

PROBLEM SET #3

1. Consider an economy with the following input-output matrix:

	Agr.	Mfg.	Con.	Inv.	Exp.	Total
Agriculture	2	3	8	2	5	20
Manufacturing	6	6	10	4	4	30
Imports	4	5				9
Tariff Revenue	2	1				3
Labor Compensation	3	10				13
Returns to Capital	3	5				8
Total	20	30	18	6	9	

- a) What are the national income and product accounts for this economy?
- b) Suppose that consumers and producers regard domestic goods and imports of goods as imperfect substitutes and that the Armington aggregators are Cobb-Douglas:

$$y_j = \gamma_j y_{j,d}^{\delta_j} y_{j,f}^{1-\delta_j}, \quad j = agr, man.$$

Calibrate these Armington aggregators. Calibrate the tariff rates τ_{agr} , τ_{man} .

- c) Suppose that all tariff revenues are transferred in lump-sum fashion to a representative consumer. Suppose that this consumer's utility function is Cobb-Douglas:

$$\theta_{agr} \log c_{agr} + \theta_{man} \log c_{man} + \theta_{inv} \log c_{inv}.$$

Calibrate the consumer's utility function and endowments $\bar{\ell}$, \bar{k} .

- d) Suppose that net domestic production of each good is governed by a nested production function that produces value added by combining labor and capital using a Cobb-Douglas function and combines intermediate inputs of the other good and value-added in fixed proportions.

$$y_{j,d} = \min [x_{agr,j} / a_{agr,j}, x_{man,j} / a_{man,j}, \beta_j k_j^{\alpha_j} \ell_j^{1-\alpha_j}], \quad j = agr, man.$$

Calibrate the two production functions.

e) Suppose that there is a production function that produces the investment good using agriculture and manufactured goods in fixed proportions:

$$y_{inv} = \min [x_{agr,inv} / a_{agr,inv}, x_{man,inv} / a_{man,inv}] .$$

Calibrate this production function.

f) Suppose that the rest of the world has income 100 and a representative consumer with the Cobb-Douglas utility function.

$$\theta_{agr,f} \log x_{agr,f} + \theta_{man,f} \log x_{man,f} + \theta_{f,f} \log x_{f,f} .$$

Suppose too that this consumer faces tariffs of $\tau_{agr,f} = 0.1$, $\tau_{man,f} = 0.1$ on imports.

Calibrate this utility function.

g) Suppose that the Armington elasticity of substitution between domestic goods and foreign goods is 5 in both the Armington aggregators in part b,

$$y_j = \gamma_j \left[\delta_j y_{j,d}^\rho + (1 - \delta_j) y_{j,f}^\rho \right]^{\frac{1}{\rho}}, \quad j = agr, man .$$

and the foreign utility function in part f,

$$\left(\theta_{agr,f} x_{agr,f}^\rho + \theta_{man,f} x_{man,f}^\rho + \theta_{f,f} x_{f,f}^\rho - 1 \right) / \rho ,$$

where $\rho = 0.8$. Recalibrate these functions.

2. a) Define an equilibrium for the economy in question 1 and calculate the benchmark equilibrium. (Hint: You know the equilibrium of all of the variables).

b) Describe how you would use this model to evaluate the impact of a trade reform.

c) Suppose that the trade reform sets $\tau_{agr} = \tau_{man} = 0.2$. Calculate the new equilibrium both in the case where the Armington elasticity is 1 and in the case where it is 5. [In the case where $1/(1 - \rho) = 1$, $\hat{w} = 1$, $\hat{r} = 0.998005$, $\hat{p}_{agr} = 0.968385$, $\hat{p}_{man} = 1.022273$,

$\hat{e} = 1.111094$, $\hat{T} = 1.999969$, $\hat{y}_{agr} = 20.547691$, $\hat{y}_{man} = 29.495026$. In the case where $1/(1 - \rho) = 5$, $\hat{w} = 1$, $\hat{r} = 0.974039$, $\hat{p}_{agr} = 0.918309$, $\hat{p}_{man} = 0.991281$, $\hat{e} = 1.032627$,

$\hat{T} = 2.530111$, $\hat{y}_{agr} = 24.042197$, $\hat{y}_{man} = 30.096442$.]

d) Describe how to modify this model to include monopolistic competition in the manufacturing sector. In particular, explain how the specification of the environment and the definition of equilibrium would change.

3. Download data on bilateral trade by sector at the 4 digit SITC level from the OECD web site, <http://oberon.sourceoecd.org>. Follow the methodology in Kehoe and Ruhl, “How Important is the New Goods Margin in International Trade?” to create a set of least traded goods and carry out one of the two following exercises:

a) Consider trade between two countries over time. Construct diagrams with fractions of trade at the end of the period by deciles of sets of goods at the beginning of the period. Graph of the fraction of trade accounted for by the least traded decile over time. Do imports and exports separately.

b) Consider exports of one country to a number of trading partners during one year. Compare the sets of least traded goods. Do you see any patterns?

4. Consider an economy where the consumers have Dixit-Stiglitz utility functions and solve the problem

$$\begin{aligned} \max \quad & (1 - \mu) \log c_0 + \frac{\mu}{\rho} \log \int_0^n c(i)^\rho di \\ \text{s.t.} \quad & p_0 c_0 + \int_0^n p(i) c(i) di = w \bar{\ell} + \pi \\ & c(i) \geq 0. \end{aligned}$$

Here π are profits of the firms, which are owned by the consumers.

a) Suppose that the producer of good i takes the prices $p(i')$, for $i' \neq i$, as given. Suppose too that this producer has the production function

$$y(i) = \max [x(i)(\ell(i) - f), 0].$$

Solve the firm's profit maximization problem to derive an optimal pricing rule.

b) Suppose that there is a measure m of potential firms. Firm productivities are distributed on the interval $x \geq 1$ according to the Pareto distribution with distribution function

$$F(x) = 1 - x^{-\gamma}.$$

Define an equilibrium for this economy.

c) Suppose that $\mu = 0.5$, $\rho = 0.5$, $\bar{\ell} = 40$, $n = 10$, $\gamma = 4$, and $f = 2$. Calculate the equilibrium of this economy.

d) Suppose now that there are two countries that engage in free trade. Each country i , $i = 1, 2$, has a population of $\bar{\ell}_i$ and a measure of potential firms of n_i . Firms' productivities are again distributed according to the Pareto distribution, $F(x) = 1 - x^{-\gamma}$. A firm in country i faces a fixed cost of exporting to country j , $j \neq i$, of f_e where

$f_e > f_d = f$ and an iceberg transportation cost of $\tau_i^j - 1 \geq 0$. Define an equilibrium for this economy.

- e) Suppose that $\mu = 0.5$, $\rho = 0.5$, $\bar{\ell}_1 = \bar{\ell}_2 = \bar{\ell} = 40$, $n_1 = n_2 = n = 10$, $\gamma = 4$, $f_d = 2$, $f_e = 3$, and $\tau_2^1 = \tau_1^2 = \tau = 1.2$. Calculate the equilibrium of this economy.
- f) Suppose now that a free trade agreement sets $\tau_2^1 = \tau_1^2 = \tau = 1$. Recalculate the equilibrium in part e.
- g) Suppose now that a different reform sets $f_e = 2$. Again recalculate the equilibrium in part e. Contrast the results with those in part f. What sort of reform can lower f_e ?