### Applied general equilibrium models that put the standard theory to work do not well in predicting the impact of trade liberalization experiences like NAFTA.

Applied general equilibrium models were the only analytical game in town when it came to analyzing the impact of NAFTA in 1992-1993.

Typical sort of model: Static applied general equilibrium model with large number of industries and imperfect competition (Dixit-Stiglitz or Eastman-Stykolt) and finite number of firms in some industries. In some numerical experiments, new capital is placed in Mexico owned by consumers in the rest of North America to account for capital flows.

Examples: Brown-Deardorff-Stern model of Canada, Mexico, and the United States Cox-Harris model of Canada Sobarzo model of Mexico T. J. Kehoe, "An Evaluation of the Performance of Applied General Equilibrium Models of the Impact of NAFTA," in T. J. Kehoe, T. N. Srinivasan, and J. Whalley, editors, *Frontiers in Applied General Equilibrium Modeling: Essays in Honor of Herbert Scarf*, Cambridge University Press, 2005, 341-77.

Research Agenda:

- Compare results of numerical experiments of models with data.
- Determine what shocks besides NAFTA policies were important.
- Construct a simple applied general equilibrium model and perform experiments with alternative specifications to determine what was wrong with the 1992-1993 models.

## **Applied GE Models Can Do a Good Job!**

**Spain:** Kehoe-Polo-Sancho (1992) evaluation of the performance of the Kehoe-Manresa-Noyola-Polo-Sancho-Serra MEGA model of the Spanish economy: A Shoven-Whalley type model with perfect competition, modified to allow government and trade deficits and unemployment (Kehoe-Serra). Spain's entry into the European Community in 1986 was accompanied by a fiscal reform that introduced a value-added tax (VAT) on consumption to replace a complex range of indirect taxes, including a turnover tax applied at every stage of the production process. What would happen to tax revenues? Trade reform was of secondary importance.

**Canada-U.S.:** Fox (1999) evaluation of the performance of the Brown-Stern (1989) model of the 1989 Canada-U.S. FTA.

### **Other changes besides policy changes are important!**

### Changes in Consumer Prices in the Spanish Model (Percent)

	data	model	model	model
sector	1985-1986	policy only	shocks only	policy&shocks
food and nonalcoholic beverages	1.8	-2.3	4.0	1.7
tobacco and alcoholic beverages	3.9	2.5	3.1	5.8
clothing	2.1	5.6	0.9	6.6
housing	-3.3	-2.2	-2.7	-4.8
household articles	0.1	2.2	0.7	2.9
medical services	-0.7	-4.8	0.6	-4.2
transportation	-4.0	2.6	-8.8	-6.2
recreation	-1.4	-1.3	1.5	0.1
other services	2.9	1.1	1.7	2.8
weighted correlation with data		-0.08	0.87	0.94
variance decomposition of change		0.30	0.77	0.85
regression coefficient a		0.00	0.00	0.00
regression coefficient b		-0.08	0.54	0.67

### **Measures of Accuracy of Model Results**

- 1. Weighted correlation coefficient.
- 2. Variance decomposition of the (weighted) variance of the changes in the data:

$$vardec(y^{data}, y^{model}) = \frac{var(y^{model})}{var(y^{model}) + var(y^{data} - y^{model})}.$$

3, 4. Estimated coefficients *a* and *b* from the (weighted) regression

$$x_i^{data} = a + bx_i^{model} + e_i.$$

	data	model	model	model
sector	1985-1986	policy only	shocks only	policy&shocks
agriculture	-0.4	-1.1	8.3	6.9
energy	-20.3	-3.5	-29.4	-32.0
basic industry	-9.0	1.6	-1.8	-0.1
machinery	3.7	3.8	1.0	5.0
automobile industry	1.1	3.9	4.7	8.6
food products	-1.8	-2.4	4.7	2.1
other manufacturing	0.5	-1.7	2.3	0.5
construction	5.7	8.5	1.4	10.3
commerce	6.6	-3.6	4.4	0.4
transportation	-18.4	-1.5	1.0	-0.7
services	8.7	-1.1	5.8	4.5
government services	7.6	3.4	0.9	4.3
weighted correlation with	n data	0.16	0.80	0.77
variance decomposition of	of change	0.11	0.73	0.71
regression coefficient a		-0.52	-0.52	-0.52
regression coefficient b		0.44	0.75	0.67

#### **Changes in Value of Gross Output/GDP in the Spanish Model (Percent)**

#### Changes in Trade/GDP in the Spanish Model (Percent)

	data	model	model	model
direction of exports	1985-1986	policy only	shocks only	policy&shocks
Spain to rest of E.C.	-6.7	-3.2	-4.9	-7.8
Spain to rest of world	-33.2	-3.6	-6.1	-9.3
rest of E.C. to Spain	14.7	4.4	-3.9	0.6
rest of world to Spain	-34.1	-1.8	-16.8	-17.7
weighted correlation wi	th data	0.69	0.77	0.90
variance decomposition	of change	0.02	0.17	0.24
regression coefficient a		-12.46	2.06	5.68
regression coefficient b		5.33	2.21	2.37

	data	model	model	model
variable	1985-1986	policy only	shocks only	policy&shocks
wages and salaries	-0.53	-0.87	-0.02	-0.91
business income	-1.27	-1.63	0.45	-1.24
net indirect taxes and tariffs	1.80	2.50	-0.42	2.15
correlation with data		0.998	-0.94	0.99
variance decomposition of char	nge	0.93	0.04	0.96
regression coefficient a		0.00	0.00	0.00
regression coefficient b		0.73	-3.45	0.85
private consumption	-0.81	-1.23	-0.51	-1.78
private investment	1.09	1.81	-0.58	1.32
government consumption	-0.02	-0.06	-0.38	-0.44
government investment	-0.06	-0.06	-0.07	-0.13
exports	-3.40	-0.42	-0.69	-1.07
-imports	3.20	-0.03	2.23	2.10
correlation with data		0.40	0.77	0.83
variance decomposition of char	nge	0.20	0.35	0.58
regression coefficient a		0.00	0.00	0.00
regression coefficient b		0.87	1.49	1.24

#### **Changes in Composition of GDP in the Spanish Model (Percent of GDP)**

### Public Finances in the Spanish Model (Percent of GDP)

	data	model	model	model
variable	1985-1986	policy only	shocks only	policy&shocks
indirect taxes and subsidies	2.38	3.32	-0.38	2.98
tariffs	-0.58	-0.82	-0.04	-0.83
social security payments	0.04	-0.19	-0.03	-0.22
direct taxes and transfers	-0.84	-0.66	0.93	0.26
government capital income	-0.13	-0.06	0.02	-0.04
correlation with data		0.99	-0.70	0.92
variance decomposition of ch	ange	0.93	0.08	0.86
regression coefficient a		-0.06	0.35	-0.17
regression coefficient b		0.74	-1.82	0.80

# Models of NAFTA Did Not Do a Good Job!

Ex-post evaluations of the performance of applied GE models are essential if policy makers are to have confidence in the results produced by this sort of model.

