INTERNATIONAL TRADE AND PAYMENTS THEORY **T. J. KEHOE**
ECON 8401 **FALL 2005**

PROBLEM SET #2

1. Consider an economy in which the consumption space is the set of functions $c: R_+ \times R_+ \to R_+$. In $c(x,t)$ the index x denotes the type of good and the index t denotes the date at which it is consumed. An individual consumer has preferences given by the functional

$$u(c) = \int_0^\infty e^{-\rho t} \left[\int_0^\infty \log(c(x,t)+1)dx \right] dt.$$

Goods are produced using a single factor of production, labor:

$$y(x,t) = \ell(x,t)/a(x,t).$$

Each consumer has an endowment of labor equal to 1, and the total number of consumers is fixed at $\bar{\ell}$. The unit labor requirement $a(x,t)$ is bounded from below, $a(x,t) > \bar{a}(x)$, where

$$\bar{a}(x) = e^{-x}.$$

At $t = 0$ there is a $z(0) > 0$ such that $a(x,0) = e^{-x}$ for all $x < z(0)$ and that $a(x,0) = e^{x-2z(0)}$ for all $x \geq z(0)$. There is learning by doing of the form

$$\frac{\dot{a}(x,t)}{a(x,t)} = \begin{cases} -\int_0^\infty b(v,t)\ell(v,t)dv & \text{if } a(x,t) > \bar{a}(x) \\ 0 & \text{if } a(x,t) = \bar{a}(x) \end{cases}.$$

Here $\dot{a}(x,t)$ denotes the partial derivative of $a(x,t)$ with respect to t and $b(v,t) = b > 0$ if $a(v,t) > \bar{a}(v)$ and $b(v,t) = 0$ if $a(v,t) = \bar{a}(v)$. There is no borrowing or lending, and there is no storage.

a) Provide a motivation for the production technology described above.

b) Define an equilibrium for this economy. Characterize the equilibrium as much as possible.

c) Consider now a two country world in which the two countries are identical except in their endowments of labor and their initial technology levels. In particular, $z^1(0) > z^2(0)$. Define an equilibrium for this economy.

d) Describe the environment of a static Ricardian model whose equilibrium has the same values of prices and quantities as $p(x,0)$, $w^1(0)$, $w^2(0)$, $y^1(x,0)$, $y^2(x,0)$, $c^1(x,0)$, $c^2(x,0)$ in the economy of part c. Illustrate and explain the (five) different possibilities for patterns of production and consumption in this model. (To make things easy assume that $z^1(0)$ and $z^2(0)$ are sufficiently large so that good $x = 0$ is not produced in equilibrium.)

e) Describe the dynamics of the model, explaining the crucial role played by the sizes of the two countries, $\overline{\ell}^1$ and $\overline{\ell}^2$.