INTERNATIONAL ECONOMIC REVIEW
Vol. 38, No. 2, May 1997

CHANGES IN SECTORAL COMPOSITION ASSOCIATED WITH ECONOMIC GROWTH*

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In this paper I use dynamic general equilibrium methods to examine the interrelationship between sectoral composition and growth. I show that growth is affected by sectoral composition, and vice versa. The model is basically a Solow model of sustained growth with multiple consumption goods and nonhomothetic preferences. Each consumption good is produced using different factor intensities. The rate of exogenous technological change is different in each sector. Nonhomotheticity of preferences drives the result that sectoral composition affects growth rates.

1. INTRODUCTION

In the economic literature there are two main schools of thought on how sectoral composition and growth interrelate. The neoclassical view holds that sectoral composition is a relatively unimportant byproduct of growth. However, scholars associated with the World Bank, including Kuznets (1971), Rostow (1971), Chenery and Syrquin (1975), and Baumol et al. (1989) posit that growth is brought about by changes in sectoral composition. The present work attempts to integrate these two approaches and demonstrate that sectoral composition affects per capita income growth rates, and vice versa.

The model used in this paper, a dynamic general equilibrium model, is basically a Solow model of sustained growth with three consumption goods. Each consumption good is produced using different factor intensities, the rate of exogenous technological change is different in each sector, and preferences are nonhomothetic.[2] Since productivity in each of the three sectors grows at a different rate, the growth rate of the economy is affected by changes in sectoral composition—the growth rate increases in the early stages and decreases later. Changes in sectoral composition are driven by nonhomothetic preferences.

The research described herein can predict other regularities: (1) the hump-shaped correlation between growth rates and income levels, with poor countries having the lowest rate of growth and middle-income countries having the highest growth rate; (2) the higher proportion of GDP that is agriculture value-added in poor countries,

* Manuscript received November 1993; revised December 1994.

[1] I thank Dave Backus, Julio Escolano, Wilfred Ethier, Pat Kehoe, Vernon Ruttan, Carlos Zarazaga, anonymous referees, and especially Tim Kehoe for helpful comments and suggestions. Morris Altman provided useful references. Support received through the Sloan Grant for Open Economy Macroeconomics is gratefully acknowledged.

[2] Other papers which explore the implications for growth models of nonhomothetic preferences are Christiano (1989), Rebelo (1992), and Easterly (1994). Atkeson and Ogaki (1993) provide empirical support for intertemporal elasticities of substitution that vary with wealth.

and the higher proportion of GDP that is services value-added in rich countries; (3) the comparatively more expensive services in rich countries; (4) the larger share of the labor force employed in agriculture in less-developed countries, and the larger share employed in services in developed countries; and (5) the higher share of output paid to labor in rich countries.

According to the simulation presented here, sectoral composition can account for a variation in growth rates of greater than two percentage points. The importance of this magnitude of variation is better appreciated when we consider that, historically, most countries have had average growth rates of less than 4 percent. I am also able to replicate the hump-shaped relationship between growth rates and initial income, and the other regularities associated with sectoral composition described above.[3]

The paper is organized as follows. Section 2 documents some regularities on sectoral composition and growth. Section 3 outlines and analyzes the model. In Section 4 the computational method is briefly discussed. Section 5 is the calibration of the model. Section 6 presents and discusses the results, and Section 7 summarizes my conclusions.

2. SOME REGULARITIES ON SECTORAL COMPOSITION AND GROWTH

There are many regularities in the relationship between sectoral composition and growth that economists have pointed out over the years. My model accounts for the following:

1. Sectoral composition explains 22% of the variation in the growth of per capita income across countries. Table 1 exhibits the results of regressing growth in GDP per capita in 67 countries, with data available from 1970 to 1987, against the sectoral composition of the base year, 1970. In this simple regression, R^2 is higher than 0.22. Thus, one single variable by itself, the sectoral composition of the base year, explains 22% of the variation in growth rates.

2. Poor countries have the lowest and middle-income countries the highest growth rates. This is shown in Figure 1, which plots the average annual growth rate of real GDP (1950–1980) against GDP in 1950, using Summers and Heston (1988) data for 59 countries. GDP is measured in 1980 "international dollars."

Looking at Figure 1, we see that industrialized countries may have an inverse relationship between growth rate and GDP, whereas in the poorest countries this relationship is positive. In other words, it seems that the growth rate accelerates during the first stage of the development process, eventually peaks, and then starts falling.[4] Maddison (1982, p. 44) presents evidence that growth rates for 16 industrialized countries increased since 1820, where his data series began, and peaked between 1950 and 1973.[5]

[3] Christiano (1989) and Easterly (1994) obtain a similar hump shape for the growth-rate time series in a model with nonhomothetic preferences and just one sector.

[4] The regression is similar to the one in Baumol et al. (1989). Easterly (1994) also finds a hump-shaped relationship between growth rates and initial income, with the maximum of the hump between 1366 and 2032 (1985) dollars. In this regression, the maximum of the hump is at 2135 (1980) international dollars.

[5] The countries are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, Norway, Sweden, Switzerland, U.K., and USA.

TABLE 1

GDP (AT FACTOR COST) PER CAPITA GROWTH 1970–1987 VERSUS SECTORAL COMPOSITION OF GDP AT FACTOR COST IN BASE YEAR (1970)*†

Sample Statistics on Data Series					
Series	Observations	Mean	Standard error	Minimum	Maximum
growth	67	0.015	0.02	−0.031	0.068
alfa1	67	23.795	17.719	1.81	70.632
alfa2	67	28.67	11.247	8.729	53.479
alfa3	67	47.535	10.635	19.201	73.3673
	Growth				
intercept	0.065				
	(4.483)				
alfa1	0.009				
	(−4.183)				
alfa2	0.01				
	(−2.926)				
$R^{**}2$	0.222				
F-statistic	9.1				

* *Source:* World Bank 1990 (t-statistics in parentheses).

†Growth = GDP (at factor cost) per capita growth 1970–1987; alfa1 = share of agriculture value-added of GDP at factor cost in 1970; alfa2 = share of industry value-added of GDP at factor cost in 1970; alfa3 = share of services value-added of GDP at factor cost in 1970.

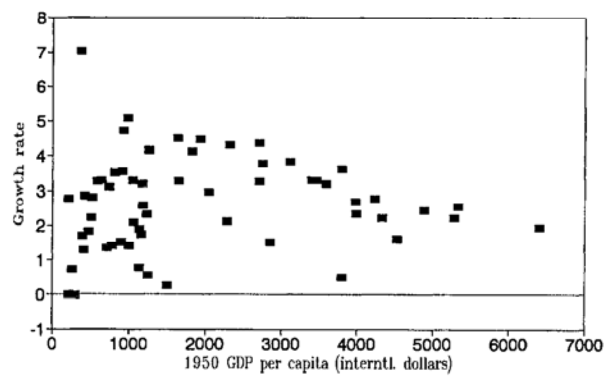

FIGURE 1

GROWTH RATES OF GDP PER CAPITA 1950–1980 (SUMMERS AND HESTON 1988)

3. The proportion of GDP that is agriculture value-added is higher in poor countries while the proportion of GDP that is services value-added is higher in rich countries. Figure 2 graphs the relationship between these proportions, as well as the share of industry value-added of GDP, and GNP per capita (in 1970 U.S. dollars) for 65 countries in 1990.[6] The inverse correlation between the share of primary goods and GNP, and the positive correlation between the share of services and GNP result in the correlation between the share of industry and GNP being hump-shaped. The *World Development Report 1987* (World Bank) points out that the historical

[6] Industry includes electricity, gas, and water; manufacturing; and construction.

