New Developments in Trade Theory and Empirics

Timothy J. Kehoe
University of Minnesota and Federal Reserve Bank of Minneapolis

University of Texas at Austin
February 2006

www.econ.umn.edu/~tkehoe

Outline:

1. Standard theory (hybrid Heckscher-Ohlin/New Trade Theory) does not well when matched with the data on the growth and composition of trade.

2. Applied general equilibrium models that put the standard theory to work do not well in predicting the impact of trade liberalization experiences like NAFTA.

3. Much of the growth of trade after a trade liberalization experience is growth on the extensive margin. Models need to allow for corner solutions or fixed costs.

4. Fixed costs seem better than Ricardian corner solutions for reconciling time series data on real exchange rate fluctuations with data on trade growth after liberalization experiences.

5. Modeling the fixed costs may explain why real exchange rate data indicate that more arbitrage across countries that have a strong bilateral trade relationship.

6. Growth theory needs to be reconsidered in the light of trade theory. In particular, a growth model that includes trade can have convergence properties that are the opposite of those of a model of closed economies.

1. Standard theory (hybrid Heckscher-Ohlin/New Trade Theory) does not well when matched with the data on the growth and composition of trade.

In the 1980s and 1990s trade economists reached a consensus that North-North trade — trade among rich countries — was driven by forces captured by the New Trade Theory and North-South trade — trade between rich countries and poor countries — was driven by forces captured by Heckscher-Ohlin theory. (South-South trade was negligible.)

A. V. Deardorff, "Testing Trade Theories and Predicting Trade Flows," in R. W. Jones and P. B. Kenen, editors, *Handbook of International Economics*, volume l, North-Holland, 1984, 467-517.

J. Markusen, "Explaining the Volume of Trade: An Eclectic Approach," *American Economic Review*, 76 (1986), 1002-1011.

In fact, a calibrated version of this hybrid model does not match the data.

R. Bergoeing and T. J. Kehoe, “Trade Theory and Trade Facts,” Federal Reserve Bank of Minneapolis, 2003.

TRADE THEORY

Traditional trade theory — Ricardo, Heckscher-Ohlin — says countries trade because they are different.

In 1990 by far the largest bilateral trade relation in the world was U.S.-Canada. The largest two-digit SITC export of the United States to Canada was 78 Road Vehicles. The largest two-digit SITC export of Canada to the United States was 78 Road Vehicles.

The New Trade Theory — increasing returns, taste for variety, monopolistic competition — explains how similar countries can engage in a lot of intraindustry trade.

Helpman and Krugman (1985)
Markusen (1986)

TRADE THEORY AND TRADE FACTS

- Some recent trade facts
- A “New Trade Theory” model
- Accounting for the facts
- Intermediate goods?
- Policy?

How important is the quantitative failure of the New Trade Theory?

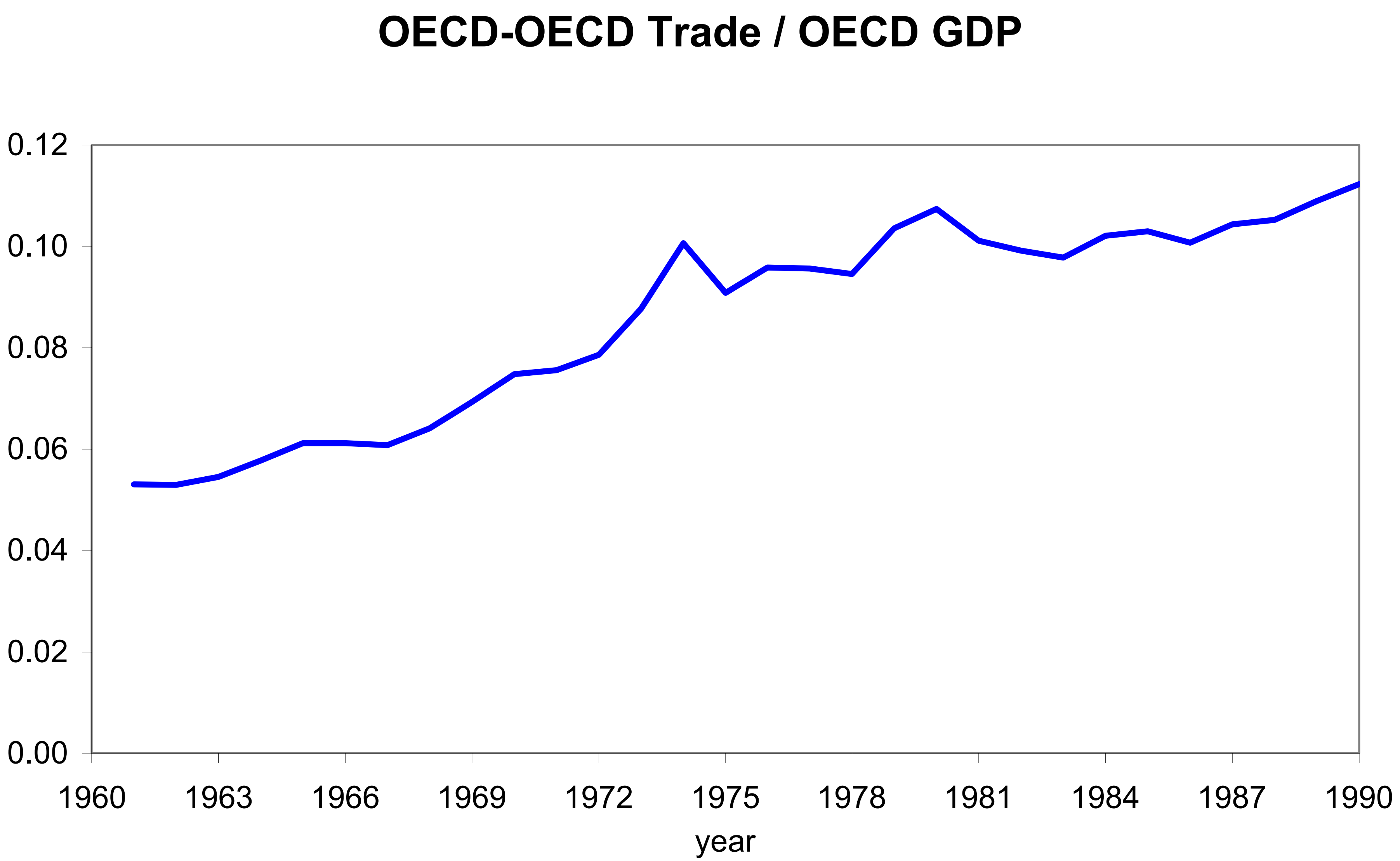
Where should trade theory and applications go from here?

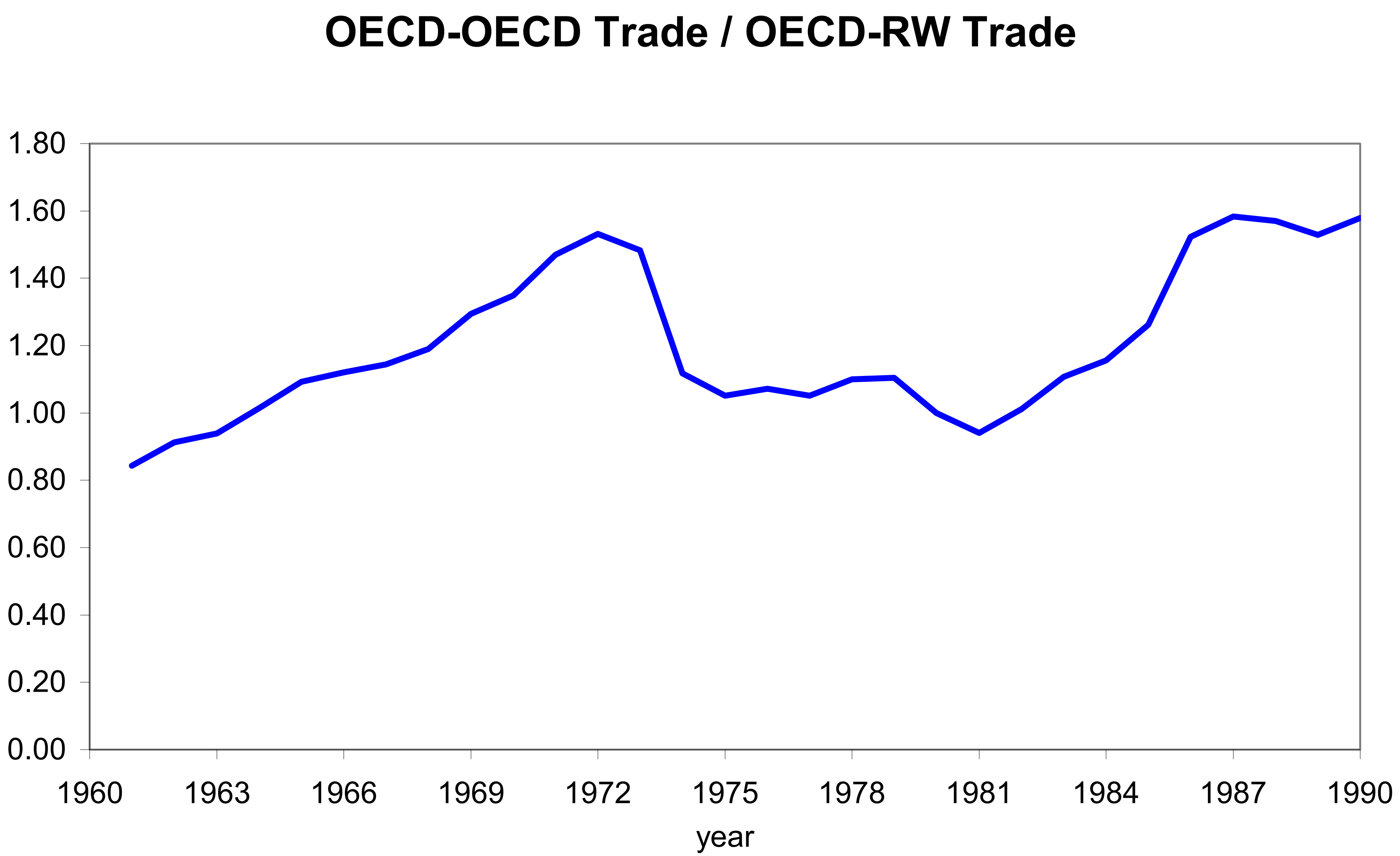
SOME RECENT TRADE FACTS

- **The ratio of trade to product has increased.**
  World trade/world GDP increased by 59.3 percent 1961-1990.
  OECD-OECD trade/OECD GDP increased by 111.5 percent 1961-1990.

- **Trade has become more concentrated among industrialized countries**
  OECD-OECD trade/OECD-RW trade increased by 87.1 percent 1961-1990.

- **Trade among industrialized countries is mostly intraindustry trade**
  Grubel-Lloyd index for OECD-OECD trade in 1990 is 68.4.
  Grubel-Lloyd index for OECD-RW trade in 1990 is 38.1.


OECD-OECD Trade / OECD GDP
0.12
0.10
0.08
0.06
0.04
0.02
0.00
1960
1963
1966
1969
1972
1975
1978
1981
1984
1987
1990
year


OECD-OECD Trade / OECD-RW Trade
1.80
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
0.00
1960
1963
1966
1969
1972
1975
1978
1981
1984
1987
1990
year

Helpman and Krugman (1985):

"These....empirical weaknesses of conventional trade theory...become understandable once economies of scale and imperfect competition are introduced into our analysis."

Markusen, Melvin, Kaempfer, and Maskus (1995):

"Thus, nonhomogeneous demand leads to a decrease in North-South trade and to an increase in intraindustry trade among the northern industrialized countries. These are the stylized facts that were to be explained."

Goal: To measure how much of the increase in the ratio of trade to output in the OECD and of the concentration of world trade among OECD countries can be accounted for by the "New Trade Theory."

PUNCHLINE

In a calibrated general equilibrium model, the New Trade Theory cannot account for the increase in the ratio of trade to output in the OECD.

Back-of-the-envelope calculations:

Suppose that the world consists of the OECD and the only trade is manufactures.

With Dixit-Stiglitz preferences, country j exports all of its production of manufactures Y_m^j except for the fraction $s^j = Y^j / Y^{oe}$ that it retains for domestic consumption.

World imports:

$$M = \sum_{j=1}^{n} \left(1 - s^j\right) Y_m^j \,.$$

World trade/GDP:

$$\frac{M}{Y^{oe}} = \frac{M}{Y_m^{oe}} \frac{Y_m^{oe}}{Y^{oe}} = \left(1 - \sum_{j=1}^{n} (s^j)^2\right) \frac{Y_m^{oe}}{Y^{oe}} \,.$$

World trade/GDP:

$$\frac{M}{Y^{oe}} = \frac{M}{Y_m^{oe}} \frac{Y_m^{oe}}{Y^{oe}} = \left(1 - \sum_{j=1}^{n} (s^j)^2\right) \frac{Y_m^{oe}}{Y^{oe}}.$$

$\left(1 - \sum_{j=1}^{n} (s^j)^2\right)$ goes from 0.663 in 1961 to 0.827 in 1990.

Y_m^{oe} / Y^{oe} goes from 0.295 in 1961 to 0.222 in 1990.

$$0.663 \times 0.295 = 0.196 \approx 0.184 = 0.827 \times 0.222.$$

Effects cancel!

A "NEW TRADE THEORY" MODEL

Environment:

- Static: endowments of factors are exogenous
- 2 regions: OECD and rest of world
- 2 traded goods: homogeneous — primaries (CRS) and differentiated — manufactures (IRS)
- 1 nontraded good — services (CRS)
- 2 factors: (effective) labor and capital
- Identical technologies and preferences (love for variety) across regions
- Primaries are inferior to manufactures

We only consider merchandise trade in both the data and in the model.

Key Features of the Model

Consumers' problem:

$$\max \quad \frac{\beta_p (c_p^j + \gamma_p)^\eta + \beta_m \left(\int_{D^w} c_m^j(z)^\rho \, dz_p \right)^{\eta/\rho} + \beta_s (c_s^j + \gamma_s)^\eta - 1}{\eta}$$

$$\text{s.t.} \quad q_p c_p^j + \int_{D^w} q_m(z) c_m^j(z) dz_p + q_s^j c_s^j \leq r^j k^j + w^j h^j .$$

Firms' problems

Primaries and Services: Standard CRS problems.

$$Y_p^j = \theta_p (K_p^j)^{\alpha_p} (H_p^j)^{1-\alpha_p}$$

$$Y_s^j = \theta_s (K_s^j)^{\alpha_s} (H_s^j)^{1-\alpha_s}$$

Manufactures: Standard (Dixit-Stiglitz) monopolistically competitive problem:

- Fixed cost.

$$Y_m(z) = \max\left[\theta_m K_m(z)^{\alpha_m} H_m(z)^{1-\alpha_m} - F, 0\right]$$

- Firm z sets its price $q_m(z)$ to max profits given all of the other prices.

$$Y_m(z) = \sum_{j=1}^{n} C_m^j(z) + C_m^{rw}(z).$$

$$C_m^j(z) = \frac{\beta_m^{\frac{1}{1-\eta}}(r^j K^j + w^j H^j + q_p \gamma_p N^j + q_s^j \gamma_s N^j)}{q_m(z)^{\frac{1}{1-\rho}} \left[\int_{D^w} q_m(z')^{\frac{-\rho}{1-\rho}} dz' \right]^{\frac{\rho-\eta}{\rho(1-\eta)}} \Delta}$$

$$\Delta = \beta_p^{\frac{1}{1-\eta}} q_p^{-\frac{\eta}{1-\eta}} + \beta_m^{\frac{1}{1-\eta}} \left[\left(\int_{D^w} q_m(z')^{\frac{-\rho}{1-\rho}} dz' \right)^{\frac{-(1-\rho)}{\rho}} \right]^{\frac{-\eta}{1-\eta}} + \beta_s^{\frac{1}{1-\eta}} q_s^{-\frac{\eta}{1-\eta}}$$

- Every firm is uniquely associated with only one variety (symmetry).
- Free entry.
- $D^w = [0, d^w]$ with d^w finite and endogenously determined.

Volume of Trade

Let s^j be the share of country j, $j = 1,\ldots,n,\ rw$, in the world production of manufactures,

$$s^j = \int_{D^j} Y_m(z)dz \ \ / \int_{D^w} Y_m(z)dz \ \ = Y_m^j / Y_m^w.$$

The imports by country *j* from the OECD are

$$M_{oe}^j = (1 - s^{rw} - s^j)C_m^j$$
$$M_{oe}^{rw} = (1 - s^{rw})C_m^{rw}.$$

Total imports in the OECD from the other OECD countries are

$$M_{oe}^{oe} = \sum_{j=1}^{n} M_{oe}^j (1 - s^{rw} - \sum_{j=1}^{n} (s^j)^2 / (1 - s^{rw}))C_m^{oe}.$$

OECD in 1990

Country	Share of GDP %	Country	Share of GDP %
Australia	1.79	Japan	18.04
Austria	0.97	Netherlands	1.72
Belgium-Lux	1.26	New Zealand	0.26
Canada	3.45	Norway	0.70
Denmark	0.78	Portugal	0.41
Finland	0.81	Spain	3.00
France	7.26	Sweden	1.40
Germany	9.96	Switzerland	0.17
Greece	0.50	Turkey	0.91
Iceland	0.04	United Kingdom	5.92
Ireland	0.28	United States	33.72
Italy	6.64		

ACCOUNTING FOR THE FACTS

Compare the changes that the model predicts for 1961-1990 with what actually took place.

Focus on key variables:

- OECD-OECD Trade/OECD GDP
- OECD-OECD Trade/OECD-RW Trade
- OECD Manfacturing GDP/OECD GDP

Calibrate to 1990 data.

Backcast to 1961 by imposing changes in parameters:

- relative sizes of countries in the OECD
- populations
- sectoral productivities
- endowments

ACCOUNTING FOR THE FACTS

Benchmark 1990 OECD Data Set
(Billion U.S. dollars)

	Primaries	Manufactures	Services	Total
H_i^{oe}	228	2,884	8,644	11,756
K_i^{oe}	441	775	3,497	4,713
Y_i^{oe}	669	3,659	12,141	16,469
C_i^{oe}	862	3,466	12,141	16,469
$Y_i^{oe} - C_i^{oe}$	-193	193	0	0

ACCOUNTING FOR THE FACTS

Benchmark 1990 Rest of the World Data Set
(Billion U.S. dollars)

	Primaries	**Manufactures**	**Services**	**Total**
Y_i^{rw}	1,223	1,159	3,447	5,829
C_i^{rw}	1,030	1,352	3,447	5,829
$Y_i^{rw} - C_i^{rw}$	193	-193	0	0

ACCOUNTING FOR THE FACTS

- $N^{oe} = 854$, $N^{rw} = 4,428$.
- $\sum_{i=p,m,s} Y_i^{rw} = \sum_{i=p,m,s} C_i^{rw} = 5,829$.
- Set $q_p = q_m(z) = q_s = w = r = 1$ (quantities are 1990 values).
- $\rho = 1/1.2$ (Morrison 1990, Martins, Scarpetta, and Pilat 1996).
- Normalize $d^w = 100$.
- Calibrate H^{rw}, K^{rw} so that benchmark data set is an equilibrium.