Just as importantly, they help make applied GE analysis a scientific discipline in which there are well-defined puzzles and clear successes and failures for alternative hypotheses.

### **Changes in Trade/GDP in Brown-Deardorff-Stern Model (Percent)**

	data	model
variable	1988-1999	
Canadian exports	52.9	4.3
Canadian imports	57.7	4.2
Mexican exports	240.6	50.8
Mexican imports	50.5	34.0
U.S. exports	19.1	2.9
U.S. imports	29.9	2.3
weighted correlation with	n data	0.64
variance decomposition of	of change	0.08
regression coefficient a		23.20
regression coefficient b		2.43

	exports to 1	Mexico	exports to United States	
sector	1988–1999	model	1988–1999	model
agriculture	122.5	3.1	106.1	3.4
mining and quarrying	-34.0	-0.3	75.8	0.4
food	89.3	2.2	91.7	8.9
textiles	268.2	-0.9	<b>97.8</b>	15.3
clothing	1544.3	1.3	237.1	45.3
leather products	443.0	1.4	-14.4	11.3
footwear	517.0	3.7	32.8	28.3
wood products	232.6	4.7	36.5	0.1
furniture and fixtures	3801.7	2.7	282.6	12.5
paper products	240.7	-4.3	113.7	-1.8
printing and publishing	6187.4	-2.0	37.2	-1.6
chemicals	37.1	-7.8	109.4	-3.1
petroleum and products	678.1	-8.5	-42.5	0.5
rubber products	647.4	-1.0	113.4	9.5
nonmetal mineral products	333.5	-1.8	20.5	1.2
glass products	264.4	-2.2	74.5	30.4
iron and steel	195.2	-15.0	92.1	12.9
nonferrous metals	38.4	-64.7	34.7	18.5
metal products	767.0	-10.0	102.2	15.2
nonelectrical machinery	376.8	-8.9	28.9	3.3
electrical machinery	633.9	-26.2	88.6	14.5
transportation equipment	305.8	-4.4	30.7	10.7
miscellaneous manufactures	1404.5	-12.1	100.0	-2.1
weighted correlation with data		-0.91		-0.43
variance decomposition of change		0.003		0.02
regression coefficient a		249.24		79.20
regression coefficient b		-15.48		-2.80

#### Changes in Canadian Exports/ GDP in the Brown-Deardorff-Stern Model (Percent)

8 · · · · · · ·				
	exports to	Canada	exports to United States	
sector	1988–1999	model	1988–1999	model
agriculture	-20.5	-4.1	-15.0	2.5
mining and quarrying	-35.5	27.3	-22.9	26.9
food	70.4	10.8	9.4	7.5
textiles	939.7	21.6	832.3	11.8
clothing	1847.0	19.2	829.6	18.0
leather products	1470.3	36.2	618.3	11.7
footwear	153.0	38.6	111.1	4.0
wood products	4387.6	15.0	145.6	-2.7
furniture and fixtures	4933.2	36.2	181.2	7.0
paper products	23.9	32.9	70.3	13.
printing and publishing	476.3	15.0	122.1	3.
chemicals	204.6	36.0	70.4	17.
petroleum and products	-10.6	32.9	66.4	34.2
rubber products	2366.2	-6.7	783.8	-5.
nonmetal mineral products	1396.1	5.7	222.3	3.7
glass products	676.8	13.3	469.8	32.
iron and steel	32.5	19.4	40.9	30.8
nonferrous metals	-35.4	138.1	111.2	156.
metal products	610.4	41.9	477.2	26.8
nonelectrical machinery	570.6	17.3	123.6	18.
electrical machinery	1349.2	137.3	744.9	178.
transportation equipment	2303.4	3.3	349.0	6.2
miscellaneous manufactures	379.4	61.1	181.5	43.2
weighted correlation with data		0.19		0.7
variance decomposition of change		0.01		0.04
regression coefficient a		120.32		38.1.
regression coefficient b		2.07		3.8'

Changes in Mexican Exports/GDP in the Brown-Deardorff-Stern Model (Percent)

	exports to (	Canada	exports to Mexico	
sector	1988–1999	model	1988–1999	model
agriculture	-24.1	5.1	6.5	7.9
mining and quarrying	-23.6	1.0	-19.8	0.5
food	62.4	12.7	37.7	13.0
textiles	177.2	<b>44.0</b>	850.5	18.6
clothing	145.5	56.7	543.0	50.3
leather products	29.9	7.9	87.7	15.5
footwear	48.8	45.7	33.1	35.4
wood products	76.4	6.7	25.7	7.0
furniture and fixtures	83.8	35.6	224.1	18.6
paper products	-20.5	18.9	-41.9	-3.9
printing and publishing	50.8	3.9	507.9	-1.1
chemicals	49.8	21.8	61.5	-8.4
petroleum and products	-6.9	0.8	-41.1	-7.4
rubber products	95.6	19.1	165.6	12.8
nonmetal mineral products	56.5	11.9	55.9	0.8
glass products	50.5	4.4	112.9	42.3
iron and steel	0.6	11.6	144.5	-2.8
nonferrous metals	-20.7	-6.7	-28.7	-55.1
metal products	66.7	18.2	301.4	5.4
nonelectrical machinery	36.2	9.9	350.8	-2.9
electrical machinery	154.4	14.9	167.8	-10.9
transportation equipment	36.5	-4.6	290.3	9.9
miscellaneous manufactures	117.3	11.5	362.3	-9.4
weighted correlation with data		-0.01		0.50
variance decomposition of change		0.14		0.02
regression coefficient a		37.27		190.89
regression coefficient b		-0.02		3.42

Changes in U.S. Exports/GDP in the Brown-Deardorff-Stern Model (Percent)