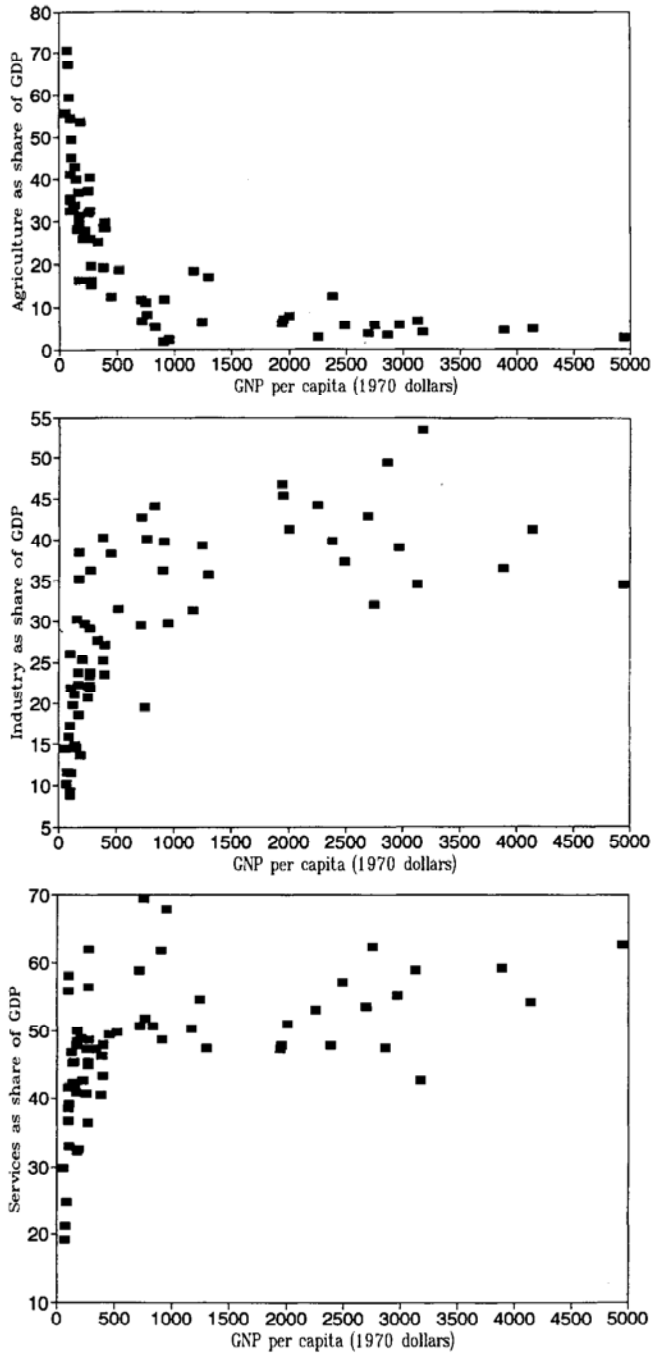

FIGURE 2

VALUE ADDED (WORLD BANK 1990)

relationship between GDP per capita and the share of manufacturing in GDP was also hump-shaped for a selected group of today's industrialized economies.[7]

At first glance the same pattern seems to be true for consumption. For instance, according to the *World Development Report 1987*, food represents 60% of total household consumption in China and Sudan and only 11% in Canada, while medical care represents 0% in Zambia and Kenya and 15% in Switzerland.

4. The relative price of services is higher in wealthier countries than in poorer ones. The first part of Figure 3 graphs the correlation between this relative price and GDP measured in 1975 "international dollars" for 34 countries in 1975. The y axis represents the price of services in each country relative to the "international" price of services, with the figures for each country normalized so the GDP price relative to the international GDP price for each country is 100. In industrialized countries today, services have become more expensive relative to manufactured goods than previously. Similarly, manufactured goods are becoming less expensive relative to primary goods in industrialized countries.

The positive correlation between relative price of services and national product seems to account for the positive correlation between services as a share of GNP and national product. The second part of Figure 3 shows the share of services in expenditure (GNP minus net exports) in national prices versus GDP in 1975 for the same 34 countries. The positive correlation between the service share and GDP is obvious. The third part of Figure 3 shows the same shares, but now both services and expenditures are measured in international prices. Once services are measured at the same prices across countries, the positive correlation between the service share and GDP disappears. The expansion of services as a share of GNP with income is explained by a rise in the relative price of services with income and not by an increase in the real services output with income.

5. The share of agriculture in the labor force decreases as GDP increases, both across countries and time. The share of services in the labor force has increased in most industrialized countries. The percentage of the labor force in agriculture in 1980 was as high as 93% in Rwanda but only 3% in Belgium. It fell from 6% in 1965 to 3% in 1980 in the latter. Conversely, services represent 4% of the labor force in Rwanda and 61% in Belgium. They increased from 48% in 1965 to 61% in 1980 in the latter (*World Development Report 1987*).

6. It appears that labor compensation represents a greater percentage of GDP in industrialized countries, although this percentage may have fallen recently. Figure 4 displays the relationship in 1970 between labor compensation as a percentage of GDP, and GDP per capita, in U.S. dollars for Latin American countries and members of the OECD. The employees' compensation share of GDP at factor cost for Latin American countries is calculated using data from *El Anuario Estadístico 1989 de la Comisión Económica para América Latina y el Caribe*. The employees' compensation share of GDP at factor cost for OECD countries is calculated using the *OECD Intersectoral Database*. Figures for GDP per capita in U.S. dollars in 1970 are from the *World Tables 1989–90*.

[7] I could have tried to construct a "typical" transition path, but the long time series' needed for the project are not available for many countries. In addition, I would need to assign each country an approximate date at which it began the transition, a date that is difficult to establish.