- Alternative calibrations of utility parameters γ_p, γ_s, and η.

OECD in 1961

Country	Share of GDP %	Country	Share of GDP %
Austria	0.75	Netherlands	1.37
Belgium-Lux	1.25	Norway	0.60
Canada	4.22	Portugal	0.32
Denmark	0.70	Spain	1.38
France	6.99	Sweden	1.62
Germany	9.71	Switzerland	1.07
Greece	0.50	Turkey	0.83
Iceland	0.03	United Kingdom	8.08
Ireland	0.21	United States	55.74
Italy	4.64		

Numerical Experiments

Calculate equilibrium in 1961:

$$\theta_{p,1961} = \theta_{p,1990}$$

$$\theta_{m,1961} = \theta_{m,1990}/1.014^{29}, \; F_{1961} = F_{1990}/1.014^{29}$$

$$\theta_{s,1961} = \theta_{s,1990}/1.005^{29} \text{ (Echevarria 1997)}$$

$$N^{oe} = 536, \; N^{rw} = 2,545$$

Numerical Experiments

Choose H^{oe}_{1961}, K^{oe}_{1961}, H^{rw}_{1961}, K^{rw}_{1961} so that

$$\frac{\sum_{i=p,m,s} Y^{oe}_{i,1990} / N^{oe}_{1990}}{\sum_{i=p,m,s} Y^{oe}_{i,1961} / N^{oe}_{1961}} = 2.400$$

$$\frac{\sum_{i=p,m,s} Y^{rw}_{i,1990} / N^{rw}_{1990}}{\sum_{i=p,m,s} Y^{rw}_{i,1961} / N^{rw}_{1961}} = 2.055$$

$$\frac{K^{oe}_{1961}}{H^{oe}_{1961}} = \frac{K^{oe}_{1990}}{H^{oe}_{1990}}$$

$$\frac{q_{p,1961}(Y^{rw}_{p,1961} - C^{rw}_{p,1961})}{\sum_{i=p,m,s} q_{i,1961} Y^{rw}_{i,1961}} = 0.050$$

How Can the Model Work in Matching the Facts?

- The ratio of trade to product has increased:

  The size distribution of countries has become more equal (Helpman-Krugman).

- Trade has become more concentrated among industrialized countries:

  OECD countries have comparative advantage in manufactures, while the RW has comparative advantage in primaries. Because they are inferior to manufactures, primaries become less important in trade as the world becomes richer (Markusen).

How Can the Model Work in Matching the Facts?

- Trade among industrialized countries is largely intraindustry trade:

  OECD countries export manufactures. Because of taste for variety, every country consumes some manufactures from every other country (Dixit-Stiglitz).

- The different total factor productivity growth rates across sectors imply that the price of manufactures relative to primaries and services has fallen sharply between 1961 and 1990. If price elasticities of demand are not equal to one, a lot can happen.

Experiment 1

$\gamma_p = \gamma_p = \eta = 0$

	1961	1990	Change
Data			
OECD-OECD Trade/OECD GDP	0.053	0.112	111.5%
OECD-OECD Trade/OECD-RW Trade	0.844	1.579	87.1%
OECD Manf GDP/OECD GDP	0.295	0.222	–24.6%
1. $\gamma_p = 0$, $\gamma_s = 0$, $\eta = 0$			
OECD-OECD Trade/OECD GDP	0.108	0.136	25.8%
OECD-OECD Trade/OECD-RW Trade	0.893	1.169	30.9%
OECD Manf GDP/OECD GDP	0.223	0.222	–0.4%

Experiment 2

$\gamma_p = -169.5$, $\gamma_s = 314.7$ to match consumption in RW in 1990, $\eta = 0$

	1961	1990	Change
Data			
OECD-OECD Trade/OECD GDP	0.053	0.112	111.5%
OECD-OECD Trade/OECD-RW Trade	0.844	1.579	87.1%
OECD Manf GDP/OECD GDP	0.295	0.222	–24.6%
2. $\gamma_p = -169.5$, $\gamma_s = 314.7$, $\eta = 0$			
OECD-OECD Trade/OECD GDP	0.103	0.132	28.1%
OECD-OECD Trade/OECD-RW Trade	0.739	1.060	43.6%
OECD Manf GDP/OECD GDP	0.225	0.222	–1.4%

Experiment 3

$\gamma_p = -169.5$, $\gamma_s = 314.7$,
$\eta = 0.559$ to match growth in OECD-OECD Trade/OECD GDP

	1961	1990	Change
Data			
OECD-OECD Trade/OECD GDP	0.053	0.112	111.5%
OECD-OECD Trade/OECD-RW Trade	0.844	1.579	87.1%
OECD Manf GDP/OECD GDP	0.295	0.222	–24.6%
3. $\gamma_p = -169.5$, $\gamma_s = 314.7$, $\eta = 0.559$			
OECD-OECD Trade/OECD GDP	0.063	0.132	111.5%
OECD-OECD Trade/OECD-RW Trade	0.738	1.060	43.7 %
OECD Manf GDP/OECD GDP	0.137	0.222	62.7%

Experiments 4 and 5

$\gamma_p = -169.5$, $\gamma_s = 314.7$, reasonable values of η $(0.5 \geq 1/(1-\eta) \geq 0.1)$

	1961	1990	Change
Data			
OECD-OECD Trade/OECD GDP	0.053	0.112	111.5%
OECD-OECD Trade/OECD-RW Trade	0.844	1.579	87.1%
OECD Manf GDP/OECD GDP	0.295	0.222	–24.6%
4. $\gamma_p = -169.5$, $\gamma_s = 314.7$, $\eta = -1$			
OECD-OECD Trade/OECD GDP	0.118	0.132	11.7%
OECD-OECD Trade/OECD-RW Trade	0.739	1.060	43.5%
OECD Manf GDP/OECD GDP	0.259	0.222	–14.1%
5. $\gamma_p = -169.5$, $\gamma_s = 314.7$, $\eta = -9$			
OECD-OECD Trade/OECD GDP	0.118	0.132	1.6%
OECD-OECD Trade/OECD-RW Trade	0.739	1.060	43.5%
OECD Manf GDP/OECD GDP	0.284	0.222	–21.8%

Sensitivity Analysis: Alternative Calibration Methodologies

- Alternative specifications of nonhomogeneity
- Gross imports calibration
- Alternative RW endowment calibration
- Alternative RW growth calibration
- Intermediate goods

INTERMEDIATE GOODS?

$$Y_p^j = \min\left[\frac{X_{pp}^j}{a_{pp}}, \frac{\int_{D^w} X_{mp}^j(z)dz}{a_{mp}}, \frac{X_{sp}^j}{a_{sp}}, \theta_p \left(K_p^j\right)^{\alpha_p} \left(H_p^j\right)^{1-\alpha_p}\right]$$

$$Y_m(z) = \min\left[\begin{matrix}\dfrac{X_{pm}^j(z)}{a_{pm}}, \dfrac{\int_{D^w} X_{mm}^j(z,z')dz'}{a_{mm}}, \dfrac{X_{sm}^j(z)}{a_{sm}}, \\ \theta_m \left(K_m(z)\right)^{\alpha_m} \left(H_m(z)\right)^{1-\alpha_m} - F\end{matrix}\right]$$

$$Y_s^j = \min\left[\frac{X_{ps}^j}{a_{ps}}, \frac{\int_{D^w} X_{ms}^j(z)dz}{a_{ms}}, \frac{X_{ss}^j}{a_{ss}}, \theta_s \left(K_s^j\right)^{\alpha_s} \left(H_s^j\right)^{1-\alpha_s}\right]$$

Results for Model with Intermediate Goods

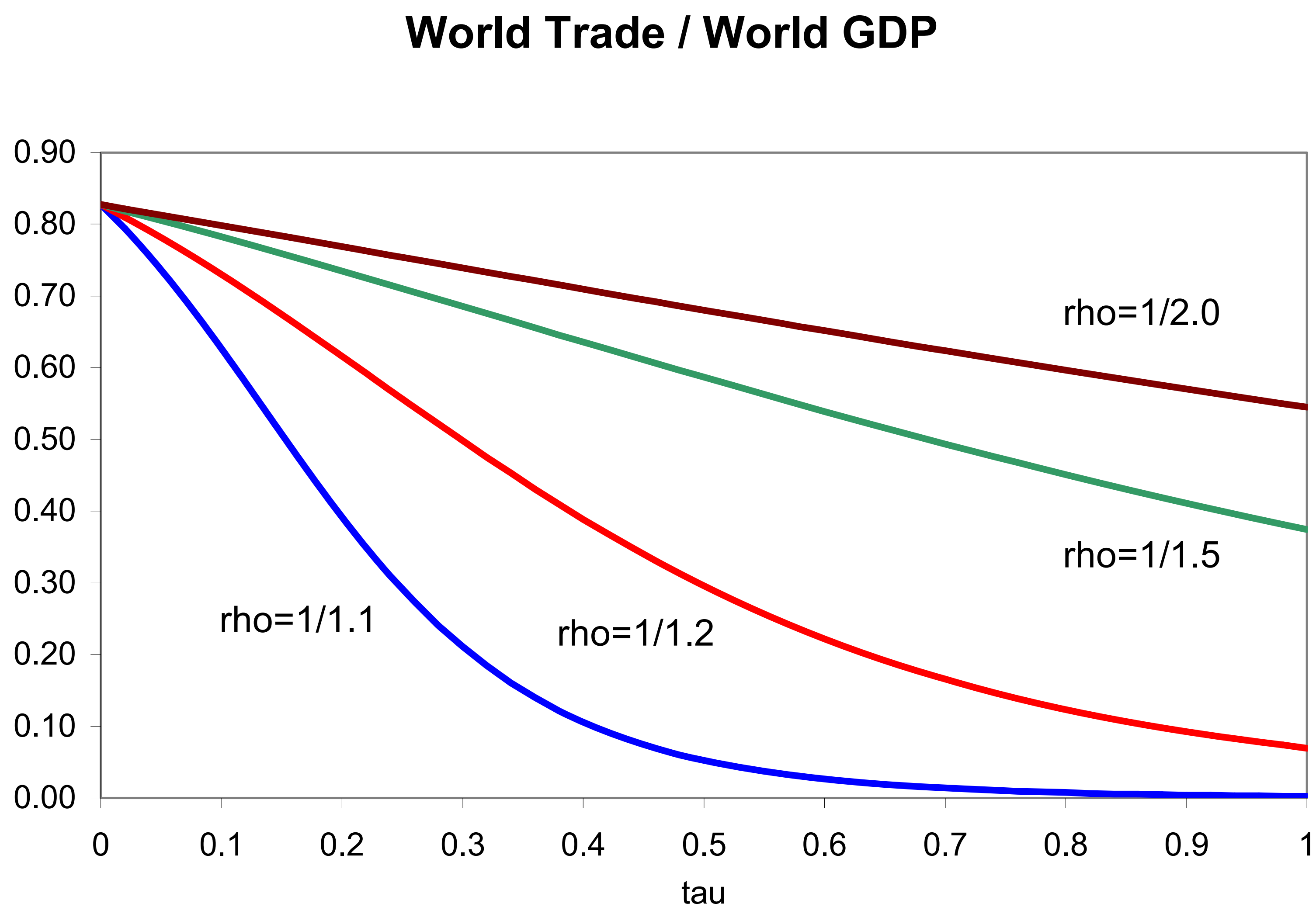
	1961	1990	Change
Data			
OECD-OECD Trade/OECD GDP	0.053	0.112	111.5%
OECD-OECD Trade/OECD-RW Trade	0.844	1.579	87.1%
OECD Manf GDP/OECD GDP	0.295	0.222	–24.6%
4. $\gamma_p = -307.8$, $\gamma_s = 262.2$, $\eta = -1$			
OECD-OECD Trade/OECD GDP	0.323	0.370	14.5%
OECD-OECD Trade/OECD-RW Trade	0.994	1.305	31.3%
OECD Manf GDP/OECD GDP	0.263	0.222	–15.6%
5. $\gamma_p = -307.8$, $\gamma_s = 262.2$, $\eta = -9$			
OECD-OECD Trade/OECD GDP	0.337	0.370	9.7%
OECD-OECD Trade/OECD-RW Trade	0.933	1.305	39.9%
OECD Manf GDP/OECD GDP	0.307	0.222	–27.5%

POLICY?

In a version of our model with n OECD countries, a manufacturing sector, and a uniform ad valorem tariff τ, the ratio of exports to income is given by

$$\frac{M}{Y} = \frac{(n-1)C_f}{Y} = \frac{n-1}{n-1+(1+\tau)^{1/(1-\rho)}}$$

Fixing n to replicate the size distribution of national incomes in the OECD, and setting $\rho = 1/1.2$, a fall in τ from 0.45 to 0.05 produces an increase in the ratio of trade to output as seen in the data.



World Trade / World GDP
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
tau
rho=1/2.0
rho=1/1.5
rho=1/1.1
rho=1/1.2

2. Applied general equilibrium models that put the standard theory to work do not well in predicting the impact of trade liberalization experiences like NAFTA.

Applied general equilibrium models were the only analytical game in town when it came to analyzing the impact of NAFTA in 1992-1993.

Typical sort of model: Static applied general equilibrium model with large number of industries and imperfect competition (Dixit-Stiglitz or Eastman-Stykolt) and finite number of firms in some industries. In some numerical experiments, new capital is placed in Mexico owned by consumers in the rest of North America to account for capital flows.

Examples:
Brown-Deardorff-Stern model of Canada, Mexico, and the United States
Cox-Harris model of Canada
Sobarzo model of Mexico

T. J. Kehoe, "An Evaluation of the Performance of Applied General Equilibrium Models of the Impact of NAFTA," in T. J. Kehoe, T. N. Srinivasan, and J. Whalley, editors, *Frontiers in Applied General Equilibrium Modeling: Essays in Honor of Herbert Scarf*, Cambridge University Press, 2005, 341-77.

Research Agenda:

- Compare results of numerical experiments of models with data.
- Determine what shocks — besides NAFTA policies — were important.
- Construct a simple applied general equilibrium model and perform experiments with alternative specifications to determine what was wrong with the 1992-1993 models.

Applied GE Models Can Do a Good Job!

Spain: Kehoe-Polo-Sancho (1992) evaluation of the performance of the Kehoe-Manresa-Noyola-Polo-Sancho-Serra MEGA model of the Spanish economy: A Shoven-Whalley type model with perfect competition, modified to allow government and trade deficits and unemployment (Kehoe-Serra). Spain's entry into the European Community in 1986 was accompanied by a fiscal reform that introduced a value-added tax (VAT) on consumption to replace a complex range of indirect taxes, including a turnover tax applied at every stage of the production process. What would happen to tax revenues? Trade reform was of secondary importance.

Canada-U.S.: Fox (1999) evaluation of the performance of the Brown-Stern (1989) model of the 1989 Canada-U.S. FTA.

Other changes besides policy changes are important!

Changes in Consumer Prices in the Spanish Model
(Percent)

sector	data 1985-1986	model policy only	model shocks only	model policy&shocks
food and nonalcoholic beverages	1.8	-2.3	4.0	1.7
tobacco and alcoholic beverages	3.9	2.5	3.1	5.8
clothing	2.1	5.6	0.9	6.6
housing	-3.3	-2.2	-2.7	-4.8
household articles	0.1	2.2	0.7	2.9
medical services	-0.7	-4.8	0.6	-4.2
transportation	-4.0	2.6	-8.8	-6.2
recreation	-1.4	-1.3	1.5	0.1
other services	2.9	1.1	1.7	2.8
weighted correlation with data		-0.08	0.87	0.94
variance decomposition of change		0.30	0.77	0.85
regression coefficient *a*		0.00	0.00	0.00
regression coefficient *b*		-0.08	0.54	0.67

Measures of Accuracy of Model Results

1. Weighted correlation coefficient.
2. Variance decomposition of the (weighted) variance of the changes in the data:

$$vardec(y^{data}, y^{model}) = \frac{var(y^{model})}{var(y^{model}) + var(y^{data} - y^{model})}.$$

3, 4. Estimated coefficients a and b from the (weighted) regression

$$x_i^{data} = a + bx_i^{model} + e_i.$$

Changes in Value of Gross Output/GDP in the Spanish Model (Percent)

sector	data 1985-1986	model policy only	model shocks only	model policy&shocks
agriculture	-0.4	-1.1	8.3	6.9
energy	-20.3	-3.5	-29.4	-32.0
basic industry	-9.0	1.6	-1.8	-0.1
machinery	3.7	3.8	1.0	5.0
automobile industry	1.1	3.9	4.7	8.6
food products	-1.8	-2.4	4.7	2.1
other manufacturing	0.5	-1.7	2.3	0.5
construction	5.7	8.5	1.4	10.3
commerce	6.6	-3.6	4.4	0.4
transportation	-18.4	-1.5	1.0	-0.7
services	8.7	-1.1	5.8	4.5
government services	7.6	3.4	0.9	4.3
weighted correlation with data		0.16	0.80	0.77
variance decomposition of change		0.11	0.73	0.71
regression coefficient *a*		-0.52	-0.52	-0.52
regression coefficient *b*		0.44	0.75	0.67

Changes in Trade/GDP in the Spanish Model (Percent)

direction of exports	data 1985-1986	model policy only	model shocks only	model policy&shocks
Spain to rest of E.C.	-6.7	-3.2	-4.9	-7.8
Spain to rest of world	-33.2	-3.6	-6.1	-9.3
rest of E.C. to Spain	14.7	4.4	-3.9	0.6
rest of world to Spain	-34.1	-1.8	-16.8	-17.7
weighted correlation with data		0.69	0.77	0.90
variance decomposition of change		0.02	0.17	0.24
regression coefficient *a*		-12.46	2.06	5.68
regression coefficient *b*		5.33	2.21	2.37

Changes in Composition of GDP in the Spanish Model (Percent of GDP)

variable	data 1985-1986	model policy only	model shocks only	model policy&shocks
wages and salaries	-0.53	-0.87	-0.02	-0.91
business income	-1.27	-1.63	0.45	-1.24
net indirect taxes and tariffs	1.80	2.50	-0.42	2.15
correlation with data		0.998	-0.94	0.99
variance decomposition of change		0.93	0.04	0.96
regression coefficient *a*		0.00	0.00	0.00
regression coefficient *b*		0.73	-3.45	0.85
private consumption	-0.81	-1.23	-0.51	-1.78
private investment	1.09	1.81	-0.58	1.32
government consumption	-0.02	-0.06	-0.38	-0.44
government investment	-0.06	-0.06	-0.07	-0.13
exports	-3.40	-0.42	-0.69	-1.07
-imports	3.20	-0.03	2.23	2.10
correlation with data		0.40	0.77	0.83
variance decomposition of change		0.20	0.35	0.58
regression coefficient *a*		0.00	0.00	0.00
regression coefficient *b*		0.87	1.49	1.24

Public Finances in the Spanish Model
(Percent of GDP)

variable	data 1985-1986	model policy only	model shocks only	model policy&shocks
indirect taxes and subsidies	2.38	3.32	-0.38	2.98
tariffs	-0.58	-0.82	-0.04	-0.83
social security payments	0.04	-0.19	-0.03	-0.22
direct taxes and transfers	-0.84	-0.66	0.93	0.26
government capital income	-0.13	-0.06	0.02	-0.04
correlation with data		0.99	-0.70	0.92
variance decomposition of change		0.93	0.08	0.86
regression coefficient *a*		-0.06	0.35	-0.17
regression coefficient *b*		0.74	-1.82	0.80

Models of NAFTA Did Not Do a Good Job!

