Notes on Substitutability and Tariffs in Ricardian Models

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Environment:

Goods are produced using labor:

$$y_j(z) = \ell_j(z) / a_j(z) .$$

where

$$a_1(z) = e^{\alpha z}$$
$$a_2(z) = e^{\alpha(1-z)} .$$

Here $y_j(z)$ is the production of good z in country j and $\ell_j(z)$ is the input of labor.

The utility function is

$$\left(\int_0^1 c_j(z)^\rho dz - 1 \right) / \rho \qquad = \int_0^1 \log c_j(z)\, dz \quad \text{if } \rho = 0 .$$

The budget constraint is

$$\int_0^1 p_j(z) c_j(z)\, dz = w_j \bar{\ell}_j + T_j ,$$

where T_j is the lump-sum rebate of tariff revenue.

Demand:

$$\max \left(\int_0^1 c_j(z)^\rho dz - 1 \right) / \rho$$
$$\text{s.t. } \int_0^1 p_j(z) c_j(z)\, dz = w_j \bar{\ell}_j + T_j$$

$$c_j(z) = \frac{w_j \bar{\ell}_j + T_j}{p_j(z)^{\frac{1}{1-\rho}} \int_0^1 p_j(\zeta)^{-\frac{\rho}{1-\rho}}\, d\zeta} \qquad = \frac{w_j \bar{\ell}_j + T_j}{p_j(z)} \quad \text{if } \rho = 0$$

Suppose that labor endowments and tariffs are equal across countries. Then

$$p_1(z) = \min\left[a_1(z)w_1,\ (1+\tau)a_2(z)w_2\right]$$

$$p_2(z) = \min\left[(1+\tau)a_1(z)w_1,\ a_2(z)w_2\right].$$

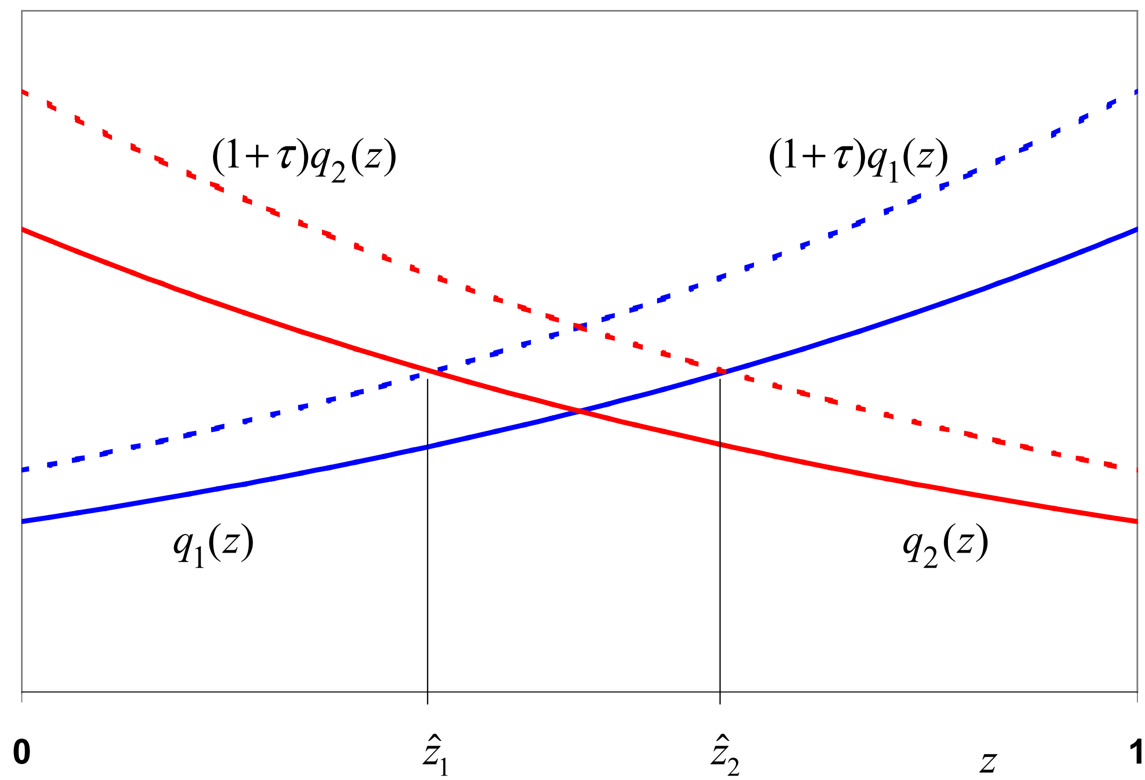
Here $p_j(z)$ is the price paid for good z in country j.