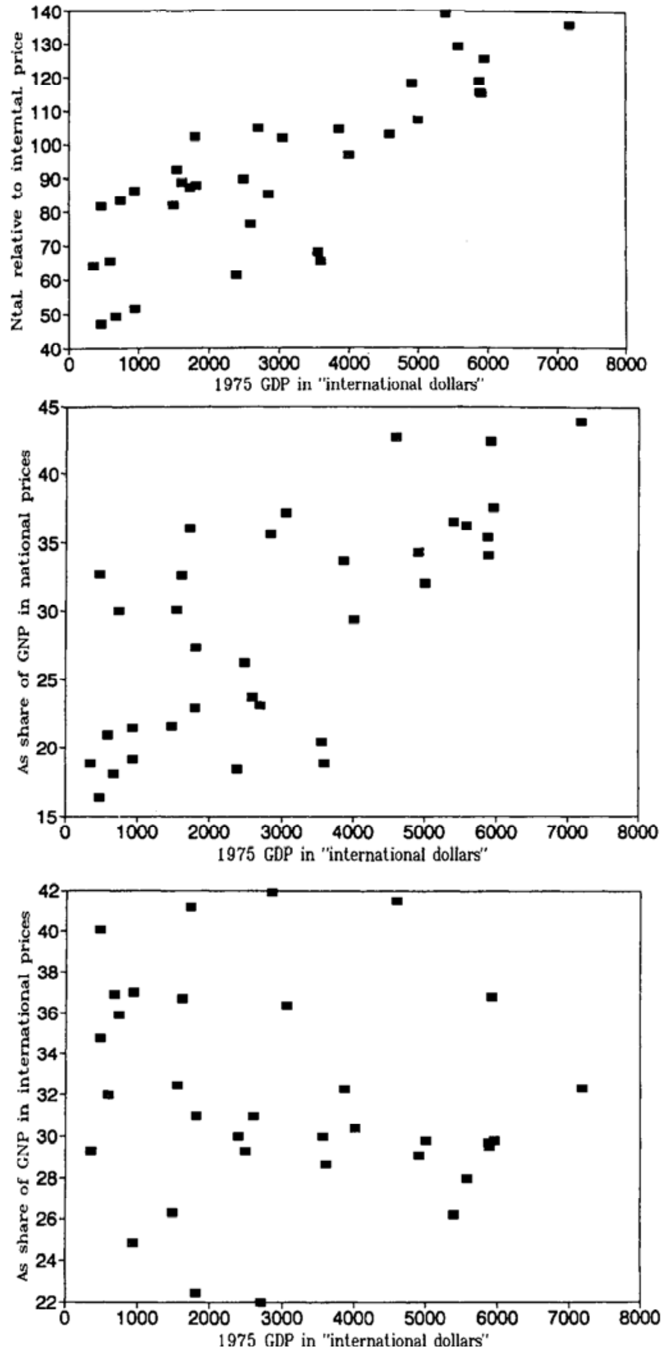

FIGURE 3

SERVICES (KRAVIS, HESTON AND SUMMERS 1982)

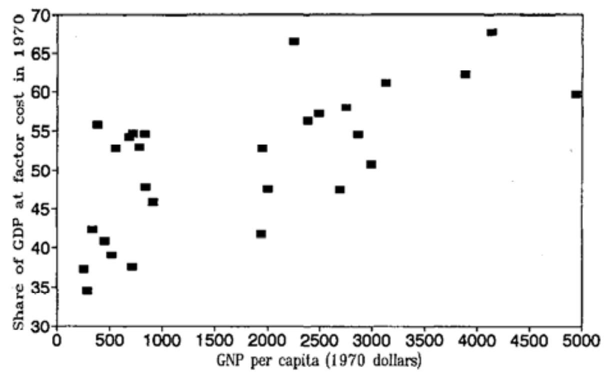

FIGURE 4

EMPLOYEES COMPENSATION AS SHARE OF GDP (CEPAL 1990, OECD 1990, AND WORLD BANK 1990)

3. THE MODEL

A key assumption in this research is that there is nothing intrinsically different between countries: all countries are the same, except for the year in which they start the development process. We have three goods: primary, manufacturing, and service. Factor intensities in the three sectors are different and each sector has a different exogenous rate of technological change. Capital is produced in the second sector (manufacturing) and distributed among the three sectors. In each country there is only one consumer and the amount of labor, normalized to one, is the same in every period. Each country is a different closed economy. In such a model the country planner's problem and the equilibrium coincide. Preferences are time separable, and leisure does not provide utility.

There are two factors of production, labor and capital, which are not sector specific. The initial level of technology in the three sectors is the same in all countries, but the level of technology for each sector at a given point in time differs across countries depending on how long they have been in the development process. For instance, the production function for primaries is $[A\mu^{t-a_i}K_{1it}^{\theta}L_{1it}^{1-\theta}]$, where K_{1it} denotes capital in primaries in country i in year t, L_{1it} represents labor in primaries in country i in year t, $(\mu - 1)$ is the rate of technological change in primaries, A is the "initial efficiency parameter" corresponding to the starting level of technology in primaries, and a_i denotes the year in which the country started the development process.

The three goods are consumption goods, although manufacturing can be used as an investment good as well. Preferences are assumed to be nonhomothetic to capture different income elasticities for the three goods. Country i's consumer utility function for each period is of the following form: $U(C_{i1}, C_{i2}, C_{i3}) = \sum_{j=1}^{3} \alpha_j \log C_{ij} - \eta C_{ij}^{-\rho_j}$; with $\rho_j > 0$.

Countries are also the same in initial endowments since it is not relevant to analyze the effect of different endowments on variation in growth rates. More specifically, capital per capita at the moment at which a given country starts this development process is the same in all countries.

By defining $t = \tau - a_i$, we can eliminate a country's subscript i and rewrite the planner's problem for every country as

$$\text{Max} \sum_{t=0}^{\infty} \beta^t U(C_{1t}, C_{2t}, C_{3t})$$

$$= \sum_{t=0}^{\infty} \beta^t \sum_{j=1}^{3} \alpha_j \log C_{jt} - \eta C_{jt}^{-\rho_j}$$

subject to

$$(1) \qquad K_{t+1} = (1-\delta)K_t + I_t$$

$$(2) \qquad C_{1t} = \mu^t A K_{1t}^{\theta} L_{1t}^{1-\theta}$$

$$(3) \qquad C_{2t} + I_t = \lambda^{(1-\gamma)t} B K_{2t}^{\gamma} L_{2t}^{1-\gamma}$$

$$(4) \qquad C_{3t} = \nu^t C K_{3t}^{\varphi} L_{3t}^{1-\varphi}$$

$$(5) \qquad K_t = K_{1t} + K_{2t} + K_{3t}$$

$$(6) \qquad 1 = L_{1t} + L_{2t} + L_{3t}$$

$$(7) \qquad K_0 = \overline{K},$$

where $\overline{K}$ is the amount of capital at a_i, equal for every country i.

The time series generated by this model show how different variables behave at different stages in the development process. Each time series can also be interpreted as a sample of cross-country values for a particular variable. Each point t in the time series corresponds to a country i whose a_i equals $(T-t)$, where T refers to the period of the cross-country observation.

The equilibrium path of this model has an asymptotic limit in which labor in the three sectors remains constant while capital in the three sectors, total capital, investment and consumption of manufactured goods all grow at the same rate, $(\lambda - 1)$. Consumption of primaries, on the other hand, grows at a rate $(\mu\lambda^{\theta} - 1)$, while consumption of services grows at a rate $(\nu\lambda^{\varphi} - 1)$.