Ex-post evaluations of the performance of applied GE models are essential if policy makers are to have confidence in the results produced by this sort of model.

Just as importantly, they help make applied GE analysis a scientific discipline in which there are well-defined puzzles and clear successes and failures for alternative hypotheses.

Changes in Trade/GDP in Brown-Deardorff-Stern Model (Percent)

variable	data 1988-1999	model
Canadian exports	52.9	4.3
Canadian imports	57.7	4.2
Mexican exports	240.6	50.8
Mexican imports	50.5	34.0
U.S. exports	19.1	2.9
U.S. imports	29.9	2.3
weighted correlation with data		0.64
variance decomposition of change		0.08
regression coefficient *a*		23.20
regression coefficient *b*		2.43

Changes in Canadian Exports/ GDP in the Brown-Deardorff-Stern Model (Percent)

sector	exports to Mexico 1988–1999	model	exports to United States 1988–1999	model
agriculture	122.5	3.1	106.1	3.4
mining and quarrying	-34.0	-0.3	75.8	0.4
food	89.3	2.2	91.7	8.9
textiles	268.2	-0.9	97.8	15.3
clothing	1544.3	1.3	237.1	45.3
leather products	443.0	1.4	-14.4	11.3
footwear	517.0	3.7	32.8	28.3
wood products	232.6	4.7	36.5	0.1
furniture and fixtures	3801.7	2.7	282.6	12.5
paper products	240.7	-4.3	113.7	-1.8
printing and publishing	6187.4	-2.0	37.2	-1.6
chemicals	37.1	-7.8	109.4	-3.1
petroleum and products	678.1	-8.5	-42.5	0.5
rubber products	647.4	-1.0	113.4	9.5
nonmetal mineral products	333.5	-1.8	20.5	1.2
glass products	264.4	-2.2	74.5	30.4
iron and steel	195.2	-15.0	92.1	12.9
nonferrous metals	38.4	-64.7	34.7	18.5
metal products	767.0	-10.0	102.2	15.2
nonelectrical machinery	376.8	-8.9	28.9	3.3
electrical machinery	633.9	-26.2	88.6	14.5
transportation equipment	305.8	-4.4	30.7	10.7
miscellaneous manufactures	1404.5	-12.1	100.0	-2.1
weighted correlation with data		-0.91		-0.43
variance decomposition of change		0.003		0.02
regression coefficient *a*		249.24		79.20
regression coefficient *b*		-15.48		-2.80

Changes in Mexican Exports/GDP in the Brown-Deardorff-Stern Model (Percent)

sector	exports to Canada 1988–1999	exports to Canada model	exports to United States 1988–1999	exports to United States model
agriculture	-20.5	-4.1	-15.0	2.5
mining and quarrying	-35.5	27.3	-22.9	26.9
food	70.4	10.8	9.4	7.5
textiles	939.7	21.6	832.3	11.8
clothing	1847.0	19.2	829.6	18.6
leather products	1470.3	36.2	618.3	11.7
footwear	153.0	38.6	111.1	4.6
wood products	4387.6	15.0	145.6	-2.7
furniture and fixtures	4933.2	36.2	181.2	7.6
paper products	23.9	32.9	70.3	13.9
printing and publishing	476.3	15.0	122.1	3.9
chemicals	204.6	36.0	70.4	17.0
petroleum and products	-10.6	32.9	66.4	34.1
rubber products	2366.2	-6.7	783.8	-5.3
nonmetal mineral products	1396.1	5.7	222.3	3.7
glass products	676.8	13.3	469.8	32.3
iron and steel	32.5	19.4	40.9	30.8
nonferrous metals	-35.4	138.1	111.2	156.5
metal products	610.4	41.9	477.2	26.8
nonelectrical machinery	570.6	17.3	123.6	18.5
electrical machinery	1349.2	137.3	744.9	178.0
transportation equipment	2303.4	3.3	349.0	6.2
miscellaneous manufactures	379.4	61.1	181.5	43.2
weighted correlation with data		0.19		0.71
variance decomposition of change		0.01		0.04
regression coefficient *a*		120.32		38.13
regression coefficient *b*		2.07		3.87

Changes in U.S. Exports/GDP in the Brown-Deardorff-Stern Model (Percent)

sector	exports to Canada 1988–1999	exports to Canada model	exports to Mexico 1988–1999	exports to Mexico model
agriculture	-24.1	5.1	6.5	7.9
mining and quarrying	-23.6	1.0	-19.8	0.5
food	62.4	12.7	37.7	13.0
textiles	177.2	44.0	850.5	18.6
clothing	145.5	56.7	543.0	50.3
leather products	29.9	7.9	87.7	15.5
footwear	48.8	45.7	33.1	35.4
wood products	76.4	6.7	25.7	7.0
furniture and fixtures	83.8	35.6	224.1	18.6
paper products	-20.5	18.9	-41.9	-3.9
printing and publishing	50.8	3.9	507.9	-1.1
chemicals	49.8	21.8	61.5	-8.4
petroleum and products	-6.9	0.8	-41.1	-7.4
rubber products	95.6	19.1	165.6	12.8
nonmetal mineral products	56.5	11.9	55.9	0.8
glass products	50.5	4.4	112.9	42.3
iron and steel	0.6	11.6	144.5	-2.8
nonferrous metals	-20.7	-6.7	-28.7	-55.1
metal products	66.7	18.2	301.4	5.4
nonelectrical machinery	36.2	9.9	350.8	-2.9
electrical machinery	154.4	14.9	167.8	-10.9
transportation equipment	36.5	-4.6	290.3	9.9
miscellaneous manufactures	117.3	11.5	362.3	-9.4
weighted correlation with data		-0.01		0.50
variance decomposition of change		0.14		0.02
regression coefficient *a*		37.27		190.89
regression coefficient *b*		-0.02		3.42

Changes in Canadian Trade/GDP in Cox-Harris Model (Percent)

variable	data 1988-2000	model
total trade	57.2	10.0
trade with Mexico	280.0	52.2
trade with United States	76.2	20.0
weighted correlation with data		0.99
variance decomposition of change		0.52
regression coefficient *a*		38.40
regression coefficient *b*		1.93

Changes in Canadian Trade/GDP in the Cox-Harris Model (Percent)

sector	total exports 1988-2000	total exports model	total imports 1988-2000	total imports model
agriculture	-13.7	-4.1	4.6	7.2
forestry	215.5	-11.5	-21.5	7.1
fishing	81.5	-5.4	107.3	9.5
mining	21.7	-7.0	32.1	4.0
food, beverages, and tobacco	50.9	18.6	60.0	3.8
rubber and plastics	194.4	24.5	87.7	13.8
textiles and leather	201.1	108.8	24.6	18.2
wood and paper	31.9	7.3	97.3	7.2
steel and metal products	30.2	19.5	52.2	10.0
transportation equipment	66.3	3.5	29.7	3.0
machinery and appliances	112.9	57.1	65.0	13.3
nonmetallic minerals	102.7	31.8	3.6	7.3
refineries	20.3	-2.7	5.1	1.5
chemicals and misc. manufactures	53.3	28.1	92.5	10.4
weighted correlation with data		0.49		0.85
variance decomposition of change		0.32		0.08
regression coefficient *a*		41.85		22.00
regression coefficient *b*		0.81		3.55

Changes in Mexican Trade/GDP in the Sobarzo Model (Percent)

sector	exports to North America 1988–2000	exports to North America model	imports from North America 1988–2000	imports from North America model
agriculture	-15.3	-11.1	-28.2	3.4
mining	-23.2	-17.0	-50.7	13.2
petroleum	-37.6	-19.5	65.9	-6.8
food	5.2	-6.9	11.8	-5.0
beverages	42.0	5.2	216.0	-1.8
tobacco	-42.3	2.8	3957.1	-11.6
textiles	534.1	1.9	833.2	-1.2
wearing apparel	2097.3	30.0	832.9	4.5
leather	264.3	12.4	621.0	-0.4
wood	415.1	-8.5	168.9	11.7
paper	12.8	-7.9	68.1	-4.7
chemicals	41.9	-4.4	71.8	-2.7
rubber	479.0	12.8	792.0	-0.1
nonmetallic mineral products	37.5	-6.2	226.5	10.9
iron and steel	35.9	-4.9	40.3	17.7
nonferrous metals	-40.3	-9.8	101.2	9.8
metal products	469.5	-4.4	478.7	9.5
nonelectrical machinery	521.7	-7.4	129.0	20.7
electrical machinery	3189.1	1.0	749.1	9.6
transportation equipment	224.5	-5.0	368.0	11.2
other manufactures	975.1	-4.5	183.6	4.2
weighted correlation with data		0.61		0.23
variance decomposition of change		0.0004		0.002
regression coefficient *a*		495.08		174.52
regression coefficient *b*		30.77		5.35

What Do We Learn from these Evaluations?

The Spanish model seems to have been far more successful in predicting the consequences of policy changes than the three models of NAFTA, but

- Kehoe, Polo, and Sancho (KPS) knew the structure of their model well enough to precisely identify the relationships between the variables in their model with those in the data;

- KPS were able to use the model to carry out numerical exercises to incorporate the impact of exogenous shocks.

KPS had an incentive to show their model in the best possible light.

3. Much of the growth of trade after a trade liberalization experience is growth on the extensive margin. Models need to allow for corner solutions or fixed costs.

T. J. Kehoe and K. J. Ruhl, "How Important is the New Goods Margin in International Trade?" Federal Reserve Bank of Minneapolis, 2002.

What happens to the **least-traded** goods:

Over the business cycle?
During trade liberalization?

Indirect evidence on the extensive margin

How Does Trade Grow?

- **Intensive Margin:** growth in goods already traded
- **Extensive Margin:** trade in goods not traded before

The Extensive Margin

- The Extensive Margin has recently gained attention
- **Models**
    - Melitz (2003)
    - Alessandria and Choi (2003)
    - Ruhl (2004)
- **Empirically**
    - Hummels and Klenow (2002)
    - Eaton, Kortum and Kramarz (2004)

What Happens to the Extensive Margin?

- During trade liberalization?
  - Large changes in the extensive margin
- Over the business cycle?
  - Little change in extensive margin

Evidence from Trade Agreements

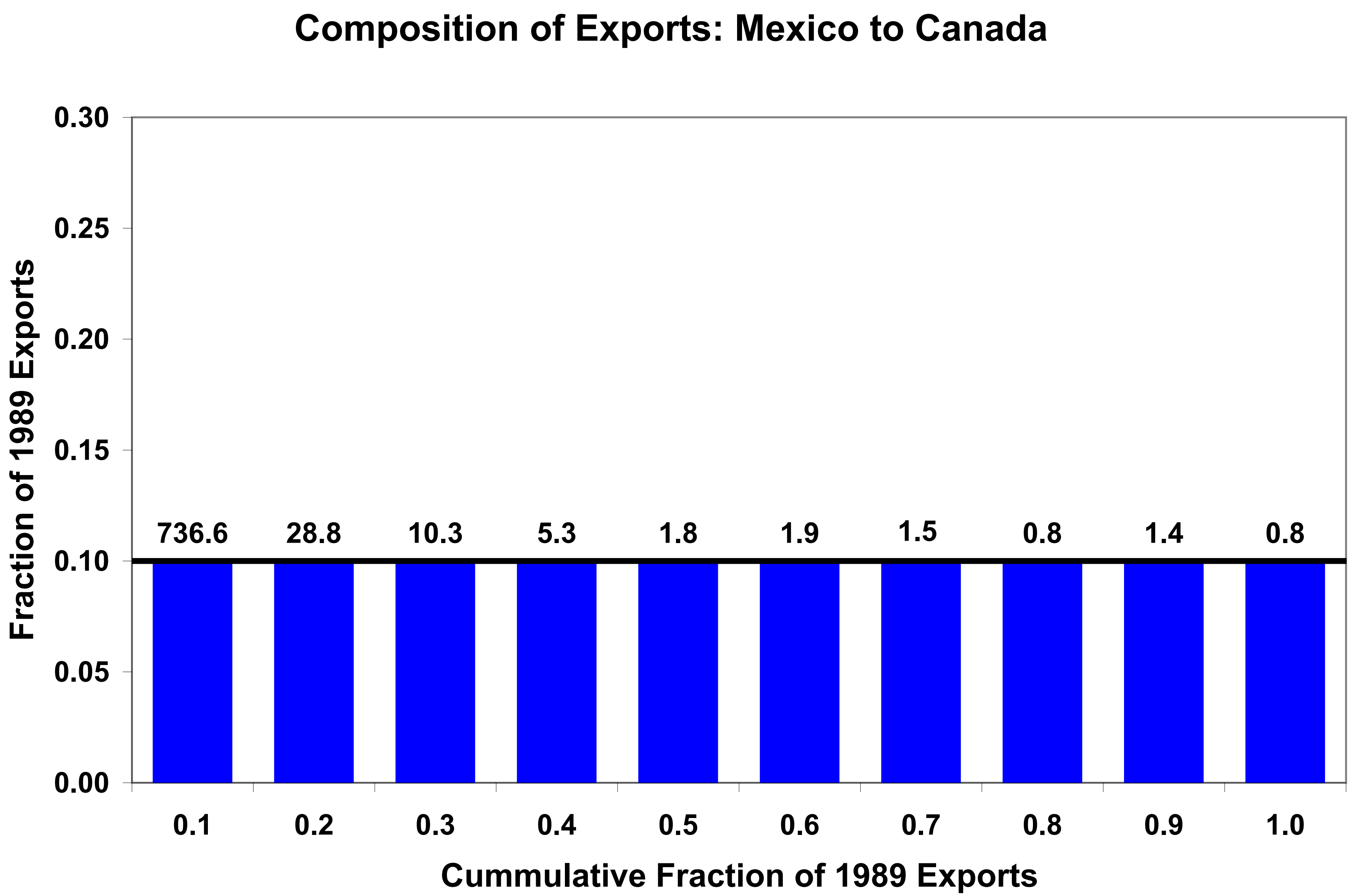
- **Events**
  - Greece's Accession to the European Econ. Community - 1981
  - Portugal's Accession to the European Community - 1986
  - Spain's Accession to the European Community - 1986
  - U.S.-Canada Free Trade Agreement - 1989
  - North American Free Trade Agreement - 1994
- **Data**
  - Four-digit SITC bilateral trade data (OECD)
    - 789 codes in revision 2
- Indirect Evidence

Measure One

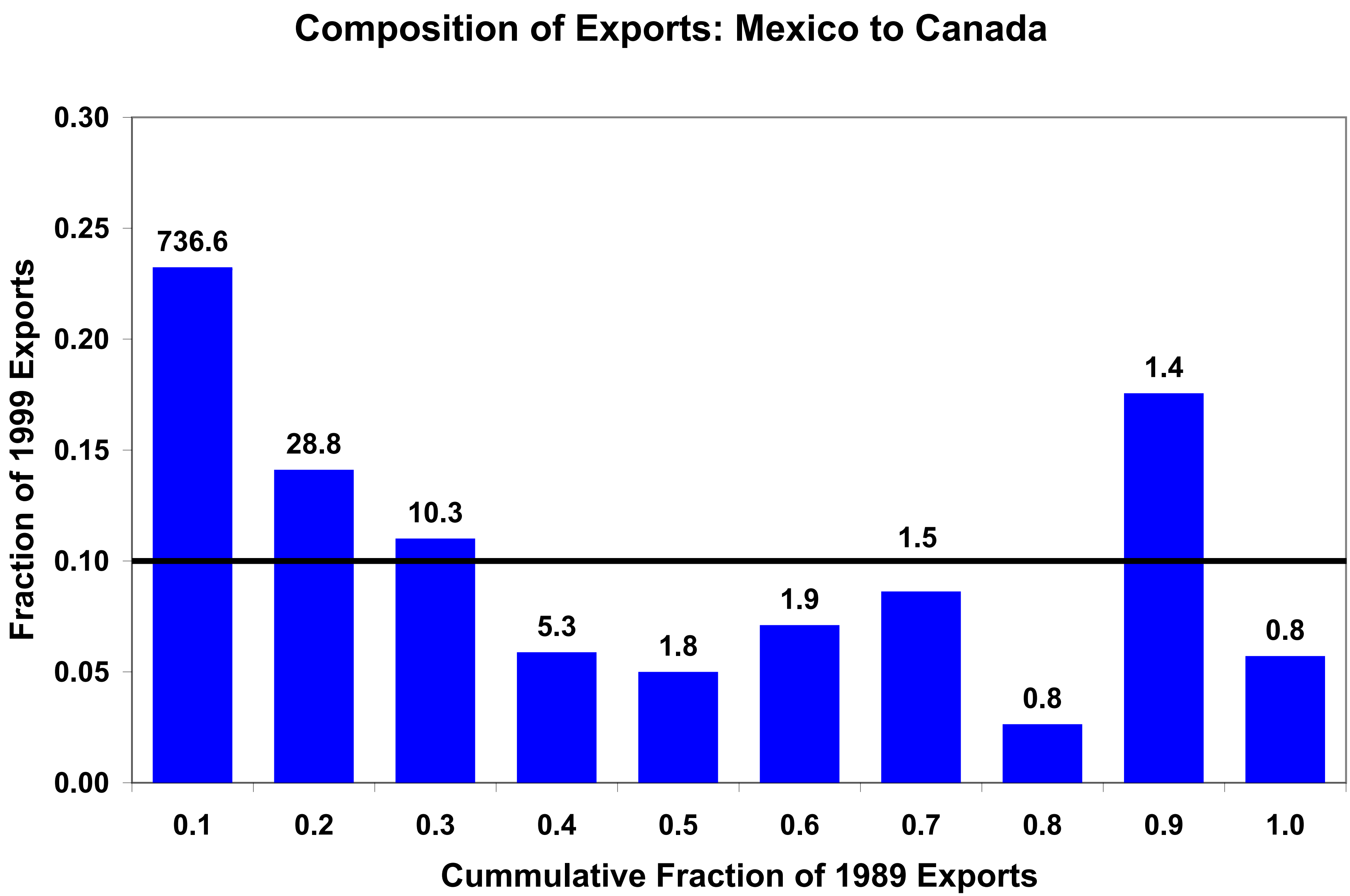
1. Rank codes from lowest value of exports to highest value of exports based on average of first 3 years

2. Form sets of codes by cumulating exports: the first 742.9 codes make up 10 percent of exports; the next 24.1 codes make up 10 percent of exports; and so on.