Symmetry implies that

$$w_1 = w_2 = 1$$
$$q_1 = q_2 = q$$

$$\hat{z}_1 = 1 - \hat{z}_2$$
$$(1+\tau)e^{\alpha \hat{z}_1} = e^{\alpha(1-\hat{z}_1)}$$
$$\hat{z}_1 = \frac{\alpha - \log(1+\tau)}{2\alpha},\quad \hat{z}_2 = \frac{\alpha + \log(1+\tau)}{2\alpha}.$$

Patterns of production and trade



Cobb-Douglas case:

Let us do case where $\rho = 0$ first. Tariff revenues are

$$T = \int_{\hat{z}_2}^{1} \tau a_2(z) \frac{\bar{\ell} + T}{(1+\tau) a_2(z)} dz = \frac{\tau(\bar{\ell} + T)}{(1+\tau)}(1 - \hat{z}_2)$$

$$T\left[1 - \frac{\tau(1-\hat{z}_2)}{(1+\tau)}\right] = \frac{\tau\bar{\ell}}{(1+\tau)}(1-\hat{z}_2)$$

$$T = \frac{\tau\bar{\ell}\left[\alpha - \log\ (1+\tau)\right]}{\left[2\alpha + \alpha\tau + \tau\log\ (1+\tau)\right]}.$$

GDP is

$$Y = \bar{\ell} + T = \frac{\bar{\ell}\left[2\alpha + \alpha\tau + \tau\log\ (1+\tau)\right] + \tau\bar{\ell}\left[\alpha - \log\ (1+\tau)\right]}{\left[2\alpha + \alpha\tau + \tau\log\ (1+\tau)\right]}$$

$$Y = \frac{2\alpha\bar{\ell}(1+\tau)}{\left[2\alpha + \alpha\tau + \tau\log\ (1+\tau)\right]}.$$

Exports are

$$X = \int_{\hat{z}_2}^{1} a_2(z) \frac{\bar{\ell} + T}{(1+\tau) a_2(z)} dz$$

$$= \frac{\bar{\ell}\left[\alpha - \log\ (1+\tau)\right]}{\left[2\alpha + \alpha\tau + \tau\log\ (1+\tau)\right]}.$$

The ratio exports/GDP is

$$\frac{X}{Y} = \frac{\bar{\ell}\left[\alpha - \log\ (1+\tau)\right]}{2\alpha\bar{\ell}(1+\tau)} = \frac{\left[\alpha - \log\ (1+\tau)\right]}{2\alpha(1+\tau)}.$$

General case:

Now let us do case where $\rho \neq 0$:

$$\int_0^1 p_j(z)^{-\frac{\rho}{1-\rho}}\, dz = \int_0^{\hat{z}_2} e^{-\frac{\alpha\rho z}{1-\rho}}\, dz + \int_{\hat{z}_2}^{1} (1+\tau)^{-\frac{\rho}{1-\rho}} e^{-\frac{\alpha\rho(1-z)}{1-\rho}}\, dz$$

$$= -\frac{1-\rho}{\alpha\rho} e^{-\frac{\alpha\rho z}{1-\rho}}\Bigg|_0^{\hat{z}_2} + \frac{1-\rho}{\alpha\rho}(1+\tau)^{-\frac{\rho}{1-\rho}} e^{-\frac{\alpha\rho(1-z)}{1-\rho}}\Bigg|_{\hat{z}_2}^{1}.$$

$$\int_0^1 p_j(z)^{-\frac{\rho}{1-\rho}}\, dz = -\frac{1-\rho}{\alpha\rho} e^{-\frac{\alpha\rho z}{1-\rho}}\Big|_0^{\hat{z}_2} + \frac{1-\rho}{\alpha\rho}(1+\tau)^{-\frac{\rho}{1-\rho}} e^{-\frac{\alpha\rho(1-z)}{1-\rho}}\Big|_{\hat{z}_2}^1$$