The asymptotic limit of the equilibrium path of this model can be constructed setting the parameter η in the utility function to zero. I choose the specification of nonhomothetic preferences because these preferences have the feature that, as C_{ij} grows, the second component of the function $(-\eta_j C_{ij}^{-\rho_j})$ becomes relatively less important. For high levels of consumption, then, the consumer behaves as one whose preferences are of the Cobb-Douglas form[8] or, equivalently, as one for whom

[8] Other utility functions that have this feature, such as the Stone-Geary function, $U(C_{i1}, C_{i2}, C_{i3}) = \sum_{j=1}^{3} \alpha_j \log(C_{ij} - \eta_j)$, have some unpleasant additional features. This function is undefined for levels of C_{ij} lower than η_j. Also, marginal utilities go to infinity as C_{ij} approaches to η_j from the right.

the parameter η in the utility function is equal to zero. With Cobb-Douglas preferences, for certain values of the parameters and $\beta < 1$, there exists a steady state and this steady state constitutes the asymptotic limit of the equilibrium path of the model with nonhomothetic preferences.

Formal proofs of the existence and uniqueness of the steady state for the model with Cobb-Douglas preferences are presented in the Appendix. I sketch out the argument as to why here. The planner's problem can be transformed into a problem without growth by dividing all variables by their growth factors. The condition $k_2 < (k_1 L_1 + k_3 L_3)/(L_1 + L_3)$, where L_i and k_i are labor and the capital/labor ratio in sector i in the steady state, respectively, guarantees a unique steady state for this transformed economy. (A sufficient, but not necessary, condition is for manufacturing to be less capital intensive than either agriculture and services.) The solution to the transformed problem, in which there is no growth, corresponds to the solution to the original problem.

Within this steady state, labor in the three sectors remains constant while capital in the three sectors, total capital, and investment all grow at the same rate, $(\lambda - 1)$; and $I_t = (\lambda + \delta)K_t$. That is, the proportion of both inputs allocated to each sector is constant. Consumption of manufactured goods also grows at the rate $(\lambda - 1)$, but consumption of primaries grows at a rate $(\mu\lambda^{\theta} - 1)$, while consumption of services grows at a rate $(\nu\lambda^{\varphi} - 1)$. The growth rates are partially due to their own exogenous rate of technological change, and partially due to the increase in capital.

Each sector's share of nominal output is constant since changes in relative prices compensate for lower or higher growth. In real terms, however, the proportions between the three goods change at a constant rate. Asymptotically, one sector dominates the whole economy while the other two goods proportionally disappear, although they grow in absolute terms. For example, the amount of food in the market may increase, but the share of food in consumption decreases. Prices of these two goods, relative to the price of the good that dominates, go to infinity. The sector that overtakes the whole economy depends on the values of the parameters. The most dynamic sector, the one with the higher rate of exogenous technological change, is not necessarily the one that overtakes the economy since the growth of each sector depends partially on its own rate of change and partially on its capital share.

4. COMPUTATION

The computational method proposed here forces the economy with nonhomothetic preferences to converge to its asymptotic limit, the steady state of the economy with homothetic preferences. We are trying to find an equilibrium for the model given an initial capital, K_0. In each period the rest of the variables are implicitly a function of K_t and K_{t+1}:

$$(C_{1t}, C_{2t}, C_{3t}, I_t, K_{1t}, K_{2t}, K_{3t}, L_{1t}, L_{2t}, L_{3t}) = \mathbf{x}(K_t, K_{t+1}).$$

The strategy is to transform this problem into a problem without growth by dividing all variables by their growth factors, solving the transformed problem, and then reconstructing the solution to the original one.

As stated above, these nonhomothetic preferences have the feature that, at high levels of income, the consumer behaves according to Cobb-Douglas preferences. Therefore, a solution to this problem will lie arbitrarily close to the solution to the problem with homothetic preferences, after a given period of time. After the same period of time, let us say n periods, the solutions to both transformed economies will lie arbitrarily close to each other; in other words, the stock of capital of this transformed economy will be arbitrarily close to the stock of capital of the transformed economy with Cobb-Douglas preferences.

Thus we are trying to solve a system of n Euler equations of the transformed economy

$$(8)\quad F(k_t, k_{t+1}, k_{t+2}, \mathbf{x}(k_t, k_{t+1}), \mathbf{x}(k_{t+1}, k_{t+2})) = 0 \qquad \text{for } t = 0, \dots, n-2$$

$$(9)\quad F(k_{n-1}, k_n, \mathbf{k}, \mathbf{x}(k_{n-1}, k_n), \mathbf{x}(k_n, \mathbf{k})) = 0 \qquad \text{for } t = n-1$$

where k_t refers to the transformed value of the stock of capital at period t and $\mathbf{k}$ the transformed stock of capital of the steady state. This method imposes the transformed stock of capital of the steady state as the final condition.

I use a nonlinear Gauss-Seidel algorithm to find the vector of values of the state variable, capital, that solves this system of nonlinear equations for the transformed economy. The Gauss-Seidel algorithm solves a system of n equations and n unknowns $G(\mathbf{z}) = 0$, by an iterative process. Given an initial vector of values for the unknowns, $\mathbf{z}^0 = (z_1^0, \dots, z_n^0)$, it generates another vector, $\mathbf{z}^1$, by solving for the ith component z_i^1 using the ith equation and taking the other components of the vector, $\mathbf{z}_{-i}^0 = (z_i^0, \dots, z_{i-1}^0, z_{i+1}^0, \dots, z_n^0)$, as given; that is, each equation is used to find a component of the new vector. $\mathbf{z}^1$ is then used to generate another vector, $\mathbf{z}^2$, and so on. The process stops when the norm of the difference between two consecutive vectors, $\mathbf{z}^k$ and $\mathbf{z}^{k+1}$, is less than an arbitrarily small value.

Instead of using the Euler equation for the last period, I use its linear approximation, calculated by taking the first order Taylor expansion of equation (8) around the value of the transformed stock of capital at the steady state, $\mathbf{k}$. This is similar to the linear quadratic approximation method proposed by Kydland and Prescott (1982). Using this linearization in the last period allows the number of periods yielding similar paths to be shortened.

Once a solution to the system is found it is easily transformed into the vector of values for capital corresponding to an equilibrium of the original economy. Since the other variables in each period are functions of capital in that period and the following period, a vector of values for capital allows us to generate any series which are of interest.

5. CALIBRATION

To study the behavior of the economy described above, we need to assign values to its parameters. These values are chosen based on features of the data following the calibration procedure of Kydland and Prescott (1982).

According to the model, some relations will only hold when the economy is close to the steady state while others will hold at any moment. For assessing the relations

TABLE 2
SHARES OF LABOR COMPENSATION ON VALUE ADDED IN THE THREE SECTORS IN DIFFERENT COUNTRIES*

	Share 1	Share 2	Share 3
Australia	0.197	0.519	0.511
Canada	0.25	0.593	0.514
Germany	0.231	0.642	0.375
Denmark	0.18	0.693	0.485
Finland	0.189	0.633	0.513
France	0.151	0.656	0.46
Great Britain	0.294	0.642	0.527
Italy	0.29	0.514	0.34
Japan	0.227	0.518	0.48
Netherlands	0.183	0.542	0.549
Norway	0.115	0.501	0.567
Sweden	0.254	0.676	0.515
United States	0.212	0.627	0.509
Average	0.213	0.597	0.488
Minimum	0.115	0.501	0.340
Maximum	0.294	0.693	0.567
Variance	0.002	0.004	0.004
Standard Deviation	0.050	0.066	0.062

**Source:* OECD (1990). Italy reports only labor compensation for Wholesale, Retail Trade, Restaurants and Hotels; and Transport, Storage and Communication in the third sector, Services.

that only hold close to the steady state I will look at the United States in the period 1976–1985, using the *Economic Report of the President 1990*, when possible. In other words, I am making the assumption that the U.S. today is a mature economy.