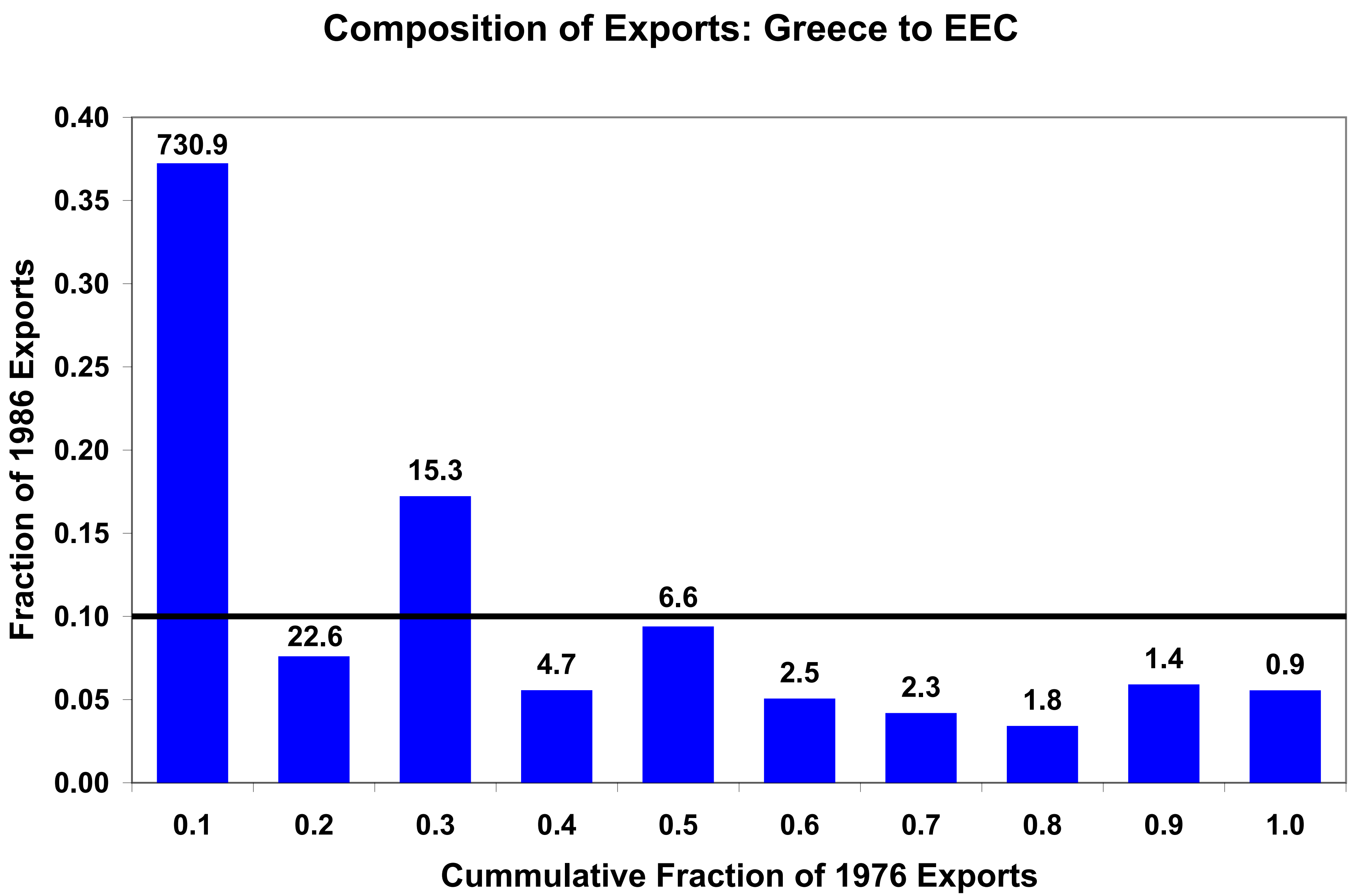
3. Calculate each set's share of export value at the end of the sample period.



Composition of Exports: Mexico to Canada
0.30
0.25
0.20
0.15
0.10
0.05
0.00
Fraction of 1989 Exports
736.6
28.8
10.3
5.3
1.8
1.9
1.5
0.8
1.4
0.8
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Cummulative Fraction of 1989 Exports



Composition of Exports: Mexico to Canada
Fraction of 1999 Exports
0.30
0.25
0.20
0.15
0.10
0.05
0.00
736.6
28.8
10.3
5.3
1.8
1.9
1.5
0.8
1.4
0.8
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Cummulative Fraction of 1989 Exports



Composition of Exports: Greece to EEC
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
Fraction of 1986 Exports
730.9
22.6
15.3
4.7
6.6
2.5
2.3
1.8
1.4
0.9
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Cummulative Fraction of 1976 Exports

Measure Two

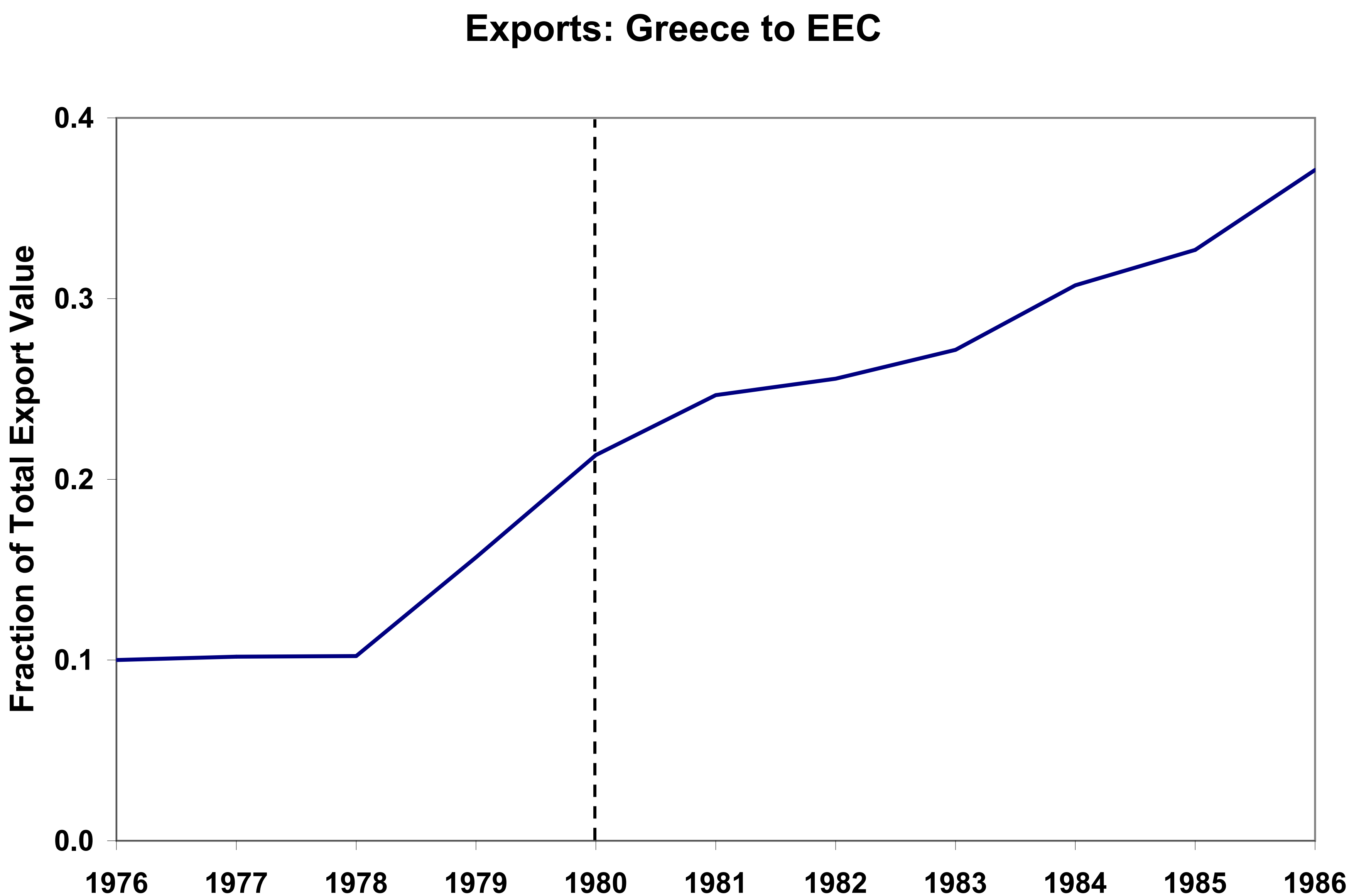
1. Order codes as before.

2. Cumulate exports as before.

3. Follow the evolution of the first (least-traded) set's share of total exports before, during, and after the liberalization.

Exports: Mexico to Canada
Fraction of Total Export Value
0.30
0.20
0.10
0.00
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999

Exports: Canada to Mexico
Fraction of Total Export Value
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999

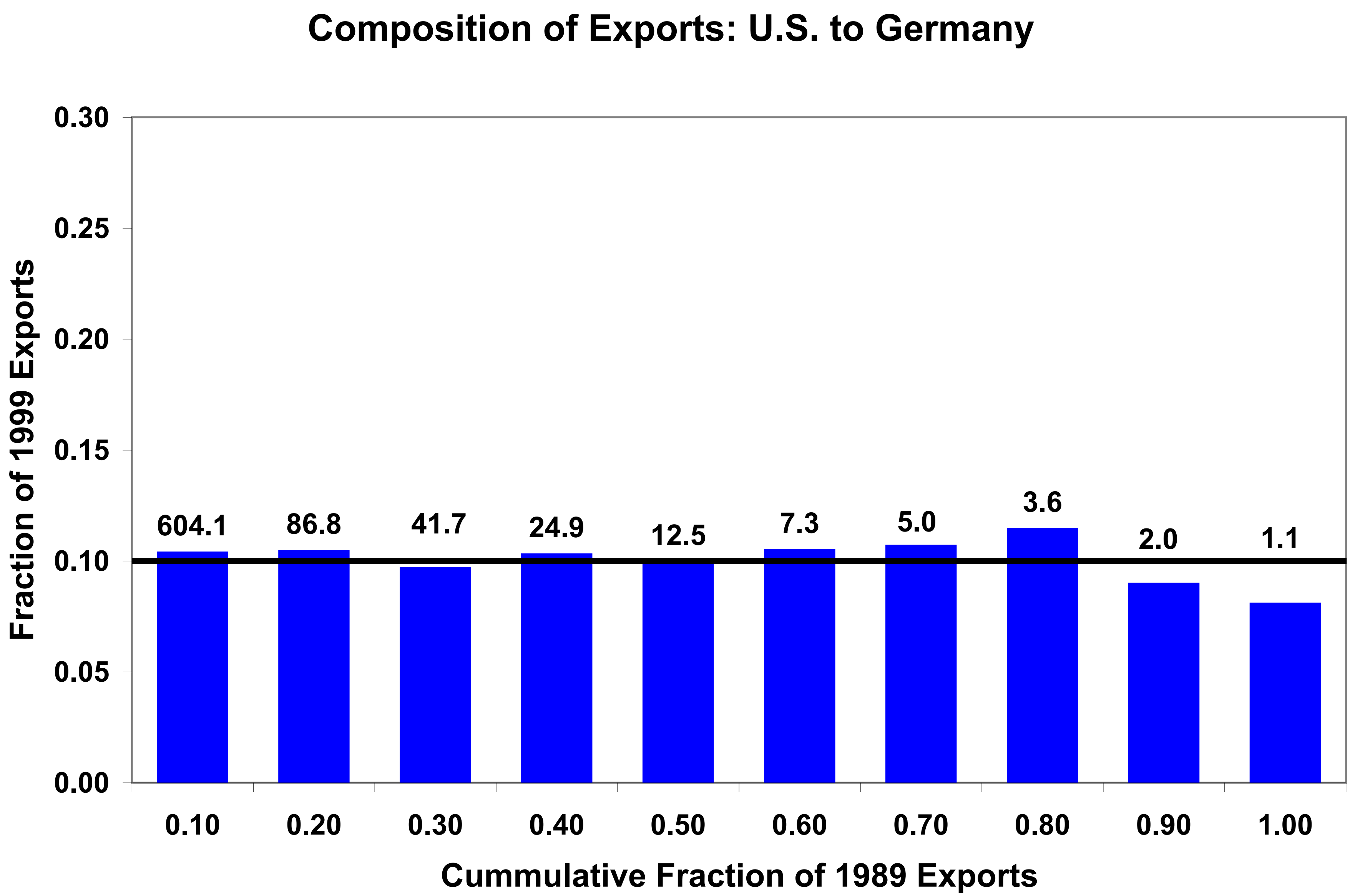

Exports: Greece to EEC
Fraction of Total Export Value
0.4
0.3
0.2
0.1
0.0
1976
1977
1978
1979
1980
1981
1982
1983
1984
1985
1986

Trade Liberalization and the Extensive Margin

Period	Trade Flow	Share of Export Growth
1989-1999	Mexico - U.S.	0.153
1989-1999	U.S. – Mexico	0.118
1989-1999	Mexico - Canada	0.231
1989-1999	Canada - Mexico	0.307
1989-1999	Canada - U.S.	0.162
1989-1999	U.S. – Canada	0.130
1978-1986	Greece to the EEC	0.371
1982-1987	Spain to the EC	0.128
1982-1987	Portugal to the EC	0.147

Business Cycles and the Extensive Margin

- Over same period, consider countries with stable policy
    - U.S. – Japan
    - U.S. – U.K.
    - U.S. – Germany



Composition of Exports: U.S. to Germany
Fraction of 1999 Exports
0.30
0.25
0.20
0.15
0.10
0.05
0.00
604.1
86.8
41.7
24.9
12.5
7.3
5.0
3.6
2.0
1.1
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00
Cummulative Fraction of 1989 Exports


Exports: United States to Germany
Fraction of Total Export Value
0.30
0.25
0.20
0.15
0.10
0.05
0.00
U.S. Exports to Germany
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999


Exports: Germany and the United States
Fraction of Total Export Value
0.30
0.25
0.20
0.15
0.10
0.05
0.00
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
German Exports to U.S.
U.S. Exports to Germany

Business Cycles and the Extensive Margin

Period	Trade Flow	Share of Export Growth
1989-1999	U.S. - U.K.	0.096
1989-1999	U.K. - U.S.	0.128
1989-1999	U.S. - Japan	0.130
1989-1999	Japan - U.S.	0.103
1989-1999	U.S. - Germany	0.104
1989-1999	Germany - U.S.	0.103

The Model

- Countries: foreign and home
- Continuum of goods:

$$y_i(x) = \frac{1}{a_i(x)} l_i(x) \qquad x \in [0,1]$$

- Stand-in consumer in each country with labor L_i.
- Preferences:

$$U = \int_0^1 \log[c_i(x)]dx$$

- *ad valorem* tariffs: τ_i

Determination of Exports

- x is exported by foreign if

$$a_h(x) > w_f a_f(x)(1+\tau_h) \quad \Leftrightarrow \quad \frac{a_h(x)}{a_f(x)} > w_f(1+\tau_h)$$

- x is exported by home if

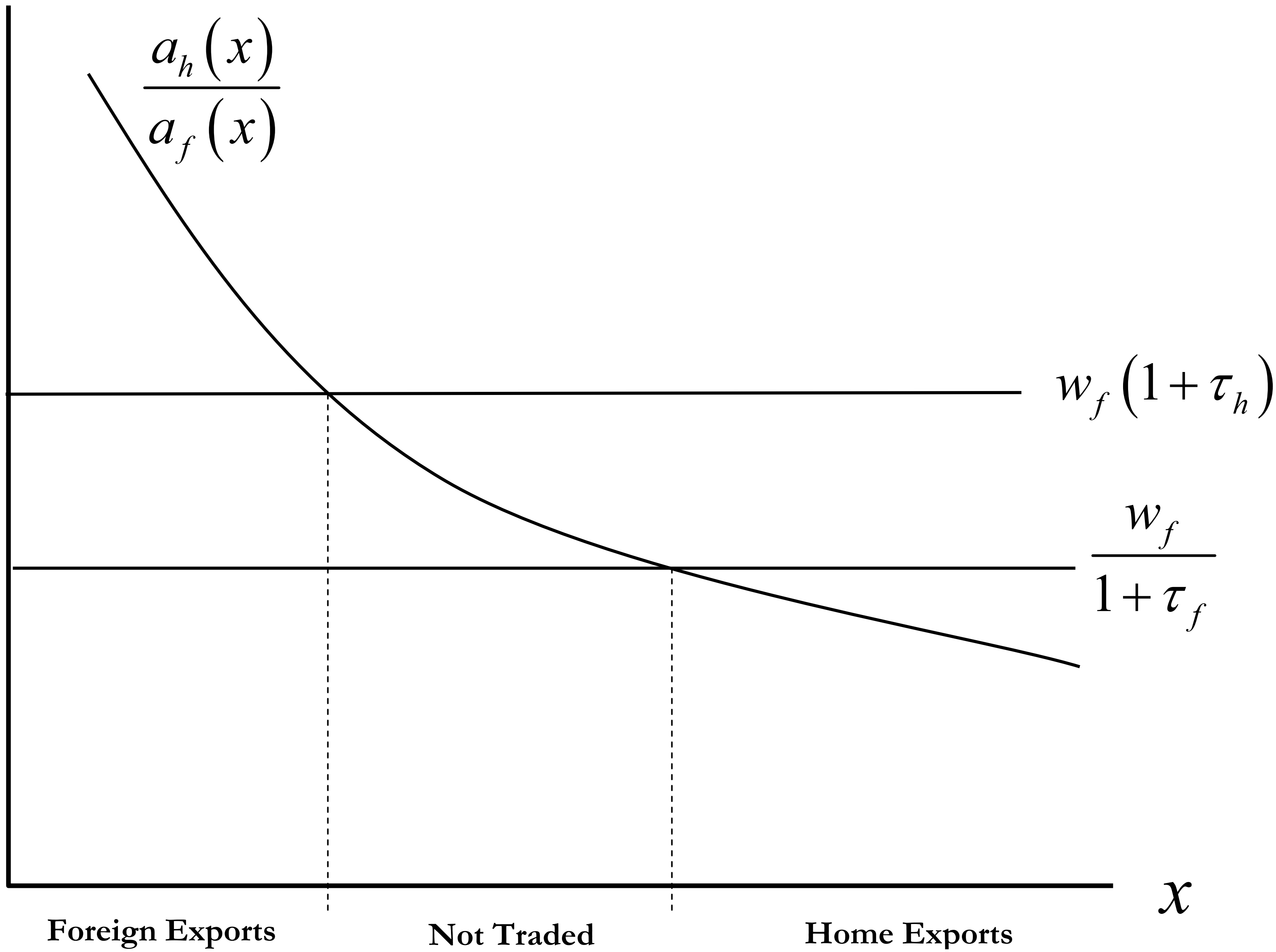
$$\frac{a_h(x)}{a_f(x)} < \frac{w_f}{1+\tau_f}$$

- x is not traded if

$$\frac{w_f}{1+\tau_h} > \frac{a_h(x)}{a_f(x)} > w_f(1+\tau_f)$$

Dornbusch, Fisher, Samuelson (1977)

- Order goods according to the relative unit costs.



