INTERNATIONAL TRADE AND PAYMENTS THEORY **T. J. KEHOE**
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PROBLEM SET #2

1. Consider an economy in which there are two countries and a continuum of goods in indexed $z \in [0,1]$. Goods are produced using labor:

$$y_j(z) = \ell_j(z) / a_j(z).$$

where

$$a_1(z) = e^{\alpha z}$$

$$a_2(z) = e^{\alpha(1-z)}.$$

Here $y_j(z)$ is the production of good z in country j and $\ell_j(z)$ is the input of labor. The stand-in consumer in each country has the utility function

$$\int_0^1 \log c_j(z)\, dz.$$

This consumer is endowed with $\overline{\ell}_j$ units of labor where $\overline{\ell}_1 = \overline{\ell}_2 = \overline{\ell}$.

a) Define an equilibrium of the economy. Calculate expressions for all of the equilibrium prices and quantities. Draw a graph that illustrates the pattern of specialization in production and trade.

b) Suppose now the each country faces iceberg transportation costs of τ to import the goods from the other country. Repeat the analysis of part a.

c) Suppose finally that the two countries engage in a tariff war in which each country imposes an *ad valorem* tariff τ on imports from the other country. Repeat the analysis of part a.

d) For the model in part c, calculate gross domestic product, exports, and the real income index

$$v_j = \exp \int_0^1 \log c_j(z)\, dz$$

as functions of τ. Suppose that in the base period $\tau = 0$ and calculate real GDP — that is, GDP in base period prices — as well as GDP in current prices.

2. Consider an economy in which the consumption space is the set of functions $c: R_+ \times R_+ \to R_+$. In $c(x,t)$ the index x denotes the type of good and the index t denotes the date at which it is consumed. An individual consumer has preferences given by the functional

$$u(c) = \int_0^\infty e^{-\rho t} \left[\int_0^\infty \log(c(x,t)+1)dx \right] dt.$$

Goods are produced using a single factor of production, labor:

$$y(x,t) = \ell(x,t) / a(x,t).$$

Each consumer has an endowment of labor equal to 1, and the total number of consumers is fixed at $\bar{\ell}$. The unit labor requirement $a(x,t)$ is bounded from below, $a(x,t) > \bar{a}(x)$, where

$$\bar{a}(x) = e^{-x}.$$

At $t = 0$ there is a $z(0) > 0$ such that $a(x,0) = e^{-x}$ for all $x < z(0)$ and that $a(x,0) = e^{x-2z(0)}$ for all $x \geq z(0)$. There is learning by doing of the form

$$\frac{\dot{a}(x,t)}{a(x,t)} = \begin{cases} -\int_0^\infty b(v,t)\ell(v,t)dv & \text{if } a(x,t) > \bar{a}(x) \\ 0 & \text{if } a(x,t) = \bar{a}(x) \end{cases}.$$

Here $\dot{a}(x,t)$ denotes the partial derivative of $a(x,t)$ with respect to t and $b(v,t) = b > 0$ if $a(v,t) > \bar{a}(v)$ and $b(v,t) = 0$ if $a(v,t) = \bar{a}(v)$. There is no borrowing or lending, and there is no storage.

a) Provide a motivation for the production technology described above.

b) Define an equilibrium for this economy. Characterize the equilibrium as much as possible.

c) Consider now a two country world in which the two countries are identical except in their endowments of labor and their initial technology levels. In particular, $z^1(0) > z^2(0)$. Define an equilibrium for this economy.

d) Describe the environment of a static Ricardian model whose equilibrium has the same values of prices and quantities as $p(x,0)$, $w^1(0)$, $w^2(0)$, $y^1(x,0)$, $y^2(x,0)$, $c^1(x,0)$, $c^2(x,0)$ in the economy of part c. Illustrate and explain the (five) different possibilities for patterns of production and consumption in this model. (To make things easy assume that $z^1(0)$ and $z^2(0)$ are sufficiently large so that good $x = 0$ is not produced in equilibrium.)

e) Describe the dynamics of the model, explaining the crucial role played by the sizes of the two countries, $\bar{\ell}^1$ and $\bar{\ell}^2$.