5.1. *Factor Intensities and Rates of Technical Change.* Shares of capital and labor in the three sectors and rate of change of total factor productivity are relations that hold at any moment. To calibrate factor intensities and rates of technological change for the three sectors, I look at OECD countries over a period of roughly fifteen years. The basic reason for limiting the sample to OECD countries is that data on labor and capital stock disaggregated by sectors is not easily available for other countries.

Table 2 shows the average over thirteen years (1976–1988) of the share of labor compensation on GDP in the three sectors for some OECD countries. These labor compensation shares correspond to $(1-\theta)$, $(1-\gamma)$, and $(1-\varphi)$, respectively, in the model. The first sector includes agriculture. The second includes mining and quarrying; electricity, gas and water; manufacturing; and construction. The third includes wholesale, retail trade, restaurants and hotels; transport, storage and communication; finance, insurance and real estate; and community, social, and personal services. Mining and quarrying is included in the second sector instead of in the first. This is done for consistency since some of these countries include mining and quarrying in manufacturing, and I do not have access to the disaggregation of manufacturing for these countries. Producers of government services are not included as the way in which output of this sector is accounted for in many countries makes the labor share of this sector close to one. The shares are fairly constant within a given country over this period.

TABLE 3
GROWTH FACTORS OF THE SOLOW RESIDUALS IN THE THREE SECTORS IN DIFFERENT COUNTRIES*

	Sector 1	Sector 2	Sector 3
Australia	1.025	1.011	1.001
Canada	0.989	1.003	1.002
Germany	1.005	1.013	1.009
Denmark	1.016	1.012	1.007
Finland	0.996	1.02	1.013
France	0.99	1.016	0.999
Great Britain	1.022	1.021	1.008
Italy	0.995	1.01	0.995
Japan	0.951	1.013	1.003
Netherlands	1.01	1.013	1.01
Norway	0.989	1.029	1.01
Sweden	0.984	1.015	1.005
United States	1.007	1.007	1.005
Average	0.998	1.014	1.005
Minimum	0.951	1.003	0.995
Maximum	1.025	1.029	1.013
Variance	0.00035	0.00004	0.00002
Standard Deviation	0.019	0.006	0.005

$t(24) = ((\text{avg}\,2 - \text{avg}\,1)^*(n-1)^{**}1/2)/((\text{var}\,2 + \text{var}\,1)^{**}1/2) = 2.7659$

$t(24) = ((\text{avg}\,2 - \text{avg}\,3)^*(n-1)^{**}1/2)/((\text{var}\,2 + \text{var}\,3)^{**}1/2) = 3.9956$

* *Source:* OECD (1989).

Table 3 shows the average of the growth factors (μ, $\lambda^{(1-\gamma)}$ and ν) of the Solow residuals in the three sectors over 15 years (1970–85) for some OECD countries. The Solow residuals are calculated using a Cobb-Douglas function with parameters implied by Table 2 and the *OECD International Sectoral Database* that reports data on total employment and capital stock for different sectors. Of course, rates of change within each country vary greatly from one year to another. Changes in total factor productivity are significantly higher in the second sector than in the other two.

Notice that the disaggregation of the services sector shows that transport, storage, and communication have a higher rate of technical change than the other components of services—a rate as high as that of manufacturing (Table 4).

The parameters that result from the estimations in this subsection for the manufacturing production function (labor share equal to 0.6 and technological change parameter equal to 1.4) are close to the usual ones found in the literature from Solow (1957) onwards. According to the data, manufacturing is more labor intensive than both agriculture and services, thus satisfying the sufficient condition for the existence of the unique steady state stated in Section 3.

5.2. *Discount Factor.* According to the model, $\beta = \lambda/(1 - \delta + R)$ in the steady state. Since this is a steady state relation, I use U.S. data to calculate β. According to the *Economic Report of the President 1990*, the average of the corporate bond rate (Moody's Aaa) minus the change in the GNP deflator over the 1976–1985 period, is 5%. Taking this figure as net return to capital implies a discount rate β for the model of 0.9743, since λ equals 1.023.

TABLE 4
DECOMPOSITION OF THE GROWTH FACTOR OF THE SOLOW RESIDUALS IN SERVICES*

	Retail	Transportation	Finance	Social Services
Australia	0.990	1.035	0.996	1.002
Canada	1.000	1.022	0.996	1.012
Germany	1.006	1.019	1.012	0.988
Denmark	1.014	0.989	1.007	0.998
Finland	1.021	1.015	1.009	0.997
France	0.993	1.014	1.000	0.990
Great Britain	0.996	1.021	1.011	1.005
Italy	1.001	1.015		
Japan	1.018	1.013	1.002	0.966
Netherlands	1.009	1.001		1.019
Norway	1.009	1.026	0.991	0.995
Sweden	1.006	1.019	0.997	1.009
United States	0.997	1.013	1.001	1.003
Average	1.005	1.016	1.002	0.999
Minimum	0.990	0.989	0.991	0.966
Maximum	1.021	1.035	1.012	1.019
Variance	0.00008	0.00012	0.00004	0.00017
Standard Deviation	0.009	0.011	0.007	0.013

* *Source:* OECD (1989). Netherlands includes Finance, Insurance and Real Estate with Community, Social and Personal Services.

5.3. *Preferences.* For calibrating preferences, I need data on prices and quantities consumed (or alternatively on expenditures and either prices or quantities consumed) for three goods—primaries, manufactured goods and services—in different countries. We have data on expenditures and on price indices for different categories of goods and many countries, but price indices are not comparable across countries and they do not give us information about differences in relative prices. Kravis, Heston and Summers (1982, Tables 6.1 and 6.5) have data on expenditures in 1975 for thirty-four countries,[9] both in national currencies and in international dollars. The purpose of measuring expenditures in international dollars is to make the quantities comparable across countries by eliminating the effects of different relative prices in different countries. I use expenditures in international dollars as quantities and expenditures in national currencies as expenditures. Expenditures in national currencies are corrected by the exchange rate (Kravis, Heston and Summers 1982, Table 1.1). This converts expenditures to similar units (1975 dollars), conserving the impact of different relative prices in different countries. I then minimize the sum of the squares of the first order conditions for utility maximization in each country.

Primaries includes food, beverages and tobacco. Manufactures includes clothing and footwear, rent and fuel, house furnishings and operations, and transportation equipment. Services includes medical care, recreation and education, transports and communication minus transportation equipment, and other expenditures.