$\frac{a_h(x)}{a_f(x)}$
$w_f(1+\tau_h)$
$\frac{w_f}{1+\tau_f}$
x
Foreign Exports
Not Traded
Home Exports

Dornbusch, Fisher, Samuelson (1977)

- Order goods according to the relative unit costs.

- Problems
  - Trade data is collected in aggregates.
  - Difficult to obtain data on relative unit costs.
  - Both countries may export the same aggregate.

Our Approach

- SITC ordering: an aggregate is an interval in $[0,1]$
- Take J evenly spaced points in $[0,1]$.
- Randomly assign log-productivities.

$$\alpha_j = \log\left[\frac{a_h(j)}{a_f(j)}\right] \qquad \alpha_j \sim u[-\bar{\alpha},\bar{\alpha}]$$

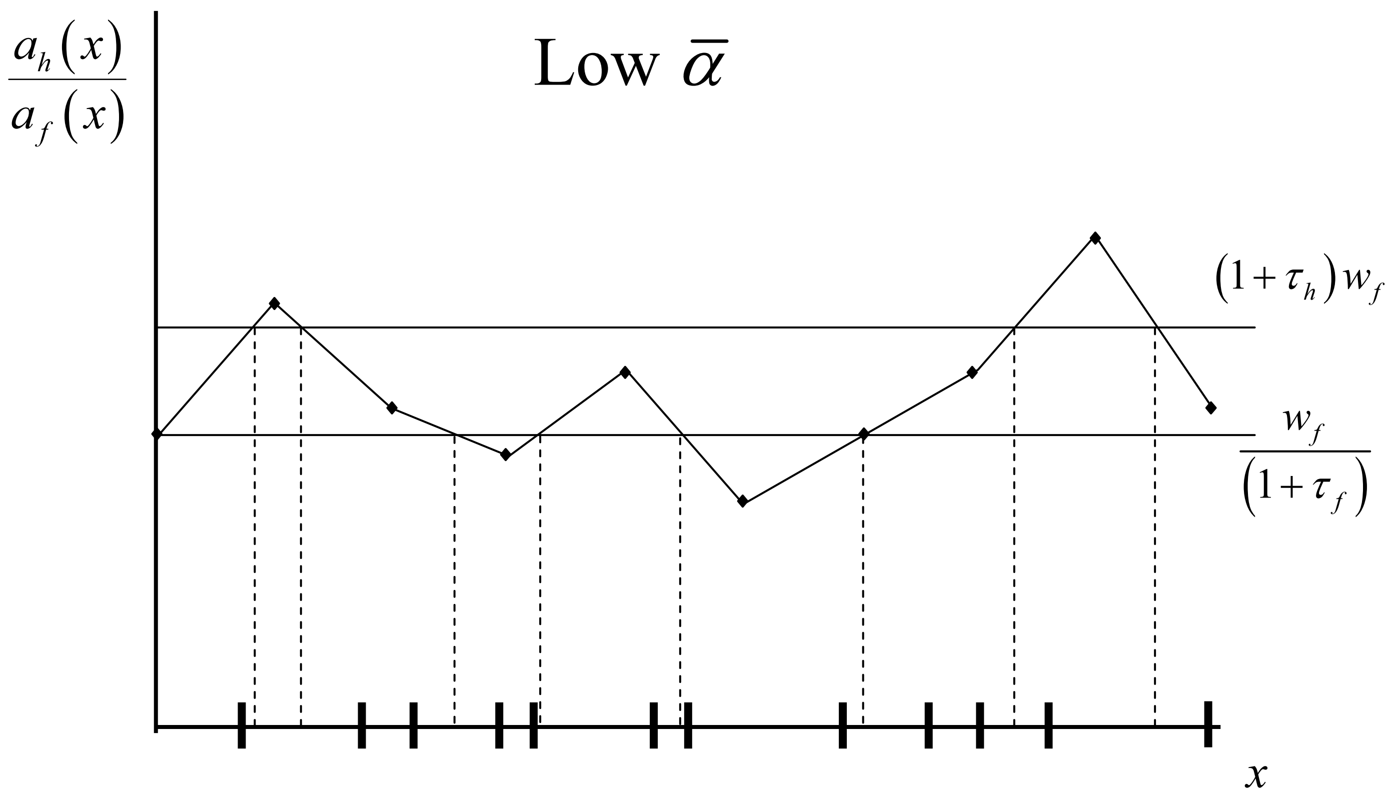
- Points not on the grid are filled in by linear interpolation.

Relative Productivity Curve

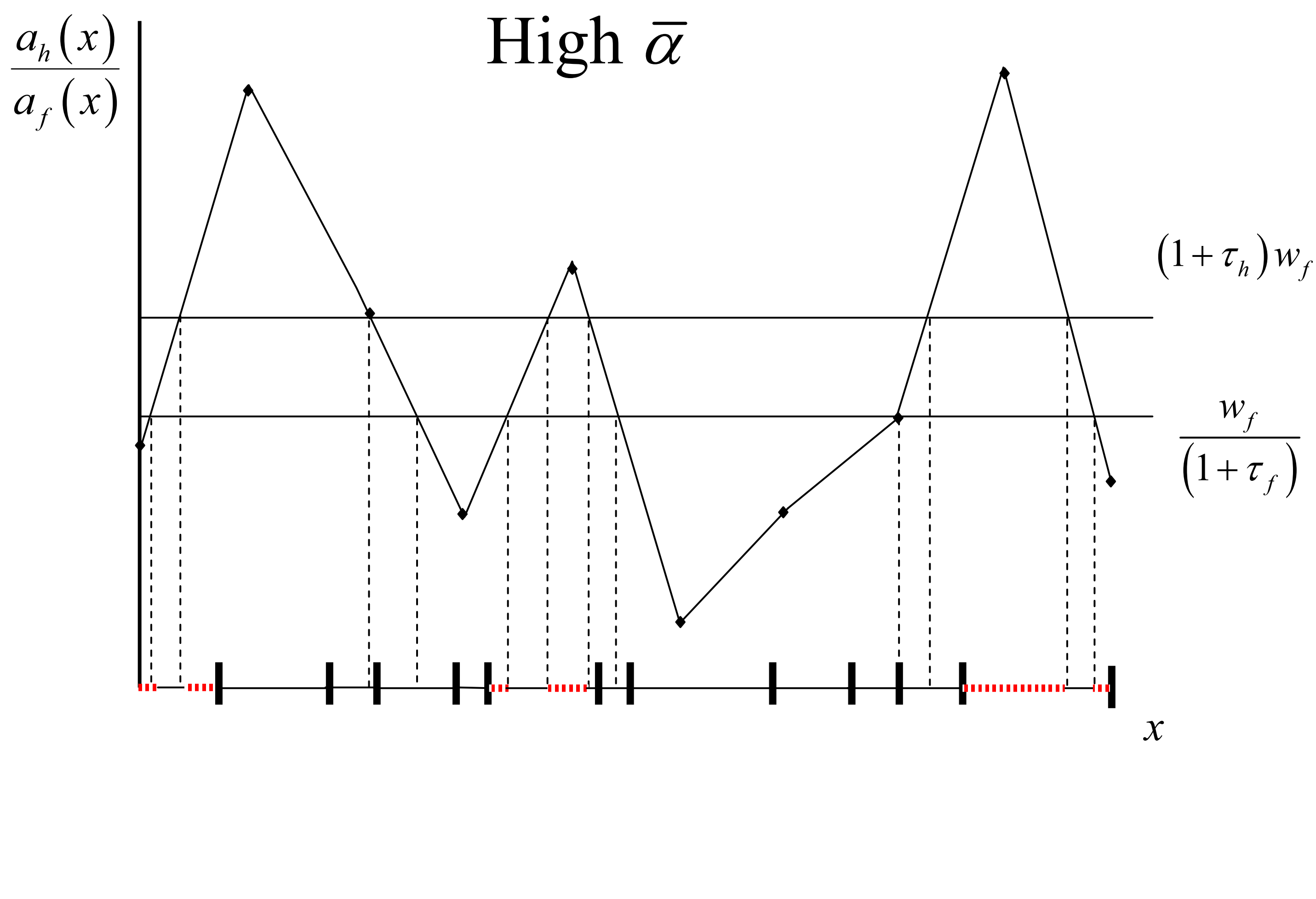
- Steeper segments
  - less trade growth
  - more intra-industry trade

- For a given J larger $\bar{\alpha}$ imply steeper segments.

- For a given $\bar{\alpha}$, larger J imply steeper segments.



$\frac{a_h(x)}{a_f(x)}$
Low $\bar{\alpha}$
$(1+\tau_h)w_f$
$\frac{w_f}{(1+\tau_f)}$
x



$\frac{a_h(x)}{a_f(x)}$
High $\bar{\alpha}$
$(1+\tau_h)w_f$
$\frac{w_f}{(1+\tau_f)}$
x

Model Solution

1. Choose J and $\bar{\alpha}$.

2. Draw a realization of the relative productivity curve.

3. Solve the model and compute extensive margin measures.

4. Repeat for 5000 simulations.

5. Calculate means over simulations.

Calibration

- Parameters: $\bar{\alpha}$, J, L_f / L_h, SITC endpoints

- Country size is measured by gross output of commodities.

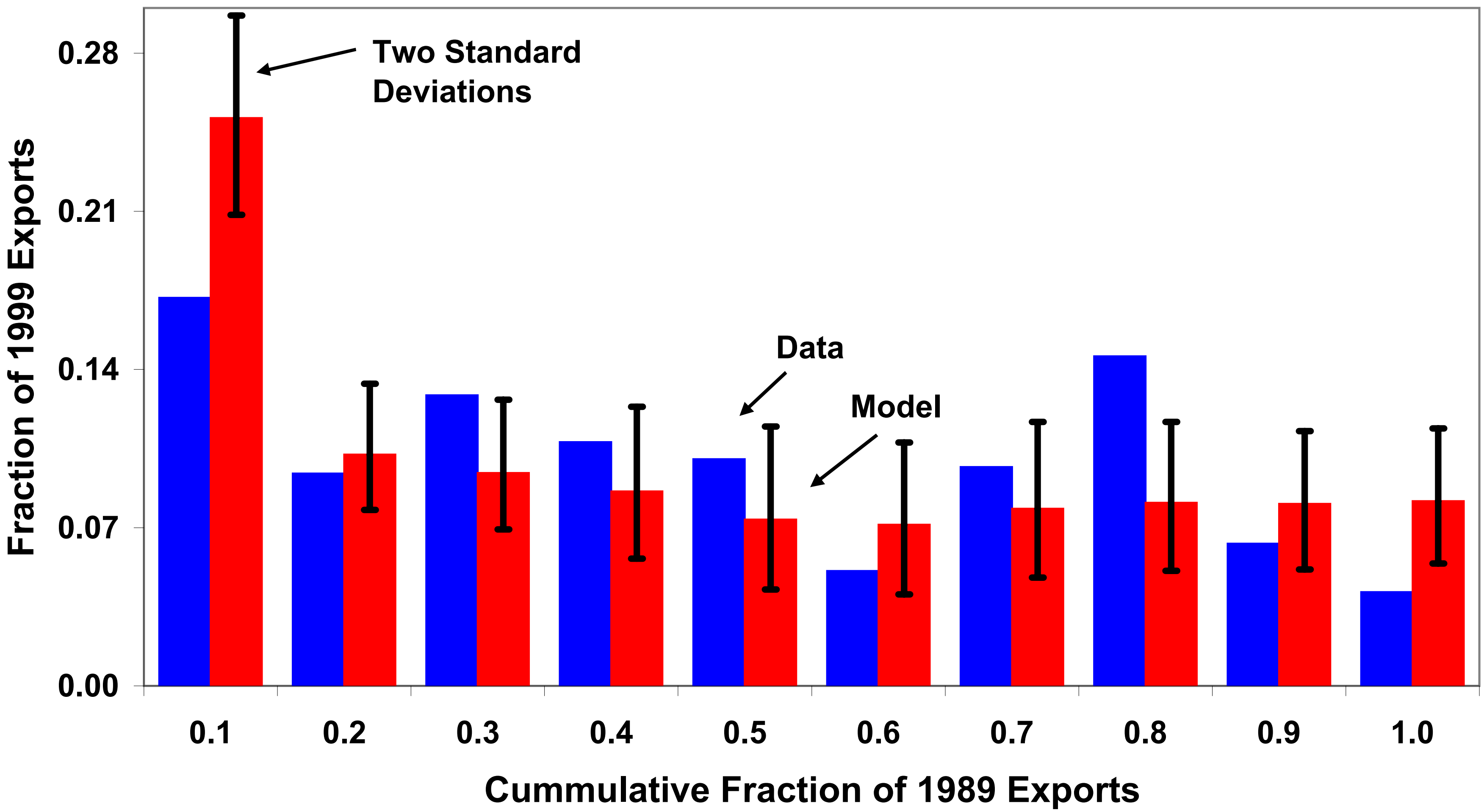
- Codes are ordered by their SITC number.

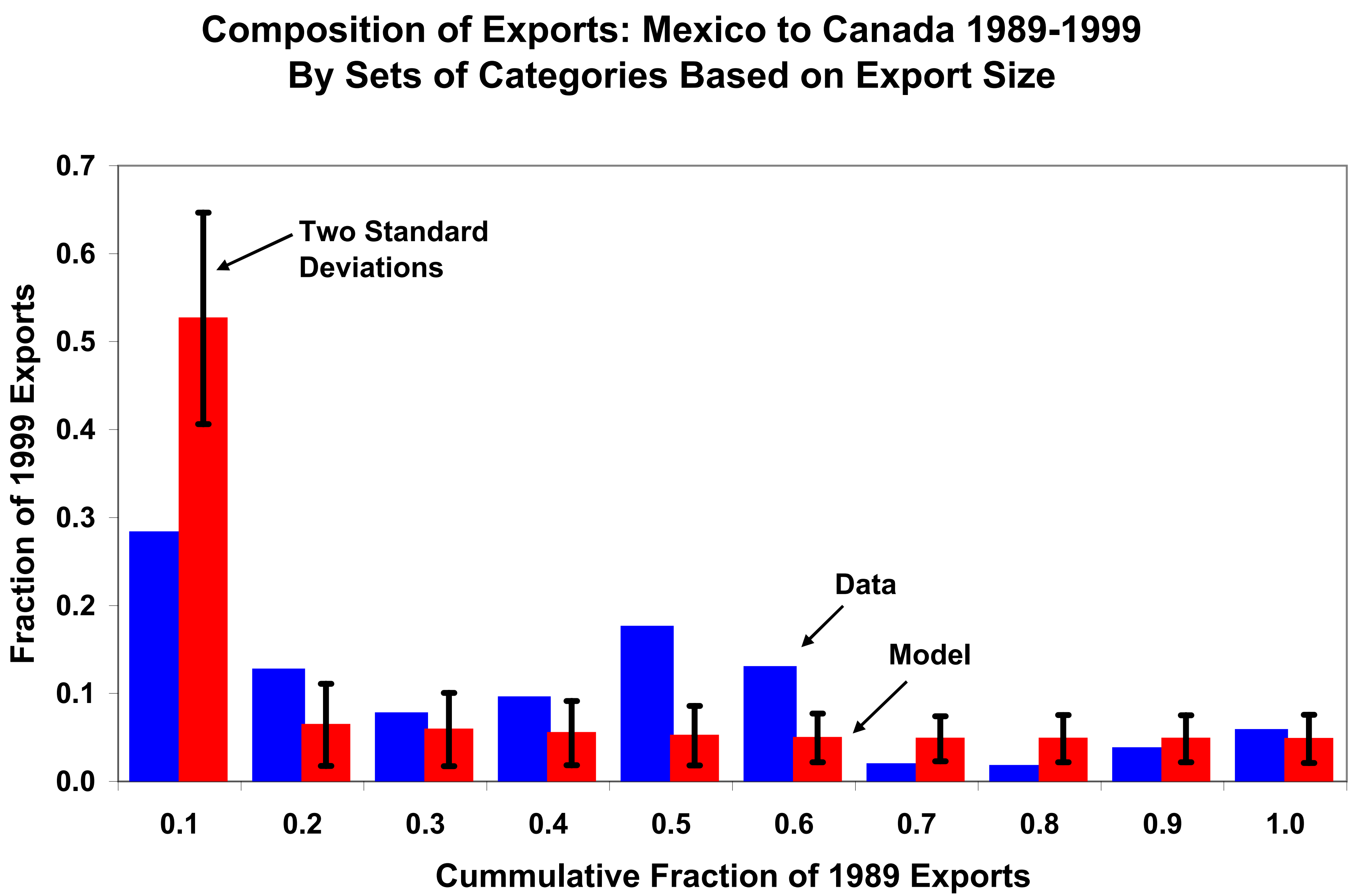
- Code size is determined by its world export value.

$$size_k = \frac{EX_{WORLD,k}^{MEX} + EX_{WORLD,k}^{US}}{\sum_k EX_{WORLD,k}^{MEX} + \sum_k EX_{WORLD,k}^{US}}$$