### Changes in Canadian Trade/GDP in Cox-Harris Model (Percent)

	data	model
variable	1988-2000	
total trade	57.2	10.0
trade with Mexico	280.0	52.2
trade with United States	76.2	20.0
weighted correlation with d	ata	0.99
variance decomposition of c	change	0.52
regression coefficient a		38.40
regression coefficient b		1.93

	total ex	ports	total imports		
sector	1988-2000	model	1988-2000	model	
agriculture	-13.7	-4.1	4.6	7.2	
forestry	215.5	-11.5	-21.5	7.1	
fishing	81.5	-5.4	107.3	9.5	
mining	21.7	-7.0	32.1	4.0	
food, beverages, and tobacco	50.9	18.6	60.0	3.8	
rubber and plastics	<b>194.4</b>	24.5	87.7	13.8	
textiles and leather	201.1	108.8	24.6	18.2	
wood and paper	31.9	7.3	97.3	7.2	
steel and metal products	30.2	19.5	52.2	10.0	
transportation equipment	66.3	3.5	29.7	3.0	
machinery and appliances	112.9	57.1	65.0	13.3	
nonmetallic minerals	102.7	31.8	3.6	7.3	
refineries	20.3	-2.7	5.1	1.5	
chemicals and misc. manufactures	53.3	28.1	92.5	10.4	
weighted correlation with data		0.49		0.85	
variance decomposition of change		0.32		0.08	
regression coefficient <i>a</i>		41.85		22.00	
regression coefficient b		0.81		3.55	

#### **Changes in Canadian Trade/GDP in the Cox-Harris Model (Percent)**

	exports to Nor	exports to North America		rth America
sector	1988–2000	model	1988-2000	model
agriculture	-15.3	-11.1	-28.2	3.4
mining	-23.2	-17.0	-50.7	13.2
petroleum	-37.6	-19.5	65.9	-6.8
food	5.2	-6.9	11.8	-5.0
beverages	42.0	5.2	216.0	-1.8
tobacco	-42.3	2.8	3957.1	-11.6
textiles	534.1	1.9	833.2	-1.2
wearing apparel	2097.3	30.0	832.9	4.5
leather	264.3	12.4	621.0	-0.4
wood	415.1	-8.5	168.9	11.7
paper	12.8	-7.9	68.1	-4.7
chemicals	41.9	-4.4	71.8	-2.7
rubber	479.0	12.8	792.0	-0.1
nonmetallic mineral products	37.5	-6.2	226.5	10.9
iron and steel	35.9	-4.9	40.3	17.7
nonferrous metals	-40.3	-9.8	101.2	9.8
metal products	469.5	-4.4	478.7	9.5
nonelectrical machinery	521.7	-7.4	129.0	20.7
electrical machinery	3189.1	1.0	749.1	9.6
transportation equipment	224.5	-5.0	368.0	11.2
other manufactures	975.1	-4.5	183.6	4.2
weighted correlation with data		0.61		0.23
variance decomposition of change		0.0004		0.002
regression coefficient a		495.08		174.52
regression coefficient b		30.77		5.35

#### Changes in Mexican Trade/GDP in the Sobarzo Model (Percent)

## What Do We Learn from these Evaluations?

The Spanish model seems to have been far more successful in predicting the consequences of policy changes than the three models of NAFTA, but

- Kehoe, Polo, and Sancho (KPS) knew the structure of their model well enough to precisely identify the relationships between the variables in their model with those in the data;
- KPS were able to use the model to carry out numerical exercises to incorporate the impact of exogenous shocks.

KPS had an incentive to show their model in the best possible light.

3. Much of the growth of trade after a trade liberalization experience is growth on the extensive margin. Models need to allow for corner solutions or fixed costs.

T. J. Kehoe and K. J. Ruhl, "How Important is the New Goods Margin in International Trade?" Federal Reserve Bank of Minneapolis, 2002.

K. J. Ruhl, "Solving the Elasticity Puzzle in International Economics," University of Texas at Austin, 2005.

What happens to the **least-traded** goods:

Over the business cycle? During trade liberalization?

Indirect evidence on the extensive margin

### How Does Trade Grow?

- Intensive Margin: growth in goods already traded
- Extensive Margin: trade in goods not traded before

## The Extensive Margin

• The Extensive Margin has recently gained attention

• Models

• Melitz (2003)

• Alessandria and Choi (2003)

• Ruhl (2004)

• Empirically

• Hummels and Klenow (2002)

• Eaton, Kortum and Kramarz (2004)

## What Happens to the Extensive Margin?

• During trade liberalization?

• Large changes in the extensive margin

• Over the business cycle?

• Little change in extensive margin

## **Evidence from Trade Agreements**

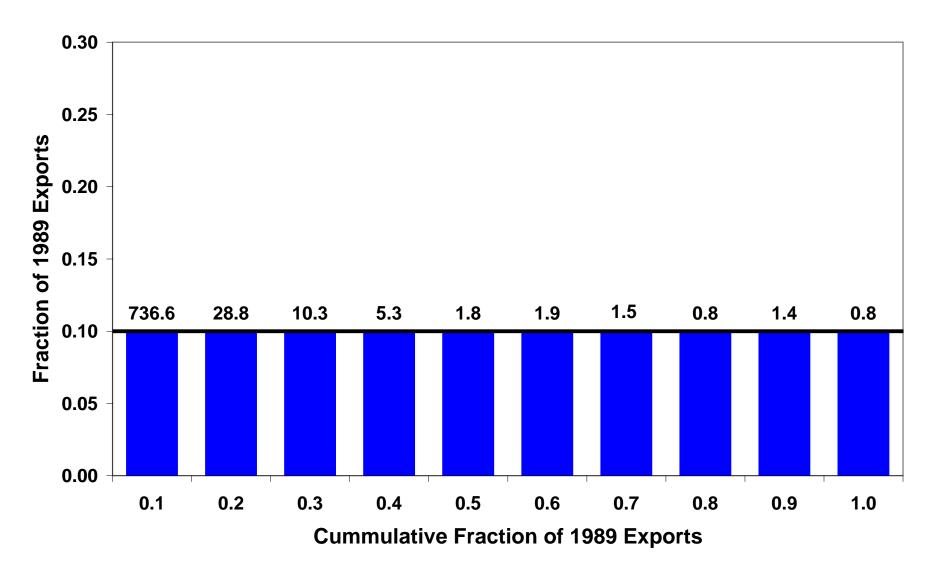
### • Events

- Greece's Accession to the European Econ. Community 1981
- Portugal's Accession to the European Community 1986
- Spain's Accession to the European Community 1986
- 0 U.S.-Canada Free Trade Agreement 1989
- 0 North American Free Trade Agreement 1994
- Data
  - Four-digit SITC bilateral trade data (OECD)
    - $\circ$  789 codes in revision 2
- Indirect Evidence

