

PROBLEM SET #3

1. Choose two countries that are important (to each other) trading partners, one with higher output per worker than the other. Use data from the Summers-Heston *Penn World Tables* (<http://pwt.econ.upenn.edu>) or from the World Bank's *Global Development Network Growth Database* (<http://www.worldbank.org/research/growth/GDNdata.htm>) to try to answer Lucas's question: To what extent can differences in output per worker be explained by differences in capital per worker? Use both data on capital per worker and IMF real interest rate data to try to make inferences about differences in capital per worker across countries. Discuss what you have learned from this exercise and what more you think would be relevant in explaining differences in output per worker across countries.

2. Consider a world with two countries. In each country  $i$ ,  $i = 1, 2$ , the consumer has the utility function

$$\sum_{t=0}^{\infty} \beta^t [z_t^i \log(a_1 c_{1t}^{i\rho} + a_2 c_{2t}^{i\rho})^{1/\rho} + (1 - z_t^i) \log(\bar{\ell}^i - \ell_t^i)].$$

Here  $z_t^i$  is a preference parameter that can vary over time,  $1 > \rho > 0$ ,  $a_i > 0$ , and  $a_1 + a_2 = 1$ . Consumer  $i$  is endowed with  $\bar{\ell}^i$  units of labor and  $\bar{k}^i = 1$  units of capital in each period.

Good  $i$ ,  $i = 1, 2$ , is produced only in country  $i$  using the production technology

$$y_{it} = \theta_{it} k_{it}^{\mu_i} \ell_{it}^{1-\mu_i}.$$

Here  $\theta_{it}$  is a technology parameter that can vary over time.

Although both  $z_t^i$  and  $\theta_{it}$  can vary over time, they do so deterministically. There is no uncertainty.

- a) Define an Arrow-Debreu equilibrium for this economy.
- b) Define a sequential markets equilibrium for this economy.
- c) Define a social planner's problem that the equilibrium allocations in both parts a and b solve.

In parts d, e, and f consider a symmetric model in which  $a_1 = a_2 = 0.5$ ,  $\bar{\ell}^1 = \bar{\ell}^2$ , and  $\mu_1 = \mu_2$ . You can go a long way in answering these questions analytically. You can

learn a lot, and have some real fun, however, writing a simple computer program that solves for equilibria for different values of the parameters.

d) Suppose that  $z_t^1 = z_t^2 = z > 0$ , but that the technology parameters  $\theta_{it}$  follow the cyclical pattern

$$(\theta_{10}, \theta_{11}, \theta_{12}, \theta_{13}, \dots) = (\bar{\theta}, \underline{\theta}, \bar{\theta}, \underline{\theta}, \dots)$$

$$(\theta_{20}, \theta_{21}, \theta_{22}, \theta_{23}, \dots) = (\underline{\theta}, \bar{\theta}, \underline{\theta}, \bar{\theta}, \dots)$$

where  $\bar{\theta} > \underline{\theta} > 0$ . Characterize the solution to the symmetric social planner's problem where welfare weights are equal. Calculate real GDP, the trade balance, and the real exchange rate. (You need to define a base year and choose a price index.)

e) Suppose now that  $\theta_{1t} = \theta_{2t} = \theta > 0$ , but that the preference parameters  $z_t^i$  follow the cyclical pattern

$$(z_0^1, z_1^1, z_2^1, z_3^1, \dots) = (\bar{z}, \underline{z}, \bar{z}, \underline{z}, \dots)$$

$$(z_0^2, z_1^2, z_2^2, z_3^2, \dots) = (\underline{z}, \bar{z}, \underline{z}, \bar{z}, \dots)$$

where  $\bar{z} > \underline{z} > 0$ . Repeat the analysis of part d.

f) Repeat the analysis of parts d and e in terms of calculating real GDP and the real exchange rate when  $(\bar{\ell}^i - \ell_t^i)$ , rather than being leisure, is a nontraded good.