- J and $\bar{\alpha}$ determined by aggregate trade growth and Intra-industry trade

Composition of Exports: Mexico to U.S. 1989-1999
By Sets if Categories Based on Export Size




Composition of Exports: Mexico to Canada 1989-1999
By Sets of Categories Based on Export Size
Fraction of 1999 Exports
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
Two Standard Deviations
Data
Model
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Cummulative Fraction of 1989 Exports

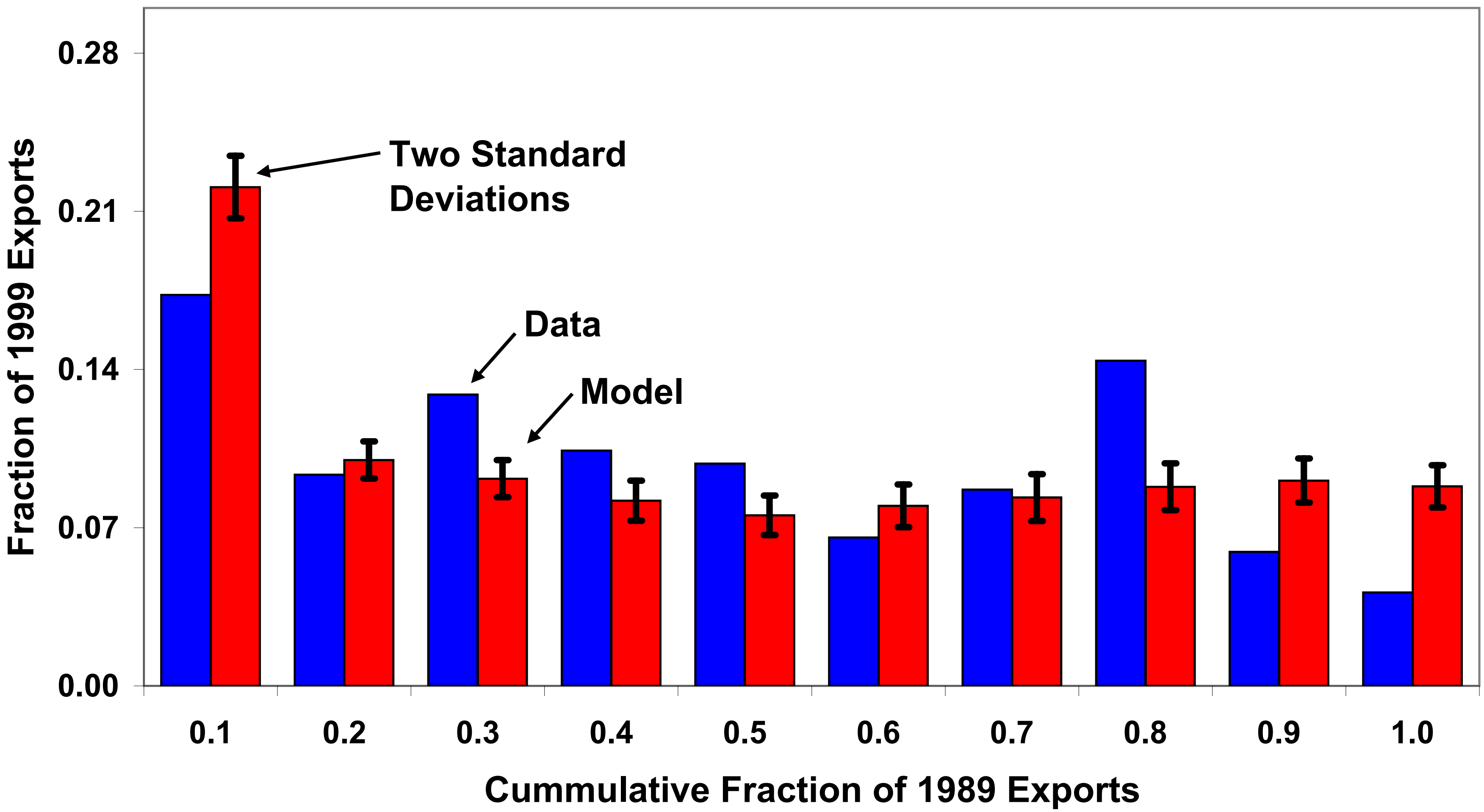
Model with Intensive and Extensive Margins

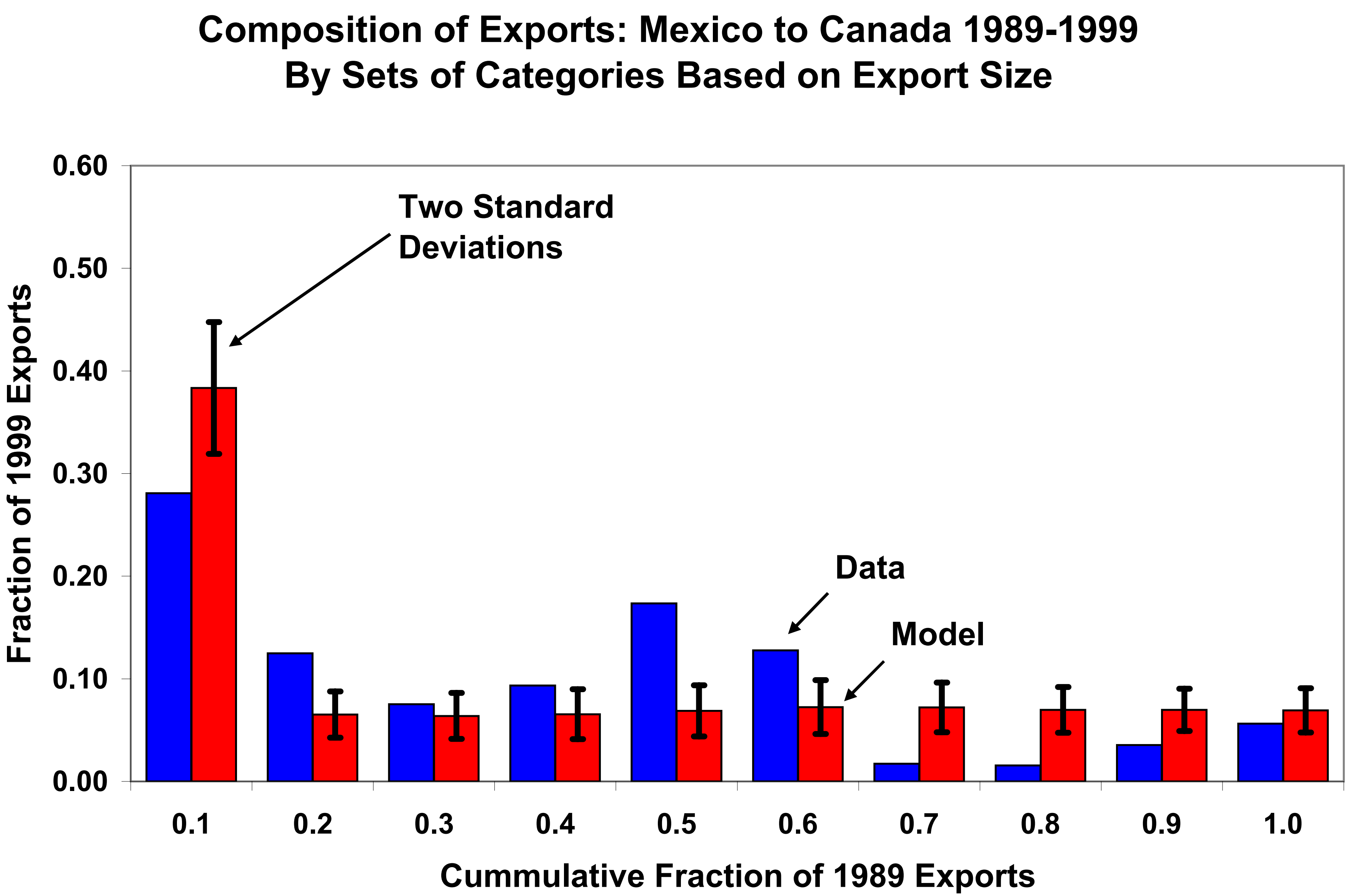
- Same Environment
- New Preferences

$$U = \left[\int_0^1 c^i(x)^\rho dx \right]^{1/\rho} \qquad \sigma = \frac{1}{1-\rho}$$

- **Expenditure on Goods**
  - Old Model $$c^i(x)p^i(x) = w^i L^i$$
  - New Model $$c^i(x)p^i(x) = w^i L^i \left(\frac{p^i(x)}{P^i} \right)^{1-\sigma}$$

Composition of Exports: Mexico to U.S. 1989-1999
By Sets of Categories Based on Export Size





Composition of Exports: Mexico to Canada 1989-1999
By Sets of Categories Based on Export Size
Two Standard Deviations
Data
Model
Fraction of 1999 Exports
0.60
0.50
0.40
0.30
0.20
0.10
0.00
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Cummulative Fraction of 1989 Exports

Conclusions

1. The extensive margin is important.

- Average increase in export share: 67%
- Correct timing

2. Simple model can produce extensive margin growth.

- Calibration uses aggregate production data.

Relative Productivity Parameters

J	Growth in Trade's Share of Production and Grubel-Lloyd Index
$\bar{\alpha}$	

- Grubel-Lloyd Index

$$GL_{MEX}^{US} = 1 - \frac{\sum_{k \in SITC} \left| EX_{MEX}^{US} - EX_{US}^{MEX} \right|}{\sum_{k \in SITC} \left[EX_{US}^{MEX} + EX_{MEX}^{US} \right]}$$

Calibration Values

	Grubel-Lloyd Index (1989)	Growth in Trade/Production (1989-1999)	Relative Output (1989)
MEX-US	.487	201%	.06
MEX-CAN	.147	299%	.66

	$\bar{\alpha}$	J	L^f / L^h
MEX-US	.223	3215	.06
MEX-CAN	.208	63	.66

Calibration Sensitivity

- Ideal SITC Measure:

$$size_k = \frac{y_k^h + y_k^f}{\sum_k y_k^h + \sum_k y_k^f}$$

- Our Proxy:

$$size_k = \frac{EX_{w,k}^h + EX_{w,k}^f}{\sum_k EX_{w,k}^h + \sum_k EX_{w,k}^f}$$

4. **Fixed costs seem better than Ricardian corner solutions for reconciling time series data on real exchange rate fluctuations with data on trade growth after liberalization experiences.**

K. J. Ruhl, “Solving the Elasticity Puzzle in International Economics,” University of Texas at Austin, 2005.

The "Armington" Elasticity

- Elasticity of substitution between domestic and foreign goods
- Crucial elasticity in international economic models
- International Real Business Cycle (IRBC) models:
    - Terms of trade volatility
    - Net exports and terms of trade co-movements
- Applied General Equilibrium (AGE) Trade models:
    - Trade response to tariff changes

The Elasticity Puzzle

- Time series (Business Cycles):
  - Estimates are low
  - Relative prices volatile
  - Quantities less volatile

- Panel studies (Trade agreement):
  - Estimates are high
  - Small change in tariffs (prices)
  - Large change in quantities

Time Series Estimates: Low Elasticity (1.5)

Study	Range
Reinert and Roland Holst (1992)	$[0.1, 3.5]$
Reinert and Shiells (1993)	$[0.1, 1.5]$
Gallaway et al. (2003)	$[0.2, 4.9]$

Trade Liberalization Estimates: High Elasticity (9.0)

Study	Range
Clausing (2001)	$[8.9, 11.0]$
Head and Reis (2001)	$[7.9, 11.4]$
Romalis (2002)	$[4.0, 13.0]$

Why do the Estimates Differ?

- Time series – no liberalization:
  - Change in trade volume from goods already traded
  - Change mostly on the *intensive margin*

- Trade liberalization:
  - Change in intensive margin *plus*
  - New types of goods being traded
  - Change on the *extensive margin*

Modeling the Extensive Margin

- Model: extensive margin from export entry costs
- Empirical evidence of entry costs
  - Roberts and Tybout (1997)
  - Bernard and Wagner (2001)
  - Bernard and Jensen (2003)
  - Bernard, Jensen and Schott (2003)

The Effects of Entry Costs

- Business cycle shocks:
  - Small extensive margin effect

- Trade liberalization:
  - Big extensive margin effect

- Asymmetry creates different empirical elasticities

Model Overview

- Two countries: $\{h, f\}$, with labor L
- Infinitely lived consumers
- No international borrowing/lending
- Continuum of traded goods plants in each country
    - Differentiated goods
    - Monopolistic competitors
    - Heterogeneous productivity
- Export entry costs
    - Differs across plants: second source of heterogeneity
- Non-traded good, competitive market: A
- Tariff on traded goods (iceberg): τ

Uncertainty

- At date t, H possible events, $\eta_t = 1, \ldots, \mathrm{H}$
- Each event is associated with a vector of productivity shocks:

$$z_t = \left[z_h\left(\eta_t\right), z_f\left(\eta_t\right) \right]$$

- First-order Markov process with transition matrix Λ

$$\lambda_{\eta\eta'} = \mathrm{pr}\left(\eta_{t+1} = \eta' \middle| \eta_t = \eta\right)$$

Traded Good Plants

- Traded good technology:

$$y(\phi,\kappa) = z\phi l$$

- Plant heterogeneity (ϕ,κ)
    - constant, idiosyncratic productivity: ϕ
    - export entry cost: κ
    - plant of type (ϕ,κ)

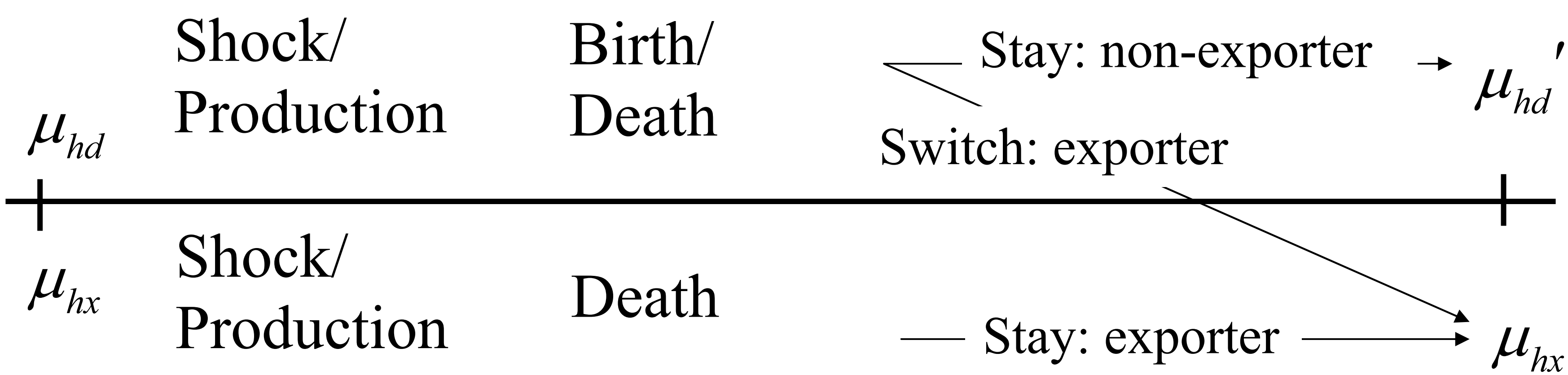
- ν plants born each period with distribution $F(\phi,\kappa)$
- Fraction δ of plants exogenously die each period

Timing

$\mu_{hx}(\phi,\kappa)$: plants of type (ϕ,κ) who paid entry cost

$\mu_{hd}(\phi,\kappa)$: plants of type (ϕ,κ) who have not paid entry cost

$\mu = (\mu_{hd}, \mu_{hx}, \mu_{fd}, \mu_{fx})$



Consumers

$$\max_{q,c_h^h(\imath),c_f^h(\imath)} \gamma \log\left(C\right)+\left(1-\gamma\right)\log\left(A\right)$$

s.t.

$$C=\left[\int_{\imath\in \mathrm{I}_h^h(\mu)} c_h^h\left(\imath\right)^{\rho} d\imath+\int_{\imath\in \mathrm{I}_f^h(\mu)} c_f^h\left(\imath\right)^{\rho} d\imath\right]^{\frac{1}{\rho}}$$

$$\int_{\imath\in \mathrm{I}_h^h(\mu)} p_h^h\left(\imath\right)c_h^h\left(\imath\right)d\imath + \int_{\imath\in \mathrm{I}_f^h(\mu)} \left(1+\tau\right)p_f^h\left(\imath\right)c_f^h\left(\imath\right)d\imath+p_{hA}A=L+\Pi_h$$

Non-traded Good

$$\max \; p_{hA}(\eta, \mu) A - l$$

$$\text{s.t. } \; A = z_h(\eta) l$$

Normalize $w_h = 1$, implying $p_{hA}(\eta, \mu) = z_h(\eta)$

Traded Goods: Static Profit Maximization

$$\pi_d\left(p_h^h, l; \phi, \kappa, \eta, \mu\right) = \max_{p_h^h,\, l} \; p_h^h z(\eta)\phi l - l$$

$$\text{s.t.} \qquad z(\eta)\phi l = \tilde{c}_h^h\left(p_h^h; \eta, \mu\right)$$

$$\pi_x\left(p_h^f, l; \phi, \kappa, \eta, \mu\right) = \max_{p_h^f,\, l} \; p_h^f z(\eta)\phi l - l$$

$$\text{s.t.} \qquad z(\eta)\phi l = \tilde{c}_h^f\left(p_h^f; \eta, \mu\right)$$

Pricing rules:

$$p_h^h(\phi, \kappa, \eta, \mu) = p_h^f(\phi, \kappa, \eta, \mu) = \frac{1}{\rho\phi z(\eta)}$$

Dynamic Choice: Export or Sell Domestically

- Exporter's Value Function:

$$V_x(\phi,\kappa,\eta,\mu) = d(\eta,\mu)\big(\pi_d(\phi,\kappa,\eta,\mu) + \pi_x(\phi,\kappa,\eta,\mu)\big)$$

$$+(1-\delta)\beta\sum_{\eta'} V_x(\phi,\kappa,\eta',\mu')\lambda_{\eta\eta'}$$

s.t. $\mu' = \mathrm{M}(\eta,\mu)$

- $d(\eta,\mu)$ = multiplier on budget constraint

- Non-exporter's Value Function:

$$V_d(\phi,\kappa,\eta,\mu) = \max\Big\{\pi_d(\phi,\kappa,\eta,\mu)d(\eta,\mu) + \beta(1-\delta)\sum_{\eta'} V_d(\phi,\kappa,\eta',\mu')\lambda_{\eta\eta'},$$

$$\big[\pi_d(\phi,\kappa,\eta,\mu) - \kappa\big]d(\eta,\mu) + \beta(1-\delta)\sum_{\eta'} V_x(\phi,\kappa,\eta',\mu')\lambda_{\eta\eta'}\Big\}$$

s.t. $\mu' = \mathrm{M}(\eta,\mu)$

Equilibrium

- Cutoff level of productivity for each value of the entry cost
- For a plant of type (ϕ, κ)