$$= \frac{1-\rho}{\alpha\rho}\left[1 - e^{-\frac{\alpha\rho\hat{z}_2}{1-\rho}} + (1+\tau)^{-\frac{\rho}{1-\rho}} - (1+\tau)^{-\frac{\rho}{1-\rho}} e^{-\frac{\alpha\rho(1-\hat{z}_2)}{1-\rho}}\right]$$

$$= \frac{1-\rho}{\alpha\rho}\left[1 + (1+\tau)^{-\frac{\rho}{1-\rho}} - 2e^{-\frac{\alpha\rho\hat{z}_2}{1-\rho}}\right]$$

$$= \frac{1-\rho}{\alpha\rho}\left[1 + (1+\tau)^{-\frac{\rho}{1-\rho}} - 2e^{-\frac{\alpha\rho}{1-\rho}\left(\frac{1}{2}+\frac{\log(1+\tau)}{2\alpha}\right)}\right]$$

$$= \frac{1-\rho}{\alpha\rho}\left[1 + (1+\tau)^{-\frac{\rho}{1-\rho}} - 2\left[(1+\tau)e^{\alpha}\right]^{-\frac{\rho}{2(1-\rho)}}\right] .$$

We define

$$P(\tau) = \left[\int_0^1 p_j(z)^{-\frac{\rho}{1-\rho}}\, dz\right]^{-\frac{1-\rho}{\rho}} = \left(\frac{1-\rho}{\alpha\rho}\left[1 + (1+\tau)^{-\frac{\rho}{1-\rho}} - 2\left[(1+\tau)e^{\alpha}\right]^{-\frac{\rho}{2(1-\rho)}}\right]\right)^{-\frac{1-\rho}{\rho}} .$$

Tariff revenues are

$$T = \int_{\hat{z}_2}^1 \tau a_2(z) \frac{\overline{\ell} + T}{(1+\tau)^{\frac{1}{1-\rho}} a_2(z)^{\frac{1}{1-\rho}} P(\tau)^{-\frac{\rho}{1-\rho}}} dz$$

$$= \frac{\tau(\overline{\ell} + T)}{(1+\tau)^{\frac{1}{1-\rho}} P(\tau)^{-\frac{\rho}{1-\rho}}} \int_{\hat{z}_2}^1 e^{-\frac{a\rho(1-z)}{1-\rho}} dz$$

$$= \frac{\tau(\overline{\ell} + T)}{(1+\tau)^{\frac{1}{1-\rho}} P(\tau)^{-\frac{\rho}{1-\rho}}} \left(\frac{1-\rho}{\alpha\rho} - \frac{1-\rho}{\alpha\rho}\left[(1+\tau)^{-1} e^{\alpha}\right]^{-\frac{\rho}{2(1-\rho)}}\right)$$

$$= \frac{(1-\rho)\left(1 - \left[(1+\tau)^{-1} e^{\alpha}\right]^{-\frac{\rho}{2(1-\rho)}}\right)\tau}{\alpha\rho(1+\tau)^{\frac{1}{1-\rho}} P(\tau)^{-\frac{\rho}{1-\rho}}} (\overline{\ell} + T) \quad .$$

$$T\left[1 - \frac{(1-\rho)\left(1 - \left[(1+\tau)^{-1} e^{\alpha}\right]^{-\frac{\rho}{2(1-\rho)}}\right)\tau}{\alpha\rho(1+\tau)^{\frac{1}{1-\rho}} P(\tau)^{-\frac{\rho}{1-\rho}}}\right] = \frac{(1-\rho)\left(1 - \left[(1+\tau)^{-1} e^{\alpha}\right]^{-\frac{\rho}{2(1-\rho)}}\right)\tau\overline{\ell}}{\alpha\rho(1+\tau)^{\frac{1}{1-\rho}} P(\tau)^{-\frac{\rho}{1-\rho}}} .$$

$$T=\frac{(1-\rho)\left(1-\left[(1+\tau)^{-1}e^{\alpha}\right]^{-\frac{\rho}{2(1-\rho)}}\right)\tau\overline{\ell}}{\left[\alpha\rho(1+\tau)^{\frac{1}{1-\rho}}P(\tau)^{-\frac{\rho}{1-\rho}}-(1-\rho)\left(1-\left[(1+\tau)^{-1}e^{\alpha}\right]^{-\frac{\rho}{2(1-\rho)}}\right)\tau\right]}$$

$$=\frac{\left(1-\left[(1+\tau)^{-1}e^{\alpha}\right]^{-\frac{\rho}{2(1-\rho)}}\right)\tau\overline{\ell}}{\left[(1+\tau)^{\frac{1}{1-\rho}}\left[1+(1+\tau)^{-\frac{\rho}{1-\rho}}-2\left[(1+\tau)e^{\alpha}\right]^{-\frac{\rho}{2(1-\rho)}}\right]-\left(1-\left[(1+\tau)^{-1}e^{\alpha}\right]^{-\frac{\rho}{2(1-\rho)}}\right)\tau\right]}$$