3. Consider an economy with two goods that enter both consumption and investment. The utility function of the representative consumer is

$$\sum_{t=0}^{\infty} \beta^t \log(c_{1t}^{a_1} c_{2t}^{a_1}).$$

Here $0 < \beta < 1$, $a_1 \geq 0$, $a_2 \geq 0$, and $a_1 + a_2 = 1$. Investment goods are produced according to

$$k_{t+1} - (1-\delta)k_t = dx_{1t}^{a_1} x_{2t}^{a_1}.$$

Feasible consumption/investment plans satisfy the feasibility conditions

$$c_{1t} + x_{1t} = \phi_1(k_{1t}, \ell_{1t}) = k_{1t}$$

$$c_{2t} + x_{2t} = \phi_2(k_{2t}, \ell_{2t}) = \ell_{2t}$$

where

$$k_{1t} + k_{2t} = k_t$$

$$\ell_{1t} + \ell_{2t} = \ell_t.$$

The initial endowment of k_t is $\bar{k}_0$. ℓ_t is equal to 1. (In other words, all variables are expressed in per capita terms.)

a) Carefully define a competitive equilibrium for this economy.

b) Reduce the equilibrium conditions to two difference equations in k_t and c_t and a transversality condition. Here $c_t = dc_{1t}^{a_1} c_{2t}^{a_1}$ is aggregate consumption. [Here is one possible approach: Prove a version of the first and second welfare theorems for this economy. Show that the two-sector social planer's problem is equivalent to a one-sector social planner's problem. Derive the difference equations and transversality conditions from the one-sector social planner's problem.]

c) Suppose now that there is a world composed of n different countries, all with the same preferences and technologies, but with different initial endowments of capital per worker, $\bar{k}_0^i$ The countries also have different population sizes, L^i, which are constant over time. (In other words, there is a continuum of identical consumers/workers of measure L^i in country i.) Suppose that there is no international borrowing or lending and no and no international capital flows. Define an equilibrium for this world economy.

Prove that in this equilibrium the variables $c_{jt} = \sum_{i=1}^{n} L^i c_{jt}^i \Big/ \sum_{j=1}^{n} L^i$, $k_t = \sum_{i=1}^{n} L^i k_t^i \Big/ \sum_{i=1}^{n} L^i$, p_{it}, r_t, and w_t satisfy the equilibrium conditions of the economy in part a when $\bar{k}_0 = \sum_{i=1}^{n} L^i \bar{k}_0^i \Big/ \sum_{i=1}^{n} L^i$.

d) State and prove versions of the factor price equalization theorem, the Stolper-Samuelson theorem, the Rybczynski theorem, and the Heckscher-Ohlin theorem for this particular world economy.

e) What effects do the assumptions of no international borrowing and lending and no international capital flows have on your analysis? Try to be precise about which variables are uniquely determined in the equilibrium with borrowing and lending and/or capital flows are which would not be uniquely determined.

f) Consider the case where $\delta = 1$. Set $z_0 = c_0 / \left(\beta r_0 k_0\right)$ and $z_t = c_{t-1} / k_t$, $t = 1, 2, \ldots$. Transform the two difference equations in part b into two difference equations in k_t and z_t. Prove that

$$\frac{k_t^i - k_t}{k_t} = \frac{z_t}{z_{t-1}} \left(\frac{k_{t-1}^i - k_{t-1}}{k_{t-1}} \right) = \frac{z_t}{z_0} \left(\frac{\bar{k}_0^i - \bar{k}_0}{\bar{k}_0} \right).$$

g) Consider again the case where $\delta = 1$. Let $s_t = c_t / y_t$ where

$$y_t = p_{1t} k_t + p_{2t} = d k_t^{a_1} = r_t k_t + w_t.$$

Transform the two difference equations in part b into two difference equations in k_t and s_t. Prove that

$$\frac{y_t^i - y_t}{y_t} = \frac{s_t}{s_{t-1}} \left(\frac{y_{t-1}^i - y_{t-1}}{y_{t-1}} \right) = \frac{s_t}{s_0} \left(\frac{y_0^i - y_0}{y_0} \right)$$

where $y_t^i = p_{1t} y_{1t}^i + p_{2t} y_{2t}^i = r_t k_t^i + w_t$. Calculate an expression for s_t and discuss the significance of this result.