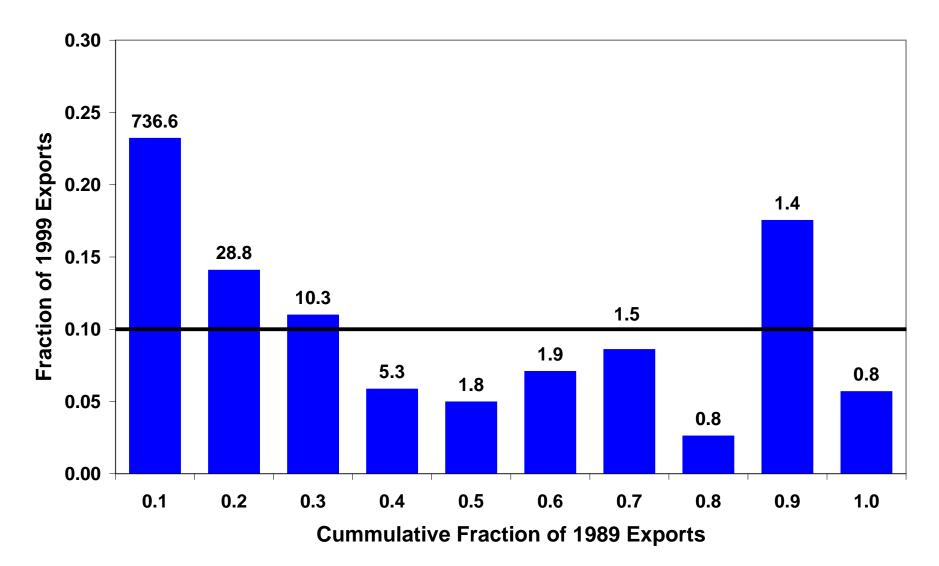
## Measure One

- 1. Rank codes from lowest value of exports to highest value of exports based on average of first 3 years
- 2. Form sets of codes by cumulating exports: the first 742.9 codes make up 10 percent of exports; the next 24.1 codes make up 10 percent of exports; and so on.
- **3.** Calculate each set's share of export value at the end of the sample period.

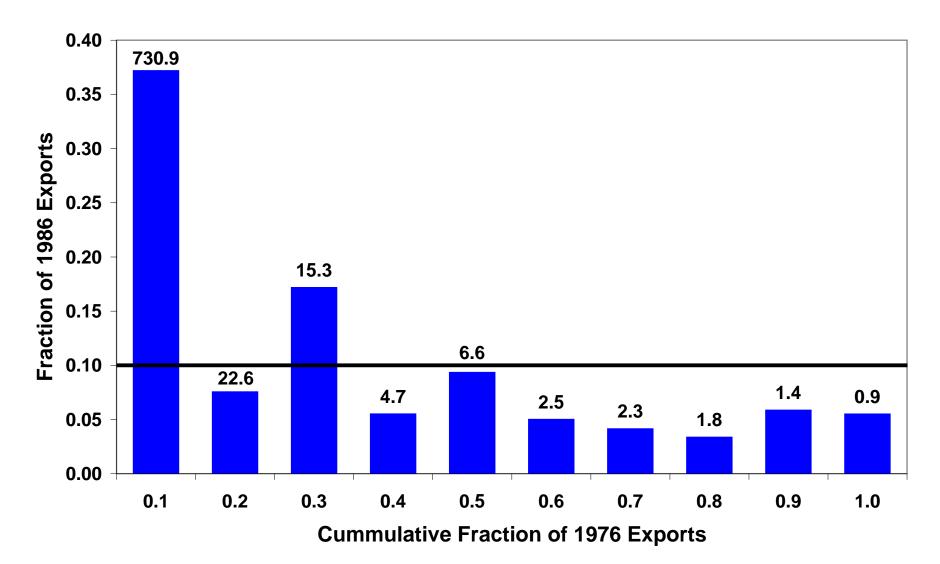
#### **Composition of Exports: Mexico to Canada**



#### **Composition of Exports: Mexico to Canada**



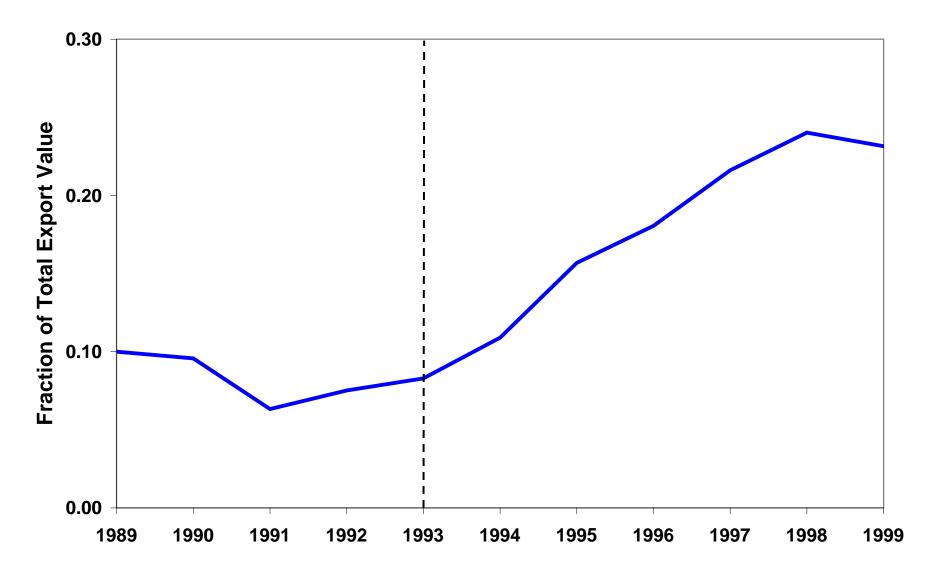
#### **Composition of Exports: Greece to EEC**