GDP is

$$Y=\overline{\ell}+T=\frac{(1+\tau)^{\frac{1}{1-\rho}}\left[1+(1+\tau)^{-\frac{\rho}{1-\rho}}-2\left[(1+\tau)e^{\alpha}\right]^{-\frac{\rho}{2(1-\rho)}}\right]\overline{\ell}}{\left[(1+\tau)^{\frac{1}{1-\rho}}\left[1+(1+\tau)^{-\frac{\rho}{1-\rho}}-2\left[(1+\tau)e^{\alpha}\right]^{-\frac{\rho}{2(1-\rho)}}\right]-\left(1-\left[(1+\tau)^{-1}e^{\alpha}\right]^{-\frac{\rho}{2(1-\rho)}}\right)\tau\right]}$$

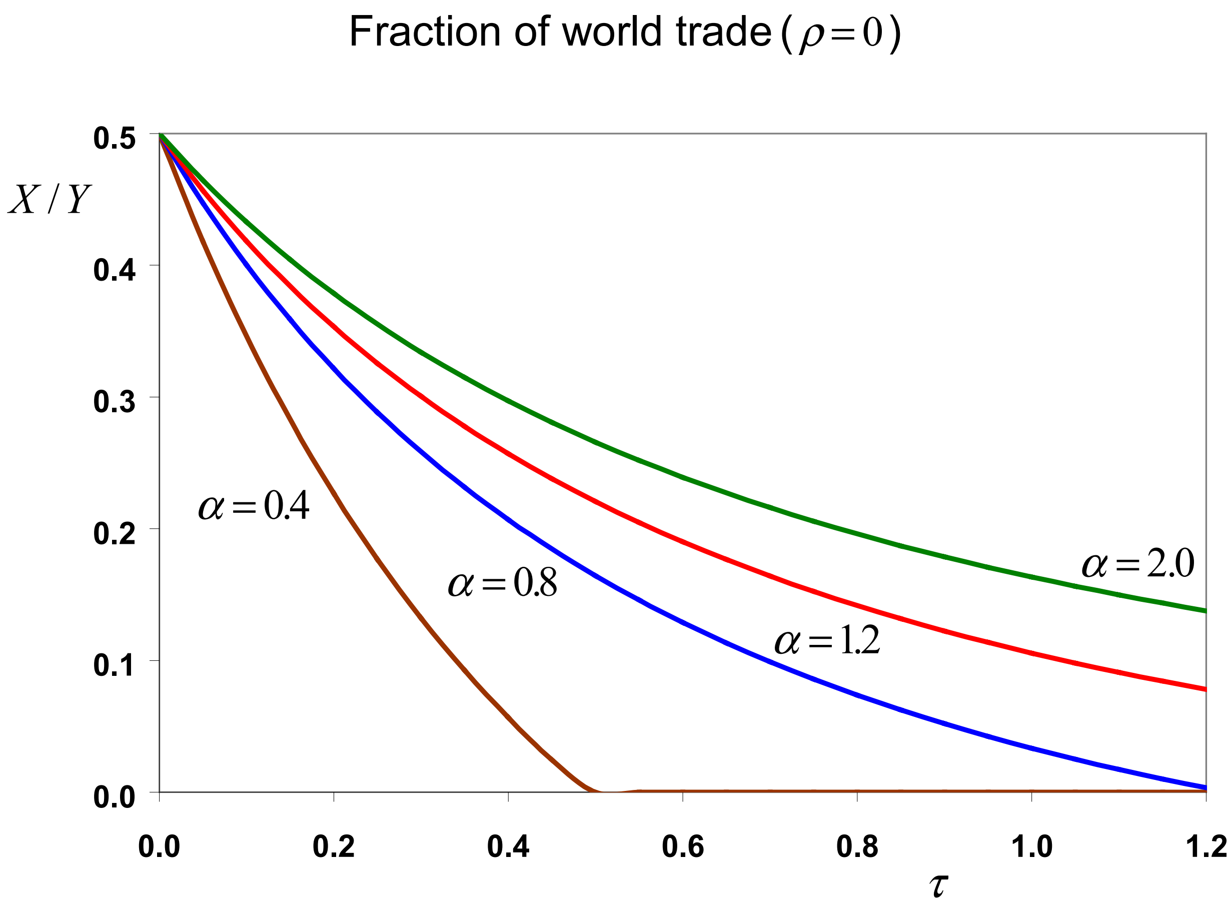
Exports are

