INTERNATIONAL TRADE AND PAYMENTS THEORY **T. J. KEHOE**
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PROBLEM SET #2

1. Consider a world with three countries. There is a representative consumer in each country who has preferences over the interval of goods $X = [0,1]$ given by the utility function

$$\int_X \log\ c(x)\ dx.$$

In each country there is a single factor, labor. Endowments are $\bar{\ell}_1 = \bar{\ell}_2 = \bar{\ell}_3 = \bar{\ell}$.

Production functions are linear but differ across countries:

$$\begin{aligned} y_j^i(x) &= \ell_j^i(x)/a_j^i(x) \\ a_1^1(x) &= a_1^2(x) = a_1^3(x) = \alpha + \beta x \\ a_2^1(x) &= a_2^2(x) = a_2^3(x) = \alpha + \beta - \beta x \\ a_3^1(x) &= a_3^2(x) = a_3^3(x) = \gamma. \end{aligned}$$

Here, for example, $y_j^i(x)$ is the amount of good x produced in country j for consumption in country i.

Initially, there are no transportation costs or tariffs.

a) Define an equilibrium for this model.

b) Characterize as much as possible the patterns of specialization and trade in the equilibrium.

c) Suppose now that there are 10 percent transportation costs between every pair of countries. Explain how your definition of equilibrium is altered and characterize as much as possible how the new equilibrium differs from that in parts a and b.

d) Suppose now that the countries engage in a trade war in which each imposes a 10 percent tariff on imports from the other two. Explain how your definition of equilibrium is altered and characterize as much as possible how the new equilibrium differs from that in part c.

2. Consider an economy in which the consumption space is the set of functions $c : R_+ \times R_+ \to R_+$. In $c(x,t)$ the index x denotes the type of good and the index t denotes the date at which it is consumed. An individual consumer has preferences given by the functional

$$u(c) = \int_0^{\infty} e^{-\rho t} \left[\int_0^{\infty} \log(c(x,t)+1) dx \right] dt.$$

Goods are produced using a single factor of production, labor:

$$y(x,t) = \ell(x,t) / a(x,t).$$

Each consumer has an endowment of labor equal to 1, and the total number of consumers is fixed at $\bar{\ell}$. The unit labor requirement $a(x,t)$ is bounded from below, $a(x,t) > \bar{a}(x)$, where

$$\bar{a}(x) = e^{-x}.$$

At $t = 0$ there is a $z(0) > 0$ such that $a(x,0) = e^{-x}$ for all $x < z(0)$ and that $a(x,0) = e^{x-2z(0)}$ for all $x \geq z(0)$. There is learning by doing of the form

$$\frac{\dot{a}(x,t)}{a(x,t)} = \begin{cases} -\int_0^{\infty} b(v,t)\ell(v,t)dv & \text{if } a(x,t) > \bar{a}(x) \\ 0 & \text{if } a(x,t) = \bar{a}(x) \end{cases}.$$

Here $\dot{a}(x,t)$ denotes the partial derivative of $a(x,t)$ with respect to t and $b(v,t) = b > 0$ if $a(v,t) > \bar{a}(v)$ and $b(v,t) = 0$ if $a(v,t) = \bar{a}(v)$.

There is no borrowing or lending, and there is no storage.

a) Provide a motivation for the production technology described above.

b) Define an equilibrium for this economy. Characterize the equilibrium as much as possible.

c) Consider now a two country world in which the two countries are identical except in their endowments of labor and their initial technology levels. In particular, $z^1(0) > z^2(0)$. Define an equilibrium for this economy.

d) Describe the environment of a static Ricardian model whose equilibrium has the same values of prices and quantities as $p(x,0)$, $w^1(0)$, $w^2(0)$, $y^1(x,0)$, $y^2(x,0)$, $c^1(x,0)$, $c^2(x,0)$ in the economy of part c. Illustrate and explain the (five) different possibilities for patterns of production and consumption in this model. (To make things easy assume that $z^1(0)$ and $z^2(0)$ are sufficiently large so that good $x = 0$ is not produced in equilibrium.)