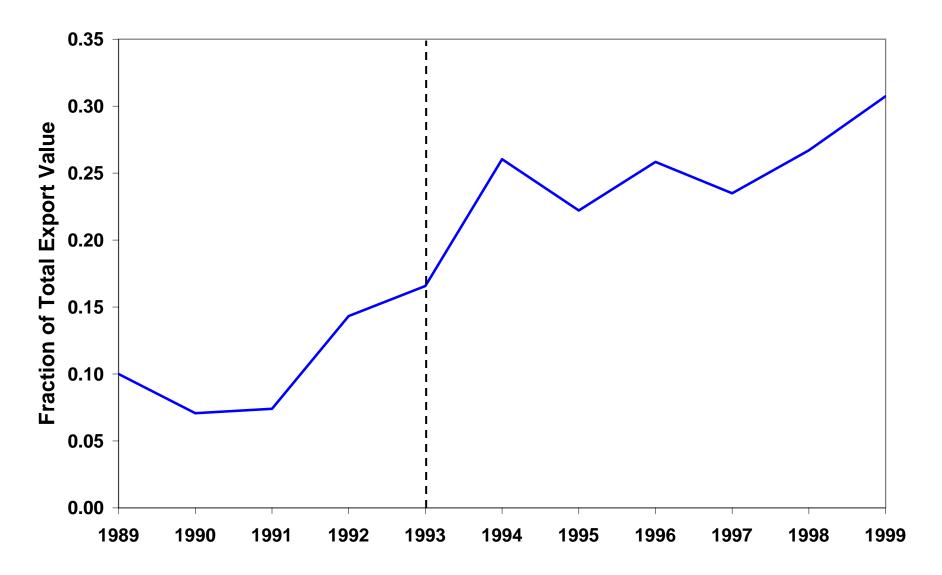


## Measure Two

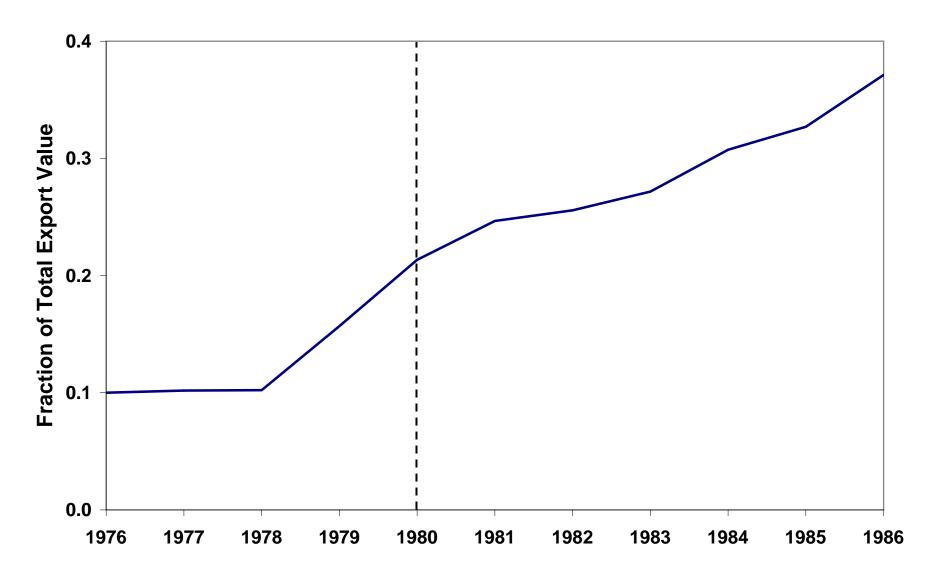
- **1.** Order codes as before.
- 2. Cumulate exports as before.
- **3.** Follow the evolution of the first (least-traded) set's share of total exports before, during, and after the liberalization.

#### **Exports: Mexico to Canada**





#### **Exports: Greece to EEC**



## Trade Liberalization and the Extensive Margin

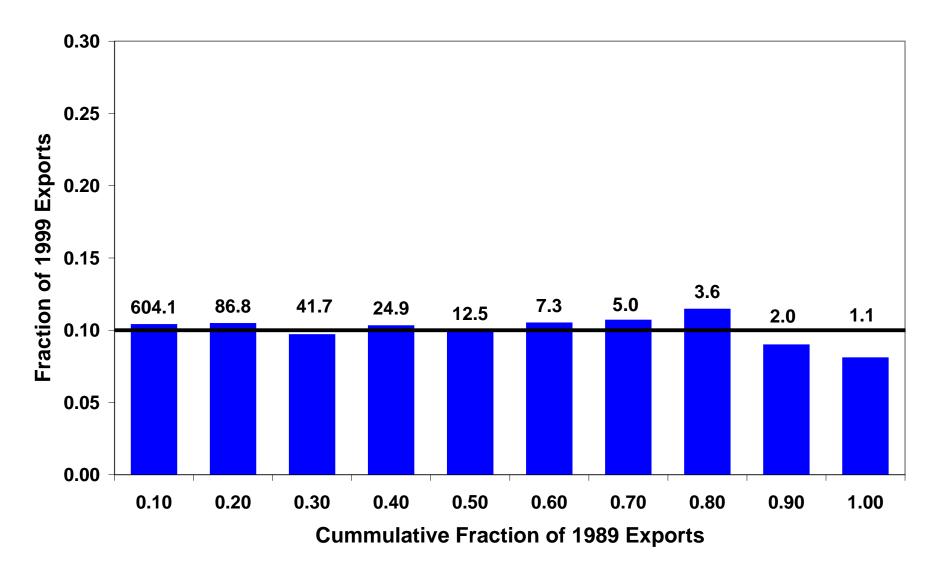
Period	Trade Flow	Share of Export Growth
1989-1999	Mexico - U.S.	0.153
1989-1999	U.S. – Mexico	0.118
1989-1999	Mexico - Canada	0.231
1989-1999	Canada - Mexico	0.307
1989-1999	Canada - U.S.	0.162
1989-1999	U.S. – Canada	0.130
1978-1986	Greece to the EEC	0.371
1982-1987	Spain to the EC	0.128
1982-1987	Portugal to the EC	0.147

### **Business Cycles and the Extensive Margin**

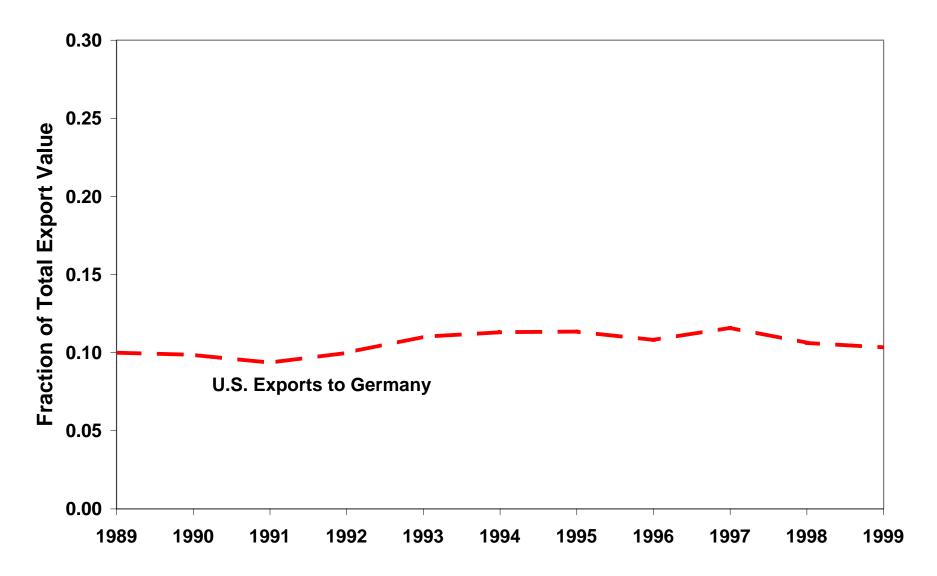
• Over same period, consider countries with stable policy