$$X=\int_{\hat{z}_2}^{1}a_2(z)\frac{\overline{\ell}+T}{(1+\tau)^{\frac{1}{1-\rho}}a_2(z)^{\frac{1}{1-\rho}}P(\tau)^{-\frac{\rho}{1-\rho}}}dz$$

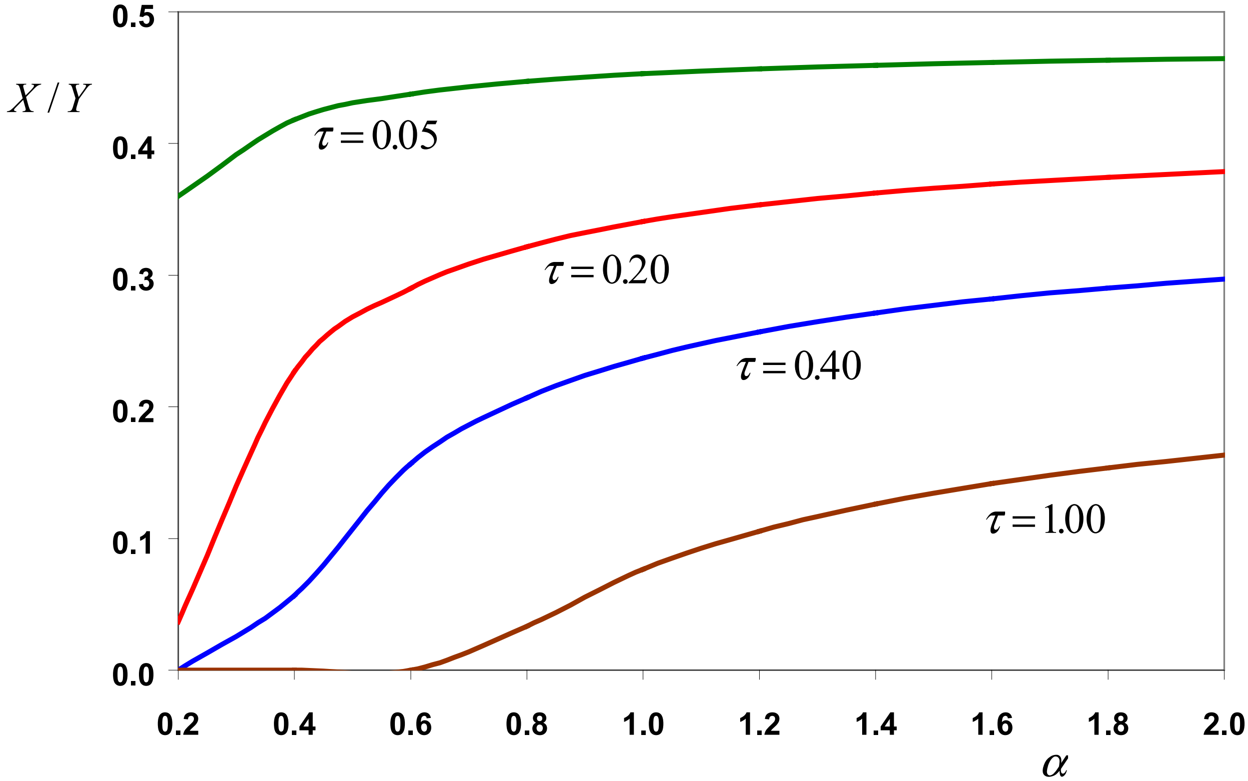
$$=\frac{\left(1-\left[(1+\tau)^{-1}e^{\alpha}\right]^{-\frac{\rho}{2(1-\rho)}}\right)\overline{\ell}}{\left[(1+\tau)^{\frac{1}{1-\rho}}\left[1+(1+\tau)^{-\frac{\rho}{1-\rho}}-2\left[(1+\tau)e^{\alpha}\right]^{-\frac{\rho}{2(1-\rho)}}\right]-\left(1-\left[(1+\tau)^{-1}e^{\alpha}\right]^{-\frac{\rho}{2(1-\rho)}}\right)\tau\right]}$$