[9] The countries are Malawi, Kenya, India, Pakistan, Sri Lanka, Zambia, Thailand, Philippines, Korea, Malaysia, Colombia, Jamaica, Syria, Brazil, Romania, Mexico, Yugoslavia, Iran, Uruguay, Ireland, Hungary, Poland, Italy, Spain, United Kingdom, Japan, Austria, Netherlands, Belgium, France, Luxembourg, Denmark, Germany, and the United States.

5.4. *Depreciation Rate*. According to the *OECD International Sectoral Database*, the proportion of stock of capital to GDP in the U.S. over the period 1976–1985 is 3.5. Using the parameters I already have, and a nonlinear system of equations representing the steady state for the transformed economy, and taking account of this proportion results in a depreciation rate of 8% for the model.

5.5. *Initial Efficiency Parameters*. The initial efficiency parameters A, B and C affect the unit of measurement of the three goods. Usually these parameters are set equal to one and the units of the three goods are accordingly chosen. The method I use for calibrating the preferences already implies units of measurement for agriculture, industry and manufacturing, so I am not free to set these parameters equal to one.

I calculate relative prices for the three units for the year 1975 in the United States, dividing expenditures in U.S. dollars (see Tables 6.1 and 6.5 in Kravis, Heston and Summers 1982) by quantities in international dollars (see Table 6.5 in the same source). The *Economic Report of the President* (1990, Table B-60) allows me to recalculate relative prices for 1990 using Consumer Price Indexes by groups. With these relative prices, the value of GNP for 1990 from the same source, and using the set of equations that characterize the steady state of the transformed economy, I am able to set values for the parameters A, B, and C.

To summarize, the set of parameters I use is as follows:

β	α_1	α_2	α_3	η	ρ_1	ρ_2	ρ_3
0.9743	0.19	0.36	0.45	1.15	10	9	0

δ	A	B	C	θ	γ	φ	μ	λ	ν
0.08	0.4315	1.6529	0.6661	0.79	0.4	0.51	0.	0.023	0.005

6. RESULTS AND DISCUSSION

Since in this paper I do not intend to assess the effect of dramatic falls in the capital-to-output ratio (due, let us say, to wars or similar disasters) when solving for the transformed economy I take as initial capital the level of the steady state. The results of the model are then compared to the data presented in Section 2.

In the simulation in this paper, the growth rate goes up initially and then decreases. The range of movement is 2.1%, from 0.5% to 2.6%. If we run a quadratic regression between growth rates of GDP per capita 1950–1980 and 1950 GDP per capita, to fit Figure 1, the range of movement in the regression curve would be 1.7%, from 1.6% to 3.3%. The range of movement in the simulation is, then, slightly larger than the range of movement in the quadratic regression curve fitting Figure 1. Figure 5 plots growth rates, measured in the standard way (using a Paasche index, equivalent to the GDP Deflator), versus National Product, measured at constant prices. International dollars are used in Figure 1 to eliminate the effects

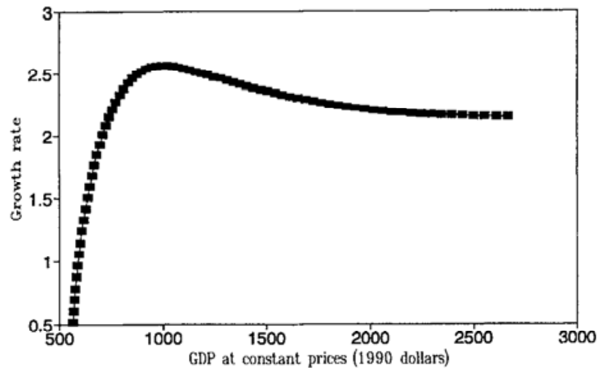

FIGURE 5

GROWTH RATE

of different relative prices across countries. Thus, if we assume that different countries are at different points in this transitional path, then Figure 5 is the equivalent of Figure 1: a plot of growth rates versus GDP measured in international dollars. In the quadratic regression curve fitting Figure 1, growth rates would peak at 2135 (1980) international dollars. According to Easterly (1994), growth rates peak between 1366 and 2032 (1985) dollars. In the simulation in this paper, growth rates peak at the equivalent of 1017 (1990) international dollars.

I conducted other simulations with different rates of technical progress in agriculture and manufacturing; different incomes elasticities and different values of the ρ parameters. In these simulations the manufacturing sector is always the sector with the highest rate of technical change and labor intensity, and primaries are always the most inelastic good. The results show the same inverted *U* shape that characterizes the relation between income per capita and growth rates.

Nonhomotheticity of preferences drive this result: a poor country, which consumes mainly necessities, cannot save (invest) much. As it gets richer it will invest or save more, thereby encouraging growth. At the same time production will shift to the second sector, which has a higher rate of technical change; thus, the first effect is reinforced. Both effects, increase in investment and increase in average total factor productivity, imply an acceleration in the growth rate.

Yet, the savings rate (net savings or investment as percentage of GDP) does not increase monotonically over time. As can be seen in Figure 6, its simulated time series is hump-shaped.[10] Eventually the savings rate falls, thus driving the growth rate down. If, at the same time, production shifts to the third sector with its lower rate of technical change, that will reinforce this effect. One should keep in mind that this is the optimal path. A simulation with the same productivity growth rate in the three sectors and the same preferences shows a range of movement of the growth rates of 0.8%. The difference between this range of movement and the range

[10] Christiano (1989) obtains a similar shape for the savings rate time series.

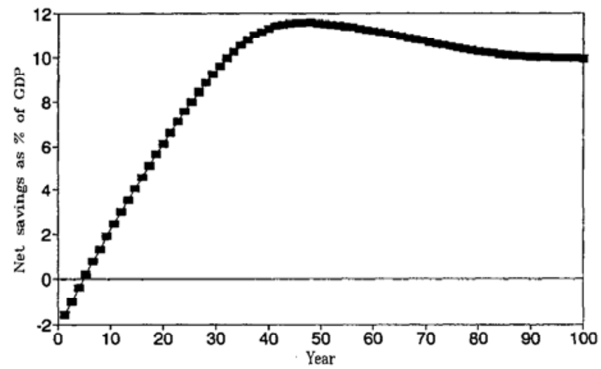

FIGURE 6

SAVINGS RATE

of movement of the present simulation is 1.3%. This difference can be interpreted as the amplifying effect of the difference in productivity growth rates across sectors, and it is larger than the effect of nonhomothetic preferences.

According to the model, development means that the national product share of agriculture falls from 26% to 12% while the national product share of manufactures rises from 53% to 60% and the share of services in the national product rises from 21% to 28%. If we plot these shares versus GDP at constant prices we obtain Figure 7, the equivalent of Figure 2.