○ U.S. – Japan
○ U.S. – U.K.
○ U.S. – Germany

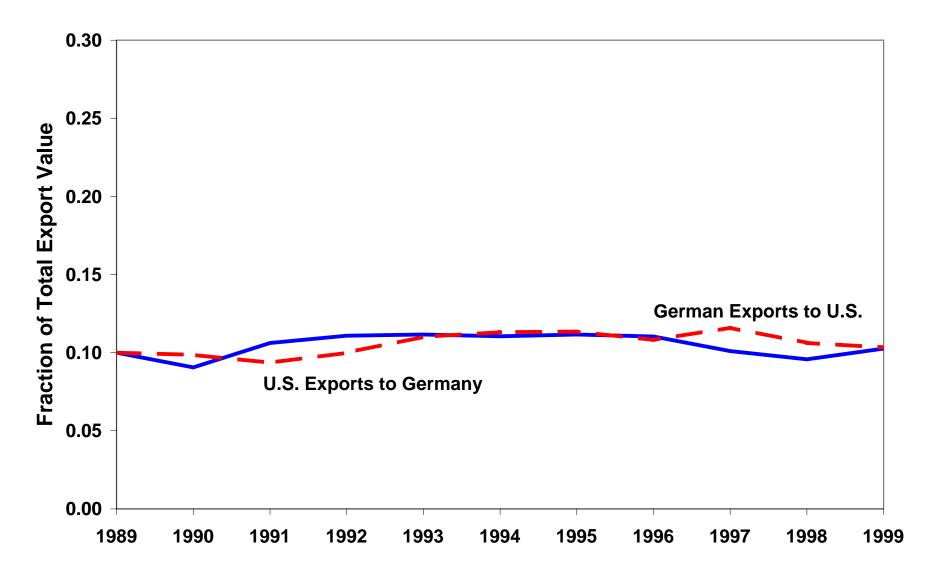
#### **Composition of Exports: U.S. to Germany**



#### **Exports: United States to Germany**



#### **Exports: Germany and the United States**



## **Business Cycles and the Extensive Margin**

Period	Trade Flow	Share of Export Growth
1989-1999	U.S U.K.	0.096
1989-1999	U.K U.S.	0.128
1989-1999	U.S Japan	0.130
1989-1999	Japan - U.S.	0.103
1989-1999	U.S Germany	0.104
1989-1999	Germany - U.S.	0.103

# The Model

- Countries: foreign and home
- Continuum of goods:

$$y_i(x) = \frac{1}{a_i(x)} l_i(x) \qquad x \in [0,1]$$

- Stand-in consumer in each country with labor  $L_i$ .
- Preferences:

$$U = \int_{0}^{1} \log[c_i(x)] dx$$

• ad valorem tariffs:  $\tau_i$ 

### **Determination of Exports**

• *x* is exported by foreign if

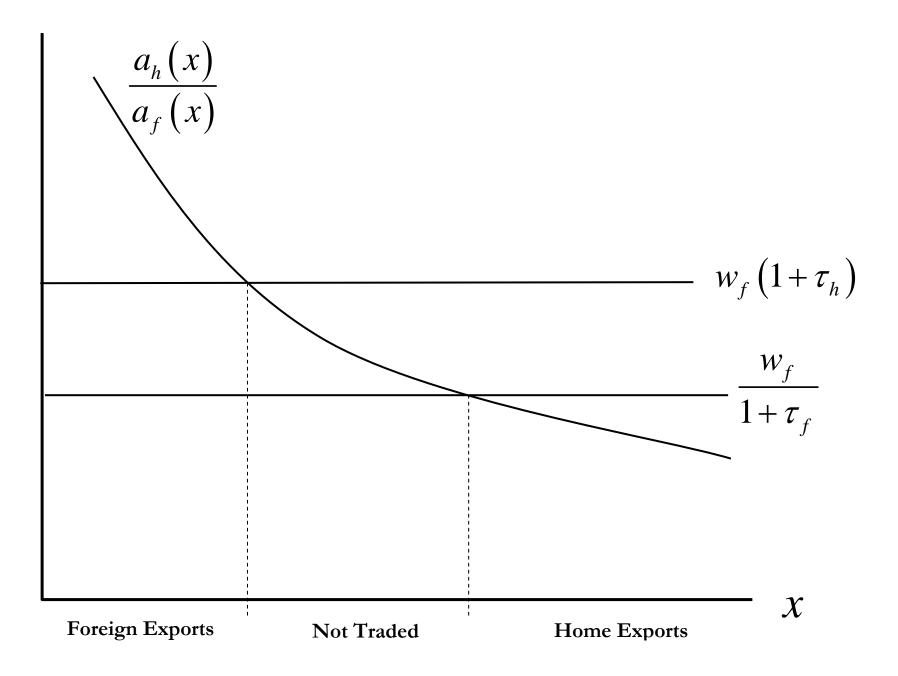
$$a_h(x) > w_f a_f(x)(1+\tau_h) \quad \Leftrightarrow \quad \frac{a_h(x)}{a_f(x)} > w_f(1+\tau_h)$$

- x is exported by home if  $\frac{a_h(x)}{a_f(x)} < \frac{w_f}{1 + \tau_f}$
- x is not traded if

$$\frac{w_f}{1+\tau_h} > \frac{a_h(x)}{a_f(x)} > w_f(1+\tau_f)$$

# Dornbusch, Fisher, Samuelson (1977)

• Order goods according to the relative unit costs.



## Dornbusch, Fisher, Samuelson (1977)

- Order goods according to the relative unit costs.
- Problems
  - Trade data is collected in aggregates.
  - Difficult to obtain data on relative unit costs.
  - Both countries may export the same aggregate.