  If $\phi \geq \hat{\phi}_{\kappa}(\eta, \mu)$ export and sell domestically

  If $\phi < \hat{\phi}_{\kappa}(\eta, \mu)$ only sell domestically

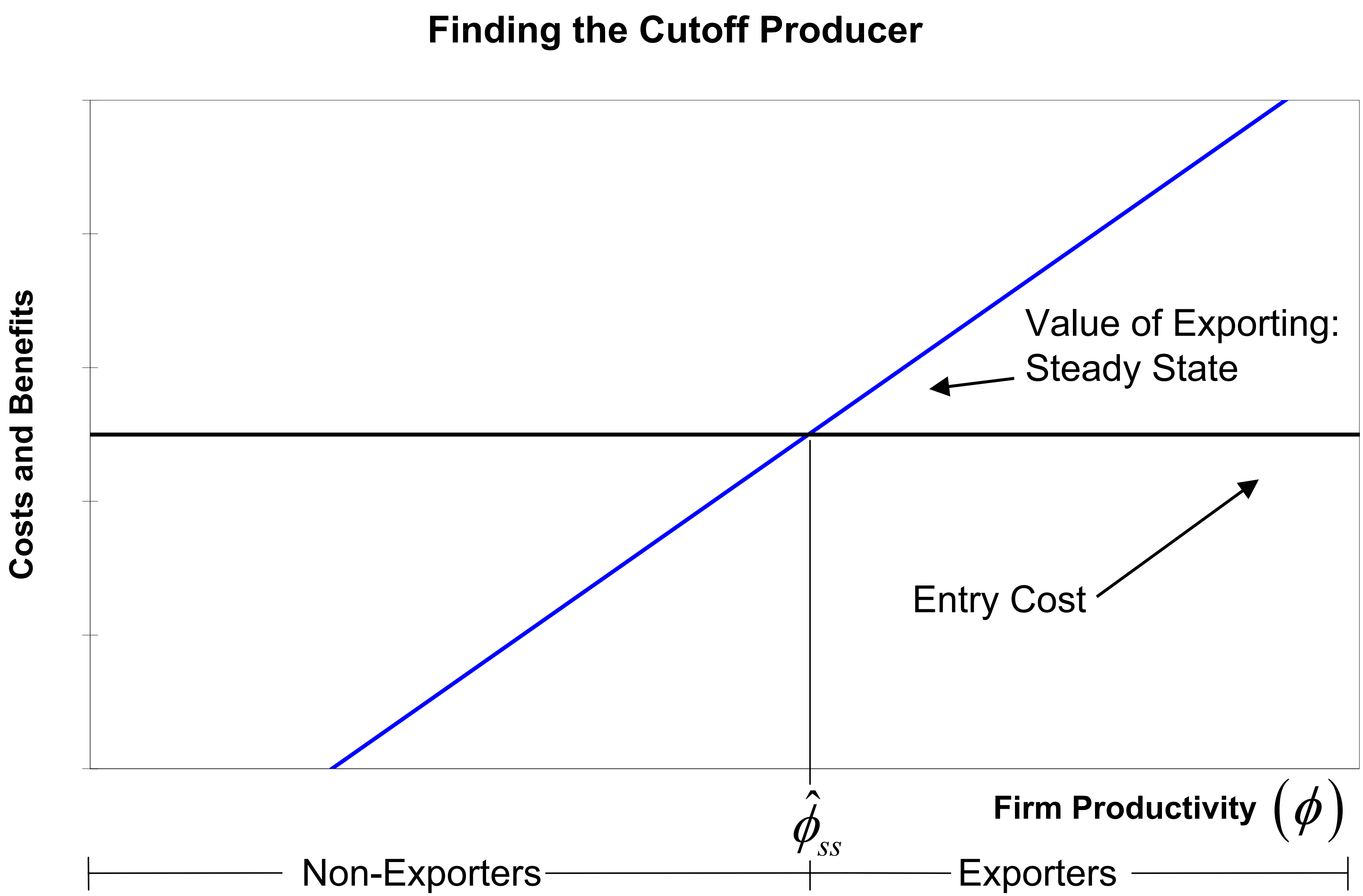
- In Equilibrium
  - "Low" productivity/"high" entry cost plants sell domestic
  - "High" productivity/"low" entry cost plants also export
  - Similar to Melitz (2003)

Determining Cutoffs

- For the cutoff plant:
    - entry cost = discounted, expected value of exporting

- $\hat{\phi}_\kappa(\eta,\mu)$ is the level of productivity, ϕ, that solves:

$$\underbrace{d\;(\eta,\mu)\kappa}_{\text{entry cost}} = \underbrace{(1-\delta)\beta\left[\sum_{\eta'} V_x(\phi,\kappa,\eta',\mu')\lambda_{\eta\eta'} - \sum_{\eta'} V_d(\phi,\kappa,\eta',\mu')\lambda_{\eta\eta'}\right]}_{\text{expected value of exporting}}$$


Finding the Cutoff Producer
Costs and Benefits
Value of Exporting:
Steady State
Entry Cost
Firm Productivity (ϕ)
$\hat{\phi}_{ss}$
Non-Exporters
Exporters

Choosing Parameters

- Set $\sigma = \dfrac{1}{1-\rho} = 2$ and $\tau = 0.15$
- Calibrate to the United States (1987) and a symmetric partner.

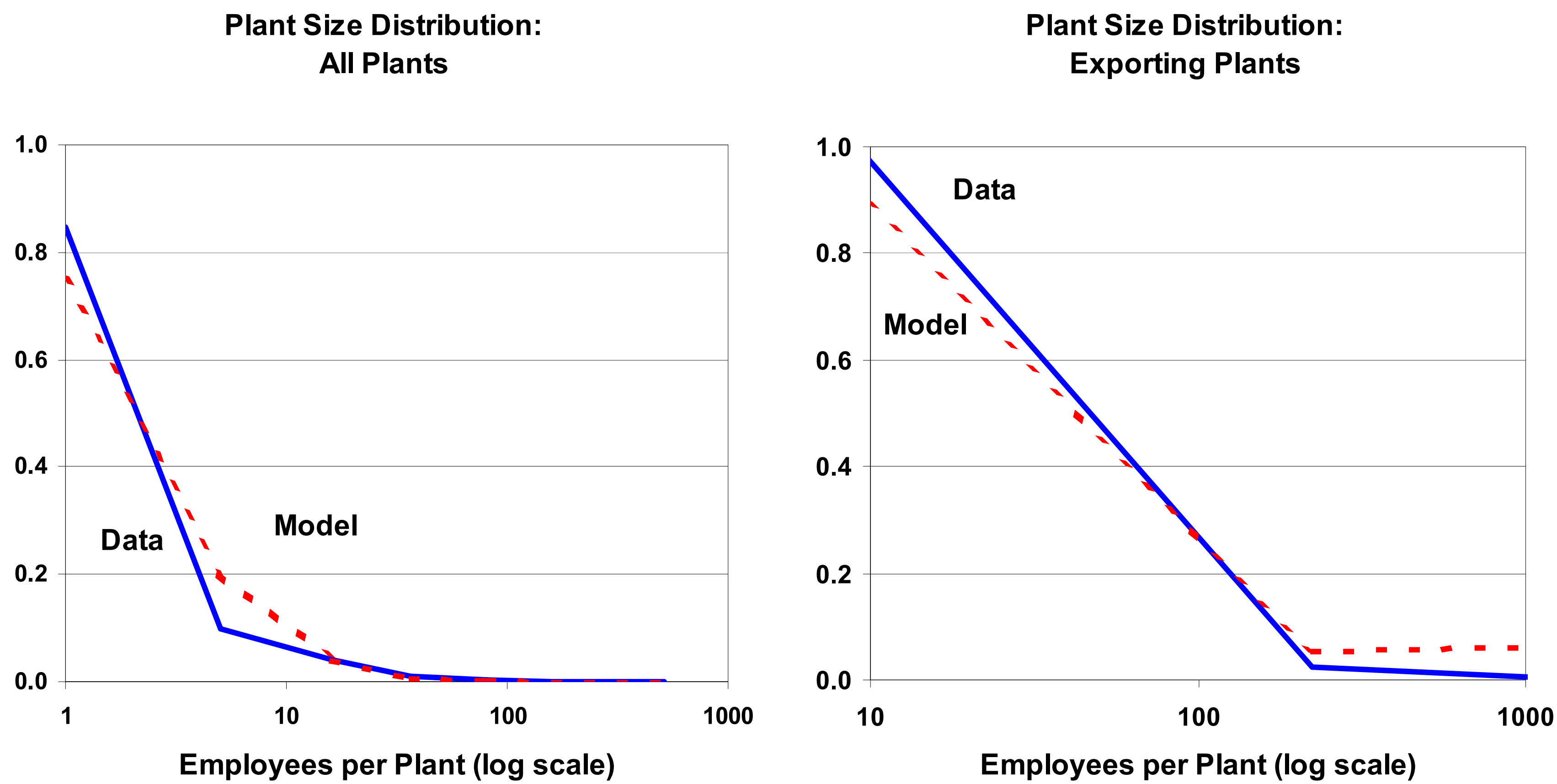
	Parameters
β	Annual real interest rate (4%)
γ	Share of manufactures in GDP (18%)
δ	Annual loss of jobs from plant deaths as percentage of employment (Davis et. al., 1996) (6%)

Other Parameters

- Distribution over new plants:

$$F_{\kappa}(\phi) = \frac{1}{\phi^{\theta_{\phi}}} \qquad\qquad F_{\phi}(\kappa) = \frac{1}{(\bar{\kappa} - \kappa)^{\theta_{\kappa}}}$$

- $\bar{\kappa}$, $\bar{\phi}$, ν, $\theta_{\phi}, \theta_{\kappa}$ jointly determine:
    - Average plant size (12 employees)
    - Standard deviation of plant sizes (892)
    - Average exporting plant size (15 employees)
    - Standard deviation of exporting plant sizes (912)
    - Fraction of production that is exported (9%)


Plant Size Distribution:
All Plants
1.0
0.8
0.6
0.4
0.2
0.0
1
10
100
1000
Data
Model
Employees per Plant (log scale)
Plant Size Distribution:
Exporting Plants
1.0
0.8
0.6
0.4
0.2
0.0
10
100
1000
Data
Model
Employees per Plant (log scale)

Productivity Process

- Two shocks, low and high:

$$z_i = 1 - \varepsilon$$
$$z_i = 1 + \varepsilon$$

- Countries have symmetric processes with Markov Matrix

$$\Lambda_i = \begin{bmatrix} \bar{\lambda} & 1-\bar{\lambda} \\ 1-\bar{\lambda} & \bar{\lambda} \end{bmatrix}$$

- ε: standard deviation of the U.S. Solow Residuals (1.0%)
- $\bar{\lambda}$: autocorrelation of the U.S. Solow Residuals (0.90)

How does Trade Liberalization Differ from Business Cycles?

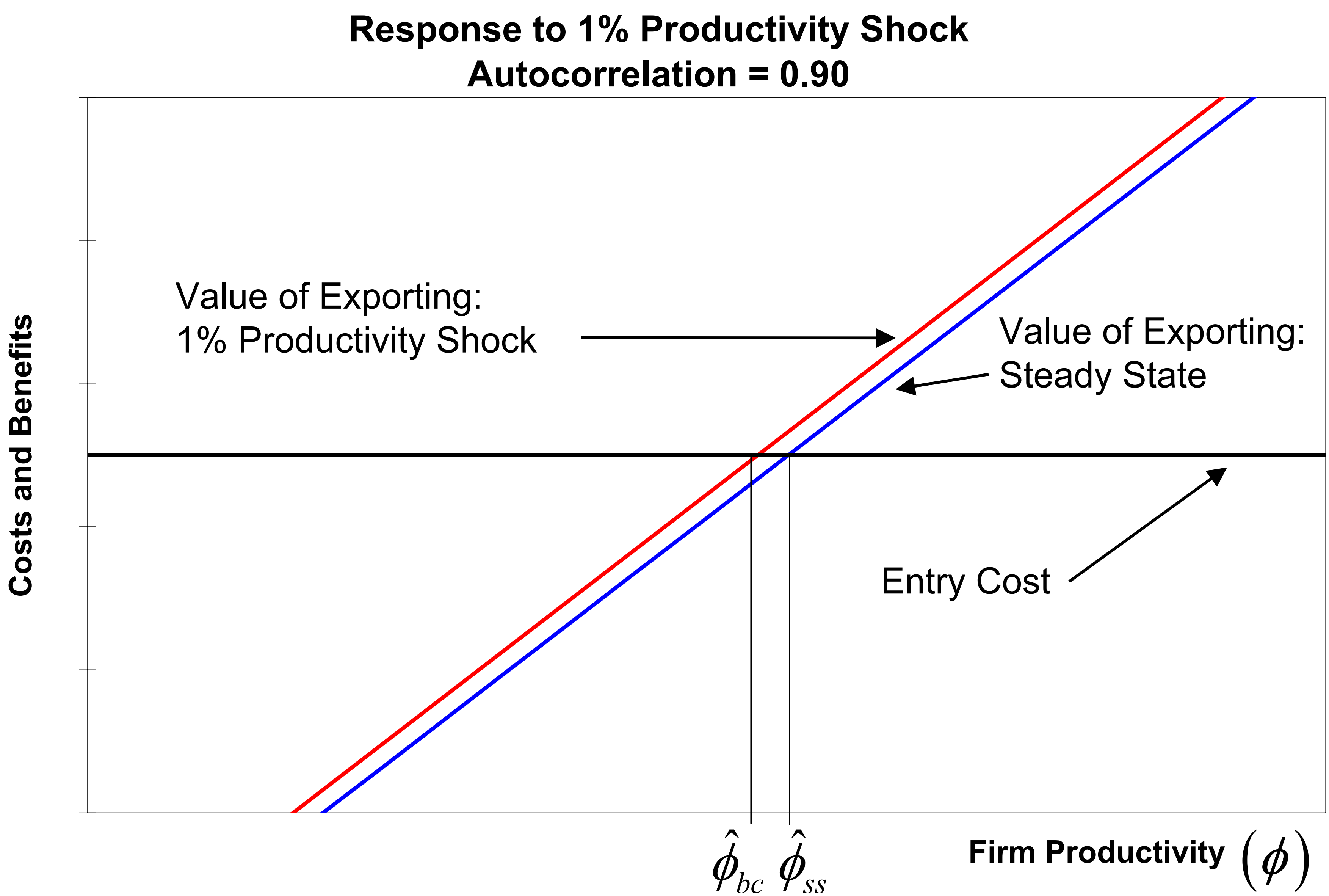
- Trade liberalization
  - Permanent changes
  - Large magnitudes

- Business cycles
  - Persistent, but not permanent changes
  - Small magnitudes

Developing Intuition: Persistent vs. Permanent Shocks

- 1% positive productivity shock in foreign country
  - Shock is persistent – autocorrelation of 0.90

- 1% decrease in tariffs
  - Change in tariffs is permanent

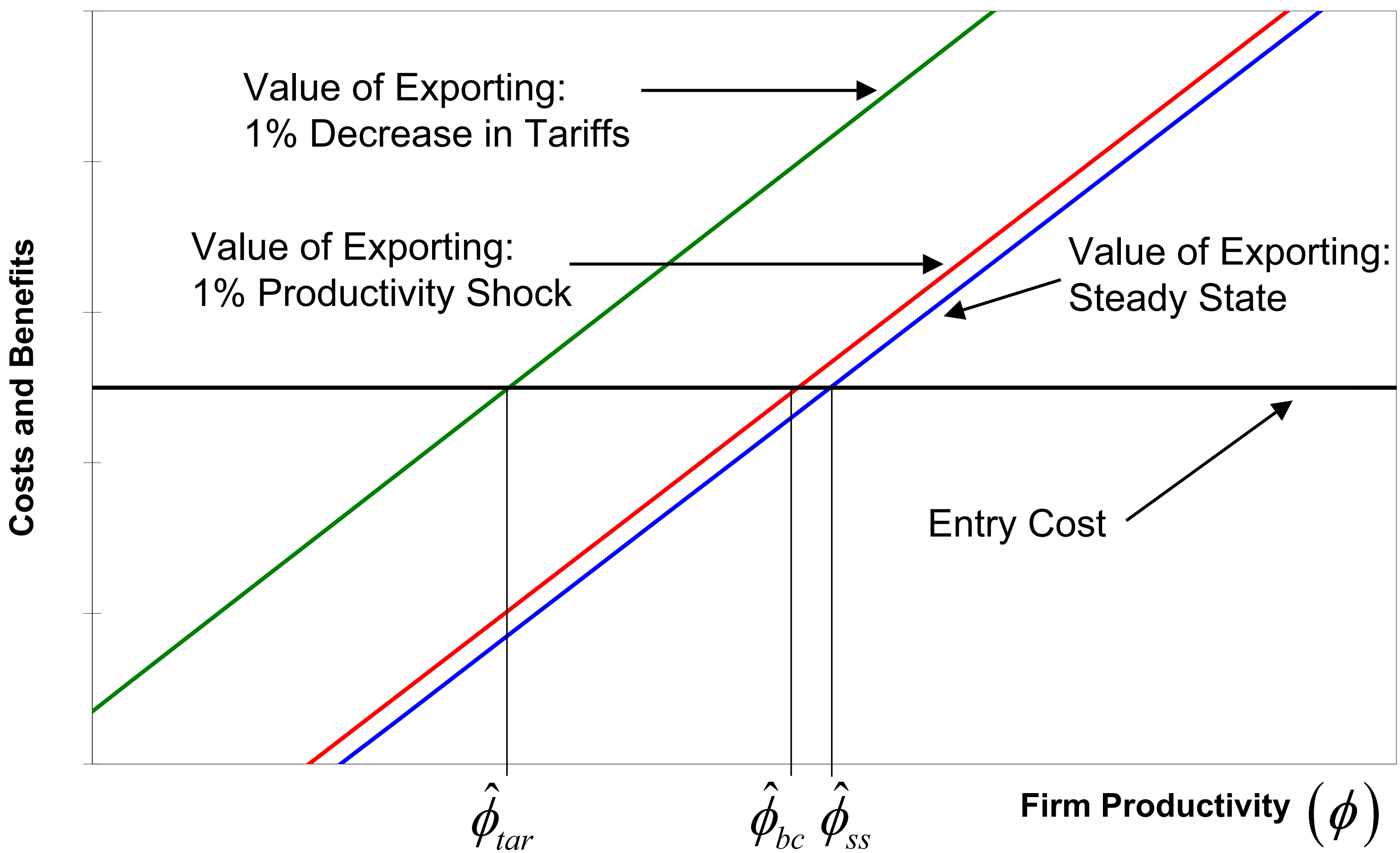

Response to 1% Productivity Shock
Autocorrelation = 0.90
Costs and Benefits
Value of Exporting:
1% Productivity Shock
Value of Exporting:
Steady State
Entry Cost
$\hat{\phi}_{bc}$
$\hat{\phi}_{ss}$
Firm Productivity (ϕ)