Development, in this case, also means that manufactured goods get cheaper with respect to other goods. The price of services relative to manufactures rises from 1.9 to 2.9, an increase of 53%, while the price of agriculture relative to manufactures rises from 1 to 1.4, an increase of 40%. If we interpret different points of the transitional path as different countries, Figure 8 depicts the equivalent of Figure 3. In part 3 of Figure 7, services as a share of GDP are plotted versus GDP at constant prices. The expansion of services as a share of GDP is accounted for by the increase in the relative price of services. Part 1 of Figure 8 plots the price of services for each point relative to an "international price" versus GDP at constant prices, the international price being an average of the price of services at each point weighted by the quantity of services produced at the same point. Once we eliminate the price effect, the proportion of the real output that services represent does not increase monotonically with income. As we can see in part 2 of Figure 8, the relation between income and services as a share of real output is positive for low levels of income and negative for higher levels of income. Part 2 shows services as a share of GDP measured in international prices, thus eliminating the effects of different relative prices at different points.

The share of employees' compensation of GDP rises (from 48% to 52%) as the economy shifts from agriculture to manufactures, which is more labor intensive. Figure 9 would be the equivalent of Figure 4 since it displays the relation between compensation of employees as a share of GDP and GDP at constant prices.

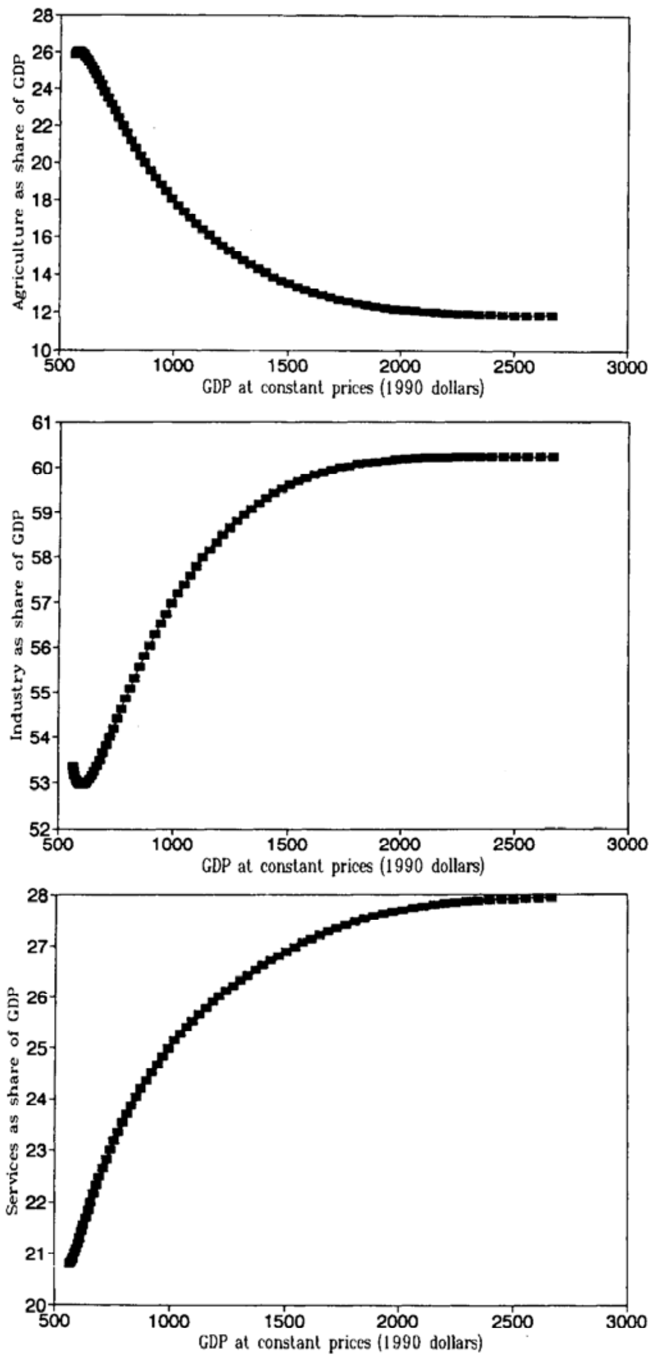

FIGURE 7

VALUE ADDED

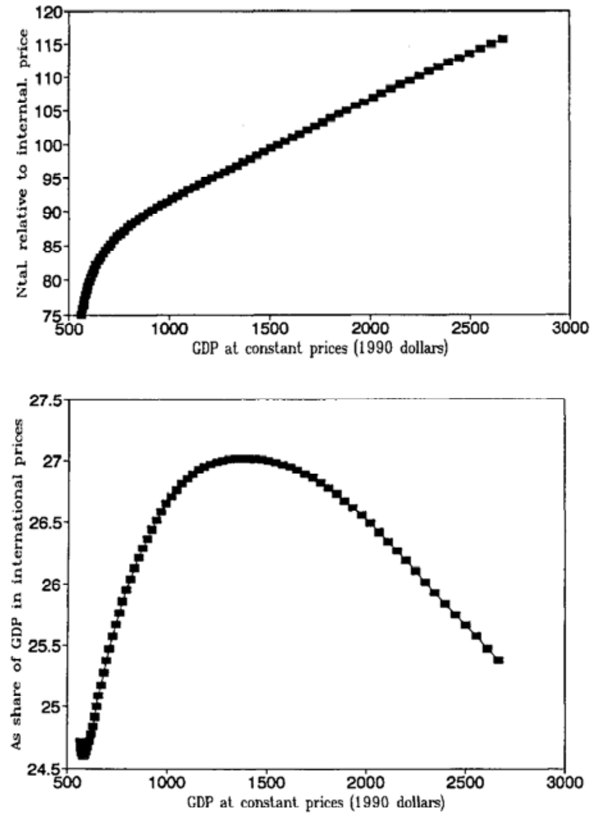

FIGURE 8

SERVICES

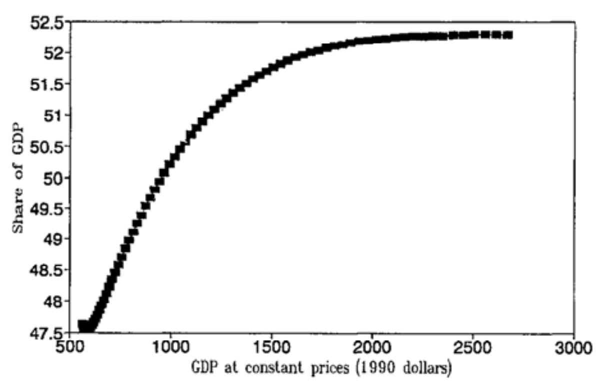

FIGURE 9

EMPLOYEES COMPENSATION AS SHARE OF GDP

Finally the share of primaries in labor goes down from 12% to 5%. This movement is compensated for by an increase in the labor share of manufactures of two points, from 67% to 69%, and an increase of the share of services in labor of five points, from 21% to 26%. In the same way the share of primaries in consumption decreases from 35% to 19%. This is compensated for by an increase of 16 points in the share of services, from 29% to 45%.

7. CONCLUSIONS

As Lucas (1988) points out: "The poorest countries tend to have the lowest growth; the wealthiest next; the middle income countries highest."