## **Our Approach**

- SITC ordering: an aggregate is an interval in [0,1]
- Take J evenly spaced points in [0,1].
- Randomly assign log-productivities.

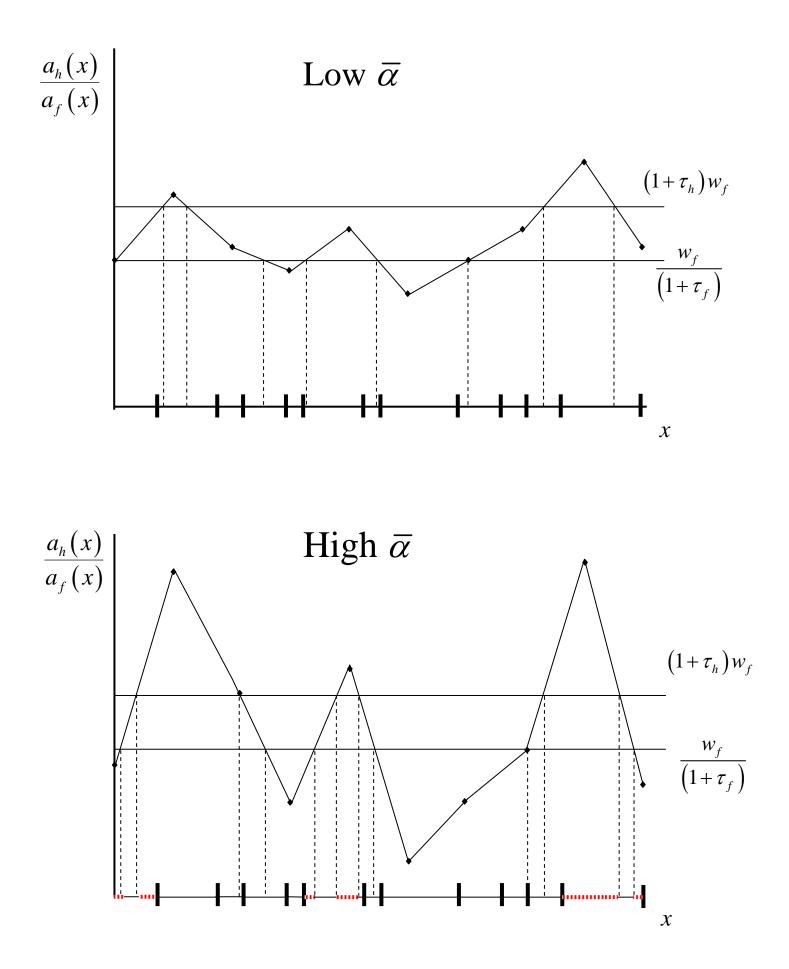
$$\alpha_{j} = \log \left[ \frac{a_{h}(j)}{a_{f}(j)} \right] \qquad \alpha_{j} \sim u \left[ -\overline{\alpha}, \overline{\alpha} \right]$$

• Points not on the grid are filled in by linear interpolation.

### **Relative Productivity Curve**

- Steeper segments
  - less trade growth
  - more intra-industry trade

- For a given J larger  $\overline{\alpha}$  imply steeper segments.
- For a given  $\overline{\alpha}$ , larger J imply steeper segments.



### **Model Solution**

- **1.** Choose J and  $\overline{\alpha}$ .
- 2. Draw a realization of the relative productivity curve.
- 3. Solve the model and compute extensive margin measures.
- 4. Repeat for 5000 simulations.
- 5. Calculate means over simulations.

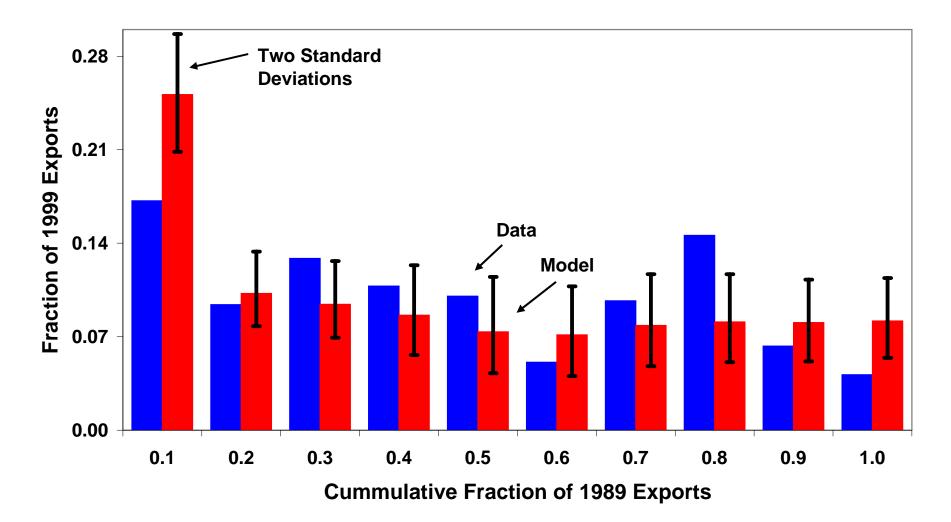
### Calibration

- Parameters:  $\overline{\alpha}$ , J,  $L_f/L_h$ , SITC endpoints
- Country size is measured by gross output of commodities.
- Codes are ordered by their SITC number.
- Code size is determined by its world export value.

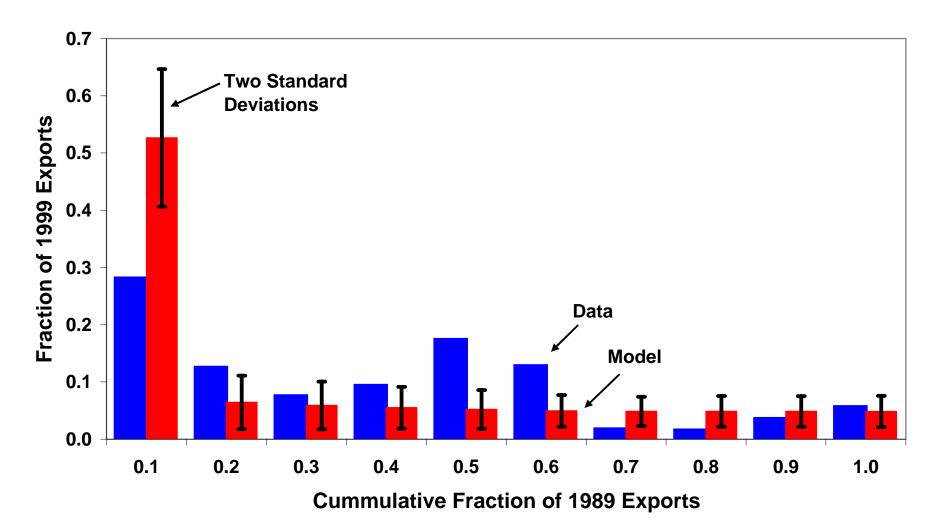
$$size_{k} = \frac{EX_{WORLD,k}^{MEX} + EX_{WORLD,k}^{US}}{\sum_{k} EX_{WORLD,k}^{MEX} + \sum_{k} EX_{WORLD,k}^{US}}$$