The ratio exports/GDP is

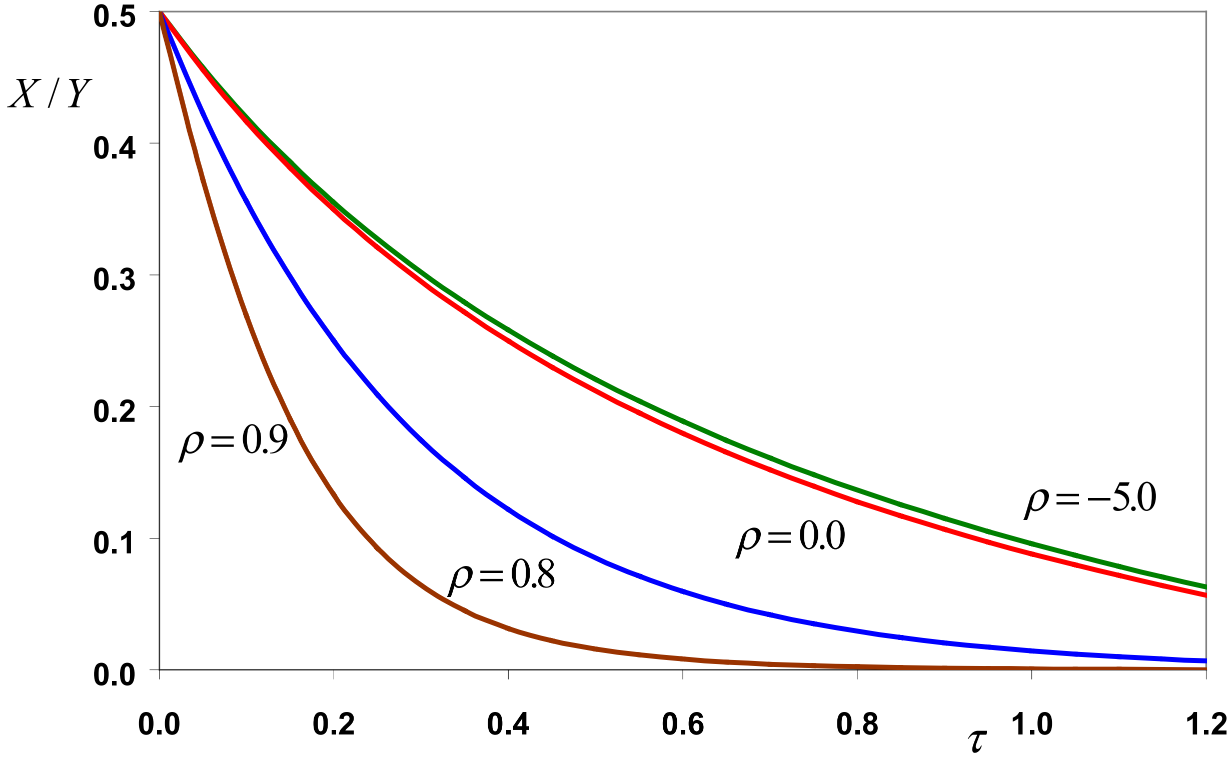
$$\frac{X}{Y}\frac{\left(1-\left[(1+\tau)^{-1}e^{\alpha}\right]^{-\frac{\rho}{2(1-\rho)}}\right)}{(1+\tau)^{\frac{1}{1-\rho}}\left[1+(1+\tau)^{-\frac{\rho}{1-\rho}}-2\left[(1+\tau)e^{\alpha}\right]^{-\frac{\rho}{2(1-\rho)}}\right]}.$$


Fraction of world trade (ρ = 0)
X / Y
0.5
0.4
0.3
0.2
0.1
0.0
α = 0.4
α = 0.8
α = 1.2
α = 2.0
0.0
0.2
0.4
0.6
0.8
1.0
1.2
τ

Fraction of world trade ($\rho = 0$)



Fraction of world trade ($\alpha = 1$)



Fraction of world trade ($\alpha = 1$)