In this paper the relation between income levels and rates of growth is explained as an effect of changes in sectoral composition driven by different income elasticities for primaries, manufacturing and services. Thus, sectoral composition explains an important part of the variation in growth rates observed across countries. The magnitude of this variation in the simulation presented is quite important; the variation in growth rates is greater than 2 points while most countries have an average growth rate smaller than 4%.

The model in this paper can also replicate some differences between rich and poor countries: different relative prices for the three goods, differences in the sectoral composition of both production and consumption, different distribution of the labor force between sectors, and differences in the distribution of the national product by factors.

The question arises as to why the rate of technical change in manufacturing is greater than in the other two sectors. To answer this question we must make technical change endogenous. In this model, the rates of technical change are assumed to be external as a simplifying assumption. Backus, Kehoe and Kehoe (1992), when looking for scale effects implied by some of the endogenous growth theories, find that these scale effects are more important in manufacturing. Further study could possibly explain why the dynamic effects of learning-by-doing, investing in human capital, research and development, and development of new products, are more important in manufacturing.

APPENDIX

EXISTENCE AND UNIQUENESS OF THE UNBALANCED GROWTH PATH

The related problem with homothetic preferences can be posed as

$$\operatorname{Max} \sum_{t=0}^{\infty} \beta^t \frac{1}{\sigma} \left(C_{1t}^{\alpha_1} C_{2t}^{\alpha_2} C_{3t}^{\alpha_3} \right)^{\sigma},$$

subject to (1)–(7). Logarithmic preferences are a special case ($\sigma = 0$) of these more general preferences. In the unbalanced growth path, the growth factor for investment, total capital, capital in the three sectors, and the second good is λ. Labor in the three sectors remains constant and the growth factor for the first good is $(\mu\lambda^{\theta})$, while that of the third good is $(\nu\lambda^{\varphi})$. These growth factors constitute the initial guess.

Let us define variables with an asterisk as "detrended" variables or variables normalized by their own growth factor and $\beta^* = \beta\mu^{\alpha_1\sigma}\lambda^{\chi\sigma}\varphi^{\alpha_3\sigma}$, where $\chi = \theta\alpha_1 + \alpha_2 + \varphi\theta\alpha_3$. Multiplying and dividing each term of the summatory of the objective function by $[\mu^{\alpha_1\sigma}\lambda^{\chi\sigma}\varphi^{\alpha_3\sigma}]^t$ we obtain $\beta^{*t}(C_{1t}^{*\alpha_1}C_{2t}^{*\alpha_2}C_{3t}^{*\alpha_3})^\sigma/\sigma$ for each term. By dividing the sets of constraints (1), (3) and (5) by λ^t, the set of constraints (2) by $[\mu\lambda^\theta]^t$ and the set of constraints (4) by $[\nu\lambda^\varphi]^t$, we obtain the new set of constraints

$$\lambda K_{t+1}^* = (1-\delta)K_t^* + I_t^* \tag{A.1}$$

$$C_{1t}^* = AK_{1t}^{*\theta}L_{1t}^{1-\theta} \tag{A.2}$$

$$C_{2t}^* + I_t^* = BK_{2t}^{*\gamma}L_{2t}^{1-\gamma} \tag{A.3}$$

$$C_{3t}^* = CK_{3t}^{*\varphi}L_{3t}^{1-\varphi} \tag{A.4}$$

$$K_t^* = K_{1t}^* + K_{2t}^* + K_{3t}^* \tag{A.5}$$

$$1 = L_{1t} + L_{2t} + L_{3t} \tag{A.6}$$

$$K_0 = \bar{K}. \tag{A.7}$$

Therefore the problem for this transformed economy is

$$\text{Max} \sum_{t=0}^{\infty} \beta^{*t}\frac{1}{\sigma}\left(C_{1t}^{*\alpha_1}C_{2t}^{*\alpha_2}C_{3t}^{*\alpha_3}\right)^\sigma,$$

subject to (A.1)–(A.7).

In what follows, I will use variables without the time subindex and without the asterisk to denote the value of the variables in the steady state of the transformed economy; that is, the value of the variables in the unbalanced growth path normalized by their own trend. If a steady state exists for this transformed economy, we know the value of R, return to capital, since according to the model, $\beta(1-\delta-R) = \lambda$ in this steady state. This return to capital implies a capital/labor ratio, k_2, for the second sector, and this ratio, in turn, implies a wage. The relation between the remuneration of the inputs implies a capital/labor ratio for the other two sectors, k_1 and k_3. These three ratios will result in relative prices for goods 1 and 3 in terms of good 2.

Denote $\varepsilon_1 = C_1/C_2$ and $\varepsilon_3 = C_3/C_2$. Five linear equations characterize the unbalanced growth path of this transformed economy:

$$\varepsilon_1 C_2 = Ak_1^\theta L_1$$

$$C_2 + (\lambda - 1 + \delta)K = Bk_2^\gamma L_2$$

$$\varepsilon_3 C_2 = Ck_3^\varphi L_3$$

$$L_1 + L_2 + L_3 = 1$$

$$k_1 L_1 + k_2 L_2 + k_3 L_3 = K.$$

The system can be reduced to two equations:

$$(\lambda - 1 + \delta)K = -\frac{Ck_3^{\varphi}}{\varepsilon_3(1+L)} + \left[Bk_2^{\gamma} + \frac{Ck_3^{\varphi}}{\varepsilon_3(1+L)}\right]L_2$$

$$K = \frac{k_1 L + k_3}{1+L} + \left[k_2 - \frac{k_1 L + k_3}{1+L}\right]L_2,$$

where $L = L_1/L_3 = (Ak_1^{\theta}\varepsilon_3)/(Ck_3^{\varphi}\varepsilon_1)$.

Clearly, the first implies that K is a monotonically increasing function of L_2. The second is monotonically decreasing in L_2 provided that $k_2 < (k_1 L + k_3)/(1+L)$. K is then positive if the line corresponding to the second equation in the (K, L) space cuts the L axis farther from the origin than the line corresponding to the first equation in the same space. So, K is positive if

$$\frac{k_1 L + k_3}{k_1 L + k_3 - k_2 - k_2 L} > \frac{Ck_3^{\varphi}}{Ck_3^{\varphi} + Bk_2^{\gamma}\varepsilon_3(1+L)},$$

which will always hold true since the left hand term is greater than one and the right hand one is smaller than one.

Thus, condition $k_2 < (k_1 L + k_3)/(1+L)$ guarantees a unique steady state. A sufficient but not necessary condition is for manufacturing to be less capital intensive than either agriculture and services. The first-order and transversality conditions are sufficient for an equilibrium for $\beta^* < 1$.[11] A solution to the transformed problem, in which there is no growth, corresponds to a solution to the original problem, that is, this steady state corresponds to an unbalanced growth path in the original problem.

The existence of an unbalanced growth path is not guaranteed for other homothetic preferences. The production side requires the relative price for two goods to move in opposite directions by the same amount as the proportion between the two goods. Therefore, if we are to sustain this unbalanced growth path as an optimum, we need preferences that also require the relative price for two goods to move in the opposite direction of their proportion and by the same amount.

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[11] See Stokey and Lucas (1989), p. 98.

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