INTERNATIONAL TRADE AND PAYMENTS THEORY **T. J. KEHOE**
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PROBLEM SET #1

1. Consider a world with two countries. The representative consumer in each country has the utility function

$$u(c_1^i,\ c_2^i) = \log c_1^i + \log c_2^i,\ i = 1,2.$$

This consumer is endowed with capital and labor in the amounts $(\bar{k}^i,\ \bar{\ell}^i)$. The production technologies in the two countries are identical.

$$y_j^i = \min[k_j^i / a_{Kj},\ \ell_j^i / a_{Lj}],\ i, j = 1,2.$$

To make things simple assume that $a_{K1} = a_{L2} = 1$ and that $a_{K2} = a_{L1} = 2$.

a) Define an autarky equilibrium for country *i*. Under what conditions on $(\bar{k}^i,\ \bar{\ell}^i)$ are both of the equilibrium factor prices positive? Under what conditions on $(\bar{k}^i,\ \bar{\ell}^i)$ are all of the factors fully employed in the sense that $k_j^i / a_{Kj} = \ell_j^i / a_{Lj}$, $j = 1,2$, $k_1^i + k_2^i = \bar{k}^i$, and $\ell_1^i + \ell_2^i = \bar{\ell}^i$?

b) Define a free trade equilibrium. Under what conditions on $(\bar{k}^1,\ \bar{\ell}^1)$ and $(\bar{k}^2,\ \bar{\ell}^2)$ do both countries produce both goods?

c) State and prove a version of the factor price equalization theorem for this particular world economy.

d) State and prove a version of the Stolper-Samuelson theorem.

e) State and prove a version of the Rybczynski theorem.

f) State and prove a version of the Heckscher-Ohlin theorem.

2. Consider a world similar to that in question 1, but in which the two production functions are

$$y_j^i = f_j(k_j^i, \ell_j^i),\ i, j = 1,2.$$

where f_j is continuously differentiable, homogeneous of degree one, and concave and satisfies the Inada conditions

$$\lim_{k \to 0} f_{jK}(k, \ell) = \lim_{\ell \to 0} f_{jL}(k, \ell) = \infty$$

where f_{jK} and f_{jL} denote partial derivatives. Assume that, for any k / ℓ,

$$\frac{f_{1K}(k/\ell,1)}{f_{1L}(k/\ell,1)} > \frac{f_{2K}(k/\ell,1)}{f_{2L}(k/\ell,1)}.$$

a) Repeat the analysis of question 1 for this world. Explain carefully where results differ qualitatively. Keep in mind that

$$\lim_{\rho\to-\infty}(a_{Kj}^{-\rho}k^{\rho}+a_{Lj}^{-\rho}\ell^{\rho})^{1/\rho}=\min[k/a_{Kj},\ell/a_{Lj}],\ j=1,2,$$

but that, for any finite value of ρ, $\rho<1$, the functions $f_j(k,\ell)=(a_{Kj}^{-\rho}k^{\rho}+a_{Lj}^{-\rho}\ell^{\rho})^{1/\rho}$ satisfy all of the above assumptions.

b) Provide an example of two functions f_1 and f_2 that violate the assumption

$$\frac{f_{1K}(k/\ell,1)}{f_{1L}(k/\ell,1)} > \frac{f_{2K}(k/\ell,1)}{f_{2L}(k/\ell,1)}.$$

for some k/ℓ no matter which good we call 1 and which we call 2. Explain the role of this assumption in your analysis. In particular, what happens if this assumption is violated?

3. Consider an economy in which there are two types of goods, agriculture and manufactured goods. Agricultural goods are homogeneous and are produced using labor according to the constant returns to scale production function

$$y_0=\ell_0.$$

Manufactured goods are differentiated by firm. The production function for firm j is

$$y_j=(1/b)\max[\ell_j-f,0].$$

Here f is the fixed cost, in terms of labor, necessary to operate the firm and b is the unit labor requirement. Suppose that there is a representative consumer with preferences

$$\log c_0+(1/\rho)\log\sum\nolimits_{j=1}^{n}c_j^{\rho}$$

where $1\geq\rho>0$. There is an endowment of $\bar{\ell}$ units of labor

a) Define a monopolistically competitive equilibrium for this economy in which firms follow Cournot pricing rules and there is free entry and exit.

b) Suppose that $b=1$, $f=3$, $\rho=1/2$, and $\bar{\ell}=50$. Calculate the autarky equilibrium.

c) Suppose now that $\bar{\ell}=200$. Calculate the equilibrium.

d) Interpret the equilibrium in part c as a trading equilibrium among two countries, one with $\bar{\ell}^1=50$ and the second with $\bar{\ell}^2=150$. Assume that production of the homogeneous good is distributed proportionally across the two countries. What impact does trade have on the number of manufacturing firms in each country? The average output of firms? The total number of products available? Consumer utility? Illustrate the efficiency gains using an average cost curve diagram.

e) Repeat the analysis of parts a-d in a model in which consumers have the utility function

$$\log c_0 + (1/\rho)\log \int_0^n c(j)^\rho \, dj.$$

Here there is a continuum $[0,n]$ of differentiated goods. (Hint: You need to be very careful in taking derivatives when solving the firms' profit maximization problems. In particular, the answers change drastically.)

f) Repeat the analysis of parts a-d in a model in which firms are Bertrand competitors, rather than Cournot competitors.

4. Consider an economy similar to that in question 3. Suppose that production of the agricultural good is governed by the function

$$y_0 = \ell_0^\alpha t_0^{1-\alpha}.$$

Here t_0 denotes inputs of land. Suppose now that there are two such countries, one with endowments $(\overline{\ell}^1, \overline{t}^1)$ and the other with endowments $(\overline{\ell}^2, \overline{t}^2)$, but otherwise identical.

a) Define a trade equilibrium.

b) Suppose that $\overline{\ell}^1 / \overline{t}^1 \neq \overline{\ell}^2 / \overline{t}^2$. Explain what changes you would expect to see in prices, average output levels, and utility levels as these two countries, initially in autarky, open to trade. Explain carefully what patterns of specialization are possible and what pattern of trade you would expect to see.