• J and  $\overline{\alpha}$  determined by aggregate trade growth and Intra-industry trade

#### Composition of Exports: Mexico to U.S. 1989-1999 By Sets if Categories Based on Export Size



#### Composition of Exports: Mexico to Canada 1989-1999 By Sets of Categories Based on Export Size



# Model with Intensive and Extensive Margins

- Same Environment
- New Preferences

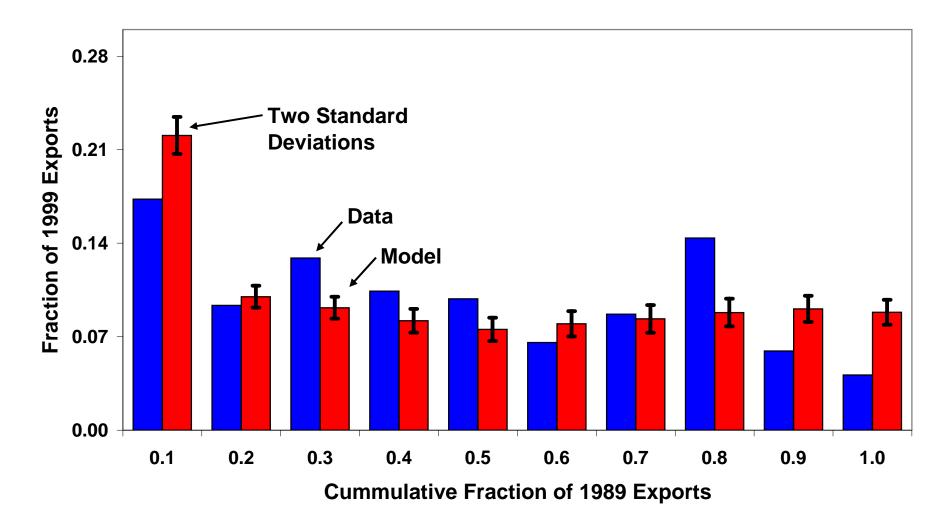
$$U = \left[\int_{0}^{1} c^{i}(x)^{\rho} dx\right]^{1/\rho} \qquad \sigma = \frac{1}{1-\rho}$$

- Expenditure on Goods
  - Old Model  $c^{i}(x)p^{i}(x)=w^{i}L^{i}$

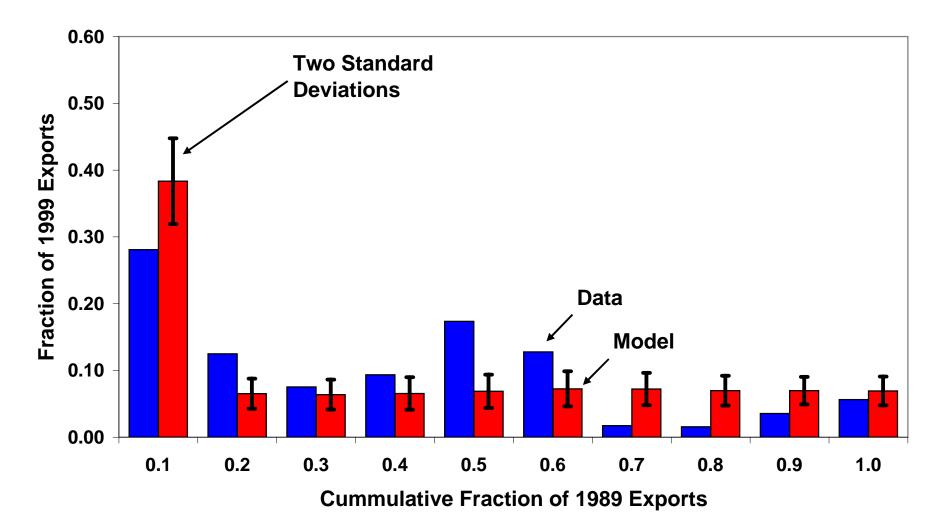
$$c^{i}(x)p^{i}(x) = w^{i}L^{i}\left(\frac{p^{i}(x)}{P^{i}}\right)^{1-\sigma}$$

• New Model

#### Composition of Exports: Mexico to U.S. 1989-1999 By Sets of Categories Based on Export Size



#### Composition of Exports: Mexico to Canada 1989-1999 By Sets of Categories Based on Export Size



### Conclusions

- **1.** The extensive margin is important.
  - Average increase in export share: 67%
  - Correct timing
- 2. Simple model can produce extensive margin growth.
  - Calibration uses aggregate production data.

### **Relative Productivity Parameters**

J	Growth in Trade's Share of Production
$\overline{\alpha}$	and Grubel-Lloyd Index

• Grubel-Lloyd Index

$$GL_{MEX}^{US} = 1 - \frac{\sum_{k \in SITC} \left| EX_{MEX}^{US} - EX_{US}^{MEX} \right|}{\sum_{k \in SITC} \left[ EX_{US}^{MEX} + EX_{MEX}^{US} \right]}$$

## **Calibration Values**

	Grubel-Lloyd	Growth in	Relative Output
	Index (1989)	Trade/Production	(1989)
		(1989-1999)	
MEX-US	.487	201%	.06
MEX-CAN	.147	299%	.66

	$\overline{lpha}$	J	$L^f / L^h$
MEX-US	.223	3215	.06
MEX-CAN	.208	63	.66

### **Calibration Sensitivity**

• Ideal SITC Measure:

$$size_{k} = \frac{y_{k}^{h} + y_{k}^{f}}{\sum_{k} y_{k}^{h} + \sum_{k} y_{k}^{f}}$$

• Our Proxy:

$$size_{k} = \frac{EX_{w,k}^{h} + EX_{w,k}^{f}}{\sum_{k} EX_{w,k}^{h} + \sum_{k} EX_{w,k}^{f}}$$