + a_2 \log c_2, \ i = 1, 2.$$

where  $a_i > 0$  and  $a_1 + a_2 = 1$ . This consumer is endowed with capital and labor in the amounts  $(\overline{k}, \overline{\ell})$ . There are two goods, which are produced with the production technologies

$$y_j = \theta_j k_j^{\alpha_j} \ell_j^{1-\alpha_j}, \quad j = 1, 2,$$

where  $1 > \alpha_1 > \alpha_2 > 0$ .

a) Define an autarky equilibrium for this economy. Calculate this equilibrium.

b) Now suppose the country opens to trade. Suppose that the country is so small that it does not affect the world prices  $p_1$ ,  $p_2$ . Define an equilibrium for this small open economy.

c) Find conditions on  $p_1$ ,  $p_2$ ,  $\overline{k}$ , and  $\overline{\ell}$  such that the country produces positive amounts of both goods in the trade equilibrium.

d) Use the answer to part c to calculate a cone of diversification that depends on  $(p_1, p_2)$ — a set of  $(\overline{k}, \overline{\ell})$  — such that small open economies with their endowments in this cone produce both goods.

e) State and prove a version of the Stolper-Samuelson theorem. [Hint: The proofs of this theorem and those in part e and in question 2, parts c and d are simple for this particular model. You should provide careful statements of the theorems. In particular, you should be careful about the restrictions on parameters needed for the theorem to hold.]

f) State and prove a version of the Rybczynski theorem.

2. Now consider a two-country version of question 1. The representative consumer in each country has the utility function

$$u(c_1^i, c_2^i) = a_1 \log c_1^i + a_2 \log c_2^i, \ i = 1, 2.$$

This consumer is endowed with capital and labor in the amounts  $(\overline{k}^i, \overline{\ell}^i)$  where  $\overline{k}^1/\overline{\ell}^1 > \overline{k}^2/\overline{\ell}^2$ . The production technologies in the two countries are identical.

$$y_{i}^{i} = \theta_{i} (k_{i}^{i})^{\alpha_{j}} (\ell_{i}^{i})^{1-\alpha_{j}}, i, j = 1, 2$$

a) Define a free trade equilibrium. Explain the four possible patterns of specialization that are possible. [Hint: You need to distinguish among three conceptually different cases: one in which both countries produce both goods, another in which one country produces both goods and the other produces only one good, and the last in which each country produces only one good.]

b) Under what conditions on  $(\overline{k}^1, \overline{\ell}^1)$  and  $(\overline{k}^2, \overline{\ell}^2)$  do both countries produce both goods? Calculate a cone of diversification that depends on  $(\overline{k}^1 + \overline{k}^2, \overline{\ell}^1 + \overline{\ell}^2)$  for the world economy such that, if both countries have endowments of labor and capital in the cone, they both produce positive amounts of both goods. [Be careful: This concept of the cone of diversification differs from that in question 1.]

c) Calculate the equilibrium in the case where both country produce both goods and for the case where country 1 specializes in the production of good 1 but country 2 produces both goods.

d) State and prove a version of the factor price equalization theorem for this particular world economy.

e) State and prove a version of the Heckscher-Ohlin theorem. Does this theorem hold when the endowments of one or both countries are outside the cone of diversification?

3. Consider an economy in which there are two types of goods, agriculture and manufactured goods. Agricultural goods are homogeneous and are produced using labor according to the constant returns to scale production function

 $y_0 = \ell_0.$ 

Manufactured goods are differentiated by firm. The production function for firm j is

$$y_i = (1/b) \max[\ell_i - f, 0].$$

Here f is the fixed cost, in terms of labor, necessary to operate the firm and b is the unit labor requirement. Suppose that there is a representative consumer with preferences

$$\log c_0 + (1/\rho) \log \sum_{j=1}^n c_j^{\rho}$$
,

where  $1 \ge \rho > 0$ . There is an endowment of  $\overline{\ell}$  units of labor

a) Define a monopolistically competitive equilibrium for this economy in which firms follow Cournot pricing rules and there is free entry and exit.

b) Suppose that b = 2, f = 4,  $\rho = 1/2$ , and  $\overline{\ell} = 36$ . Calculate the autarky equilibrium.

c) Suppose now that  $\overline{\ell} = 180$ . Calculate the equilibrium.

d) Interpret the equilibrium in part c as a trading equilibrium among two countries, one with  $\overline{\ell}^1 = 36$  and the second with  $\overline{\ell}^2 = 144$ . Assume that production of the homogeneous good is distributed proportionally across the two countries. What impact does trade have on the number of manufacturing firms in each country? The average output of firms? The total number of products available? Consumer utility and real income? Illustrate the efficiency gains using an average cost curve diagram.

4. Repeat the analysis of question 3 for two variants of the model. Compare the gains from trade in these two alternative models with those in the model in question 3.

a) Suppose that there are again a finite number of differentiated goods but that firms are now Bertrand competitors, rather than Cournot competitors.

b) Suppose that consumers have the utility function

$$\log c_0 + (1/\rho) \log \int_0^n c(j)^\rho dj.$$

Here there is a continuum [0, n] of differentiated goods. (Hint: You need to be very careful in taking derivatives when solving the firms' profit maximization problems. In particular, the answers change drastically.)

c) Compare the gains in real income in parts a and b with each other and with those in question 3, part d.

5. Consider an economy similar to that in question 3. Suppose that production of the agricultural good is governed by the function

$$y_0 = \ell_0^{\alpha} t_0^{1-\alpha}.$$

Here  $t_0$  denotes inputs of land. Suppose now that there are two such countries, one with endowments  $(\overline{\ell}^1, \overline{t}^1)$  and the other with endowments  $(\overline{\ell}^2, \overline{t}^2)$ , but otherwise identical.

a) Define a trade equilibrium.

b) Derive conditions on  $(\overline{\ell}^1, \overline{t}^1)$  and  $(\overline{\ell}^2, \overline{t}^2)$  for which factor price equalization holds.

c) Suppose that  $\overline{\ell}^1/\overline{t}^1 > \overline{\ell}^2/\overline{t}^2$ . Explain what changes you would expect to see in prices, number of firms, average output levels, and utility levels as these two countries, initially in autarky, open to trade. Explain carefully what patterns of specialization are possible and what pattern of trade you would expect to see.