Response to a 1% Foreign Productivity Shock

Increase in imports on intensive margin	=	1.89%
Increase in imports on extensive margin	=	0.16%
Total increase in imports	=	2.05%
Change in consumption of home goods	=	-0.10%

$$\frac{\text{\% Change Imports/Dom. Cons.}}{\text{\% Change Price}} = \frac{2.17}{0.99} = 2.19$$

Response to 1% Permanent Decrease in Tariffs



Response to a 1% Tariff Reduction

Increase in imports on intensive margin	=	1.42%
Increase in imports on extensive margin	=	3.04%
Total increase in imports	=	4.46%

Change in consumption of home goods	=	-0.33%

$$\frac{\text{\% Change Imports/Dom. Cons.}}{\text{\% Change Tariff}} = \frac{4.81}{1.00} = 4.81$$

Quantitative Results

- Two experiments

- Trade liberalization
  - Eliminate 15% tariff
  - Compute elasticity across tariff regimes

- Time series regressions
  - Use model to generate simulated data
  - Estimate elasticity as in the literature

Trade Liberalization Elasticity

Variable	Entry Costs (% change)	No Entry Costs (% change)
Exports	87.1	30.5
Imports/Dom. Cons.	93.0	32.2
Exporting Plants	37.7	0.0
Implied Elasticity	6.2	2.1

Elasticity in the Time Series

- Simulate: produce price/quantity time series
- Regress:

$$\log\left(C_{f,t} / C_{h,t}\right) = \alpha + \sigma \log\left(p_{h,t} / p_{f,t}\right) + \varepsilon_t$$

Parameter	Estimate
α (standard error)	-0.015 (6.36e-04)
σ (standard error)	1.39 (0.06)
R- squared	0.30

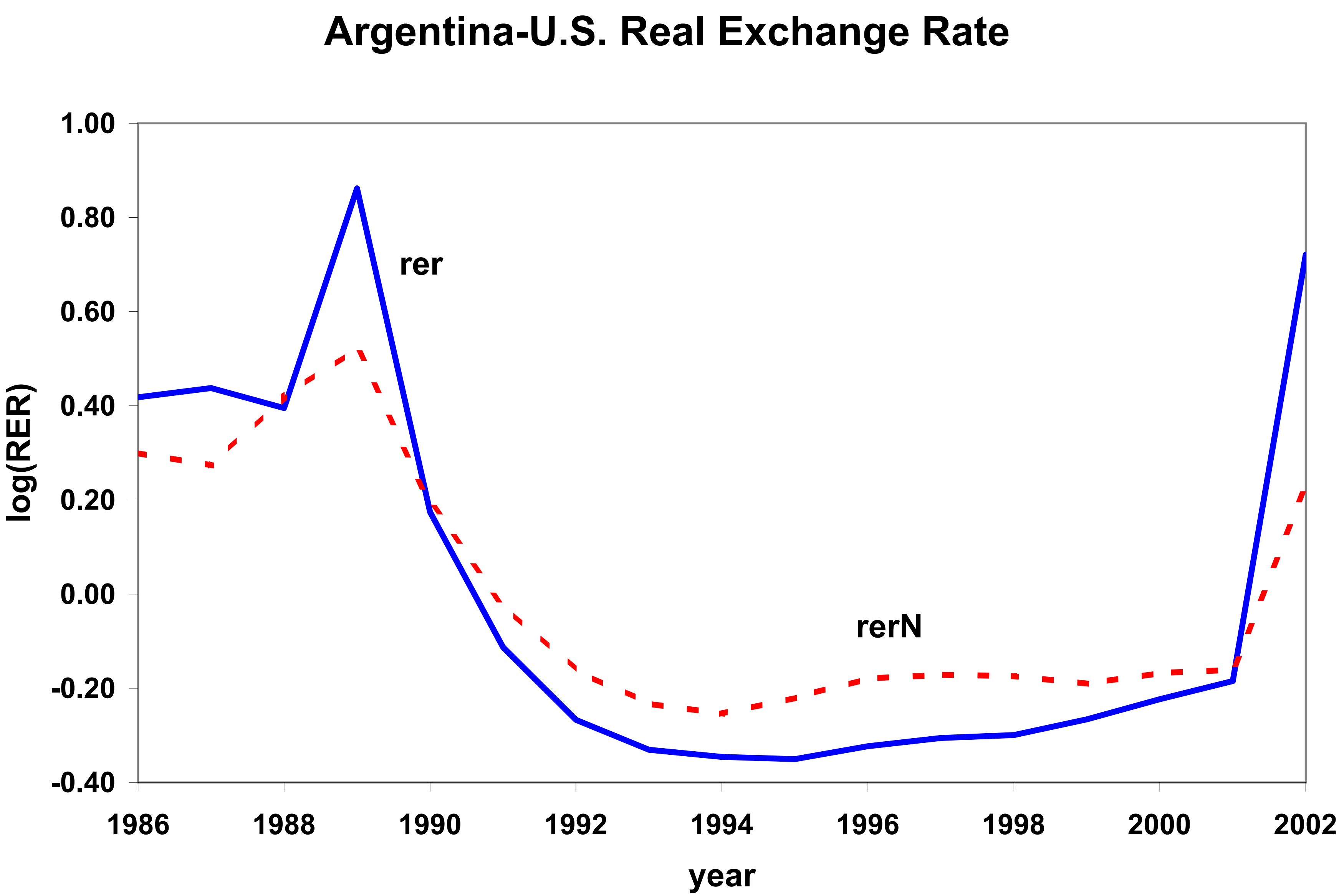
Conclusion

- Gap between dynamic macro models and trade models
  - Partially closes the gap
  - Modeling firm behavior as motivated by the data
  - Step towards better modeling of trade policy
- Single model can account for the elasticity puzzle
  - Time series elasticity of 1.4
  - Trade liberalization elasticity of 6.2

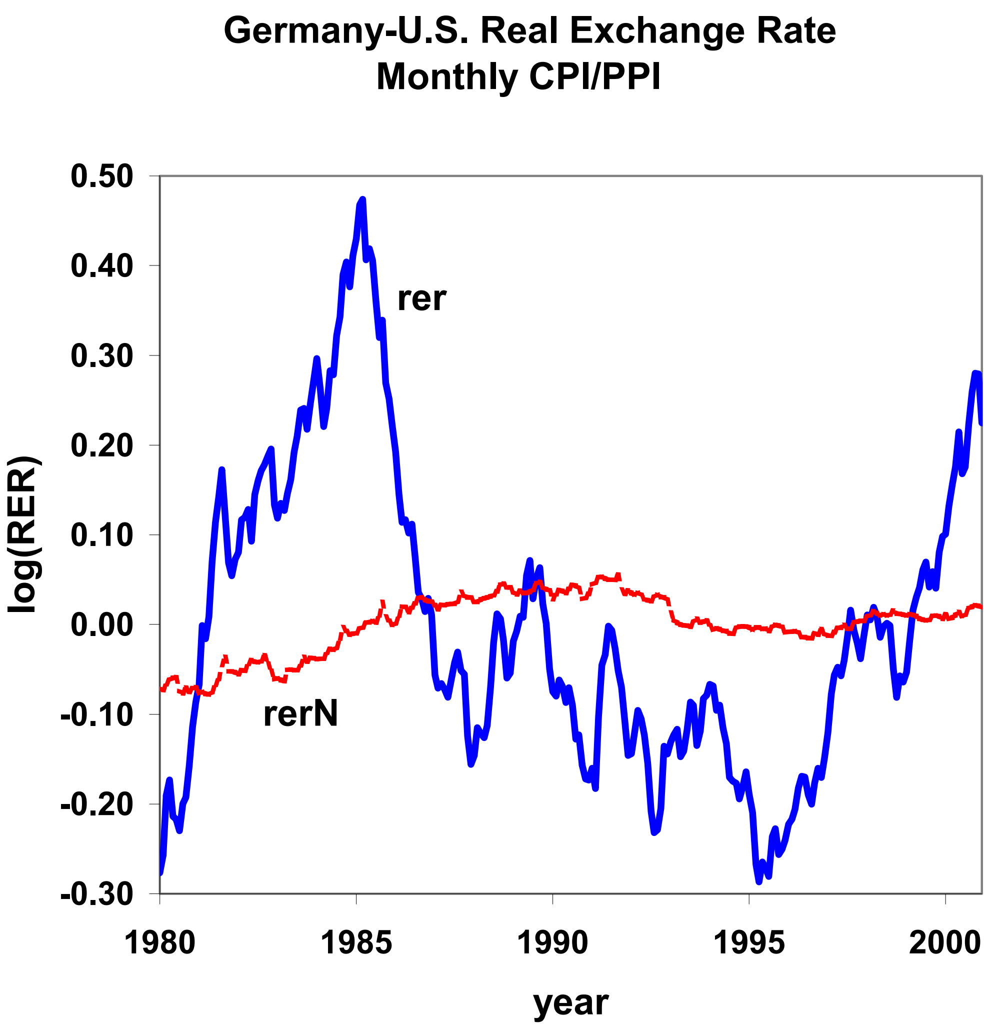
5. Modeling the fixed costs may explain why real exchange rate data indicate that more arbitrage across countries that have a strong bilateral trade relationship.

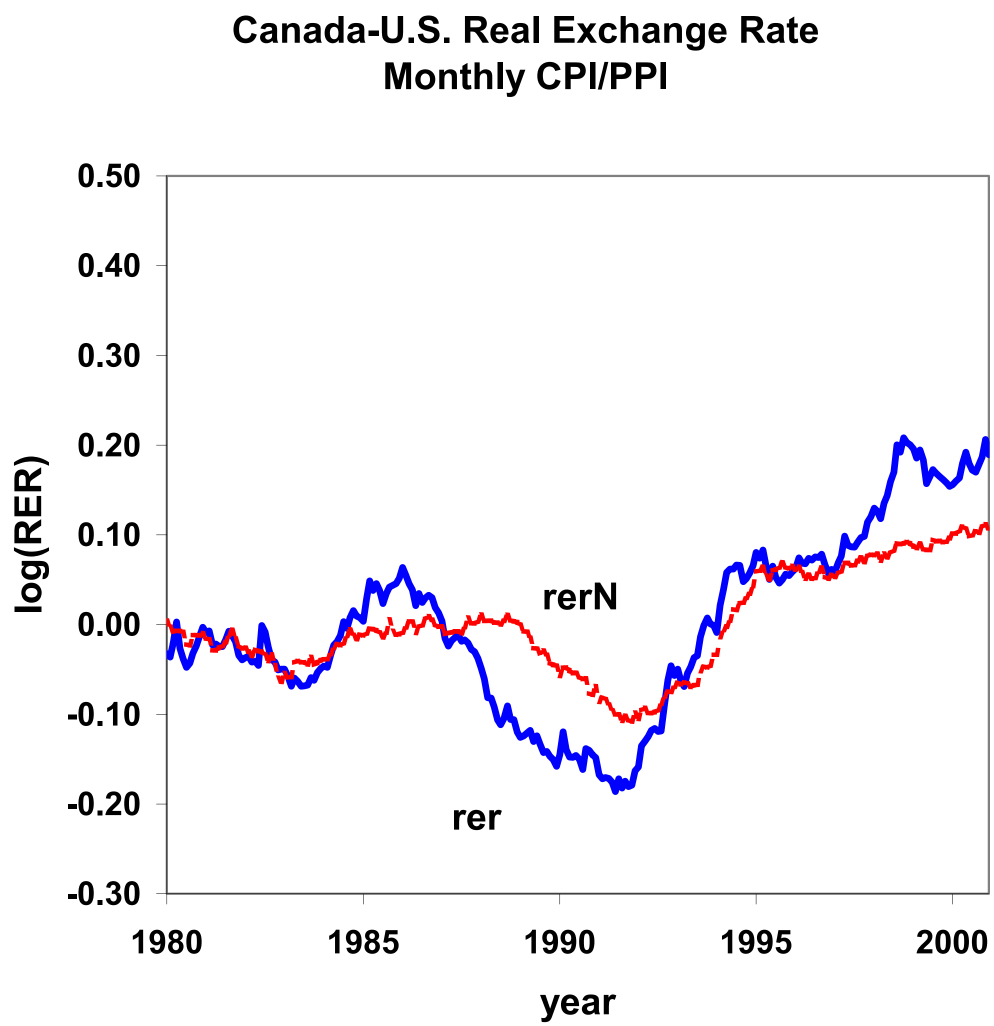
C. M. Betts and T. J. Kehoe, “Real Exchange Rate Movements and the Relative Price of Nontraded Goods,” University of Minnesota and University of Southern California, 2003.

C. M. Betts and T. J. Kehoe, “U.S. Real Exchange Rate Fluctuations and Relative Price Fluctuations,” University of Minnesota and University of Southern California, 2003.


Argentina-U.S. Real Exchange Rate
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
-0.40
log(RER)
rer
rerN
1986
1988
1990
1992
1994
1996
1998
2000
2002
year

U.S. BILATERAL REAL EXCHANGE RATES
AND RELATIVE PRICES OF NONTRADED GOODS





We investigate the empirical relation between

the U.S. bilateral real exchange rate with 5 of her largest trade partners

and

the bilateral relative price of nontraded to traded goods.

We measure the relation by

simple correlation (comovements)

relative standard deviation (volatility)

variance decompositions (percent of fluctuations)

We find the relation depends crucially on

1. the price series used to measure the relative price of nontraded goods
2. the choice of trade partner.

Specifically, the relation is stronger when

1. traded goods prices are measured using production site prices — not final consumption price data
2. the trade intensity between the U.S. and a trade partner is larger — links international relative price behavior and size of trade flows

Traditional theory real exchange rate theory (Cassel, Pigou, and many others)

dichotomy of goods into

- costlessly traded (arbitraged prices)

- entirely nontraded (domestic prices)

INAPPROPRIATE

PLAN OF DISCUSSION

- Methodology
- Data
- Results
- What We Learn
- Extended Results (paper 2)
- Theoretical Model (paper 3)
- Conclusions

METHODOLOGY

Traditional real exchange rate theory attributes all (or most) real exchange rate movements to changes in the relative price of nontraded goods.

Recent empirical analyses using variance decompositions argue there is almost no role for relative price of nontraded goods in real exchange rate movements. All movements are driven by deviations from law of one price.

REAL EXCHANGE RATE MEASUREMENT

Bilateral real exchange rate between the United States and country i:

$$RER_{i,us} = NER_{i,us} \frac{P_{us}}{P_i}$$

$NER_{i,us}$: nominal exchange rate — country i currency units per U.S. dollar

P_j: price deflator or index for the basket of goods consumed or produced in country j, $j = us,\ i$.

Traditional theory dichotomizes goods into

- costlessly traded (arbitraged prices)
- entirely nontraded (domestic prices)

$$P_j(P_j^T, P_j^N)$$

Decompose

$$RER_{i,us} = \left(NER_{i,us} \frac{P_{us}^T}{P_i^T} \right) \left(\frac{P_i^T}{P_i} \Big/ \frac{P_{us}^T}{P_{us}} \right)$$

$$RER_{i,us} = RER_{i,us}^{T} \times RER_{i,us}^{N}$$

where

$$RER_{i,us}^{T} = NER_{i,us} \frac{P_{us}^{T}}{P_{i}^{T}}$$

is the real exchange rate of traded goods — the component that measures deviations from the law of one price and

$$RER_{i,us}^{N} = \left(\frac{P_{i}^{T}}{P_{i}(P_{i}^{T}, P_{i}^{N})} \right) \Big/ \left(\frac{P_{us}^{T}}{P_{us}(P_{us}^{T}, P_{us}^{N})} \right)$$

is what we refer to as the (bilateral) relative price of nontraded (to traded) goods.

In logarithms,

$$rer_{i,us} = rer_{i,us}^{T} + rer_{i,us}^{N}.$$

In the case where

$$P_j\left(P_j^T, P_j^N\right) = \left(P_j^T\right)^{\gamma_j}\left(P_j^N\right)^{1-\gamma_j},$$

$$\begin{aligned} RER_{i,us}^N &= \left(\frac{P_i^T}{P_i(P_i^T, P_i^N)}\right) \Bigg/ \left(\frac{P_{us}^T}{P_{us}(P_{us}^T, P_{us}^N)}\right) \\ &= \left(\frac{P_i^T}{\left(P_i^T\right)^{\gamma_i}\left(P_i^N\right)^{1-\gamma_i}}\right) \Bigg/ \left(\frac{P_{us}^T}{\left(P_{us}^T\right)^{\gamma_{us}}\left(P_{us}^N\right)^{1-\gamma_{us}}}\right) \\ &= \left(\frac{P_i^T}{P_i^N}\right)^{1-\gamma_i} \Bigg/ \left(\frac{P_{us}^T}{P_{us}^N}\right)^{1-\gamma_{us}} \end{aligned}$$

SUMMARY STATISTICS

Analyze the relation between $rer_{i,us}$ and $rer_{i,us}^N$:

1. $$corr(rer_{i,us}, rer_{i,us}^N) = \frac{cov(rer_{i,us}, rer_{i,us}^N)}{\left(var(rer_{i,us})var(rer_{i,us}^N)\right)^{1/2}}.$$

2. $$\frac{std(rer_{i,us}^N)}{std(rer_{i,us})} = \left(\frac{var(rer_{i,us}^N)}{var(rer_{i,us})}\right)^{1/2}$$

3. $$vardec(rer_{i,us}, rer_{i,us}^N) = \frac{var(rer_{i,us}^N)}{var(rer_{i,us}^N) + var(rer_{i,us}^T)}$$

(Another possibility:

$$vardec^2(rer_{i,us}, rer_{i,us}^N) = \frac{var(rer_{i,us}^N) + cov(rer_{i,us}^N, rer_{i,us}^T)}{var(rer_{i,us})}.)$$

DATA

Data on 5 of largest trade partners of the United States:

- Canada (1)
- Mexico (2)
- Japan (3)
- Germany (6)
- Korea (7).

These countries account for 53 percent of U.S. trade in 2000.

Construct measures of $rer_{i,us}$:

Need aggregate price indices (and nominal exchange rates)

1. Deflator for Gross Output at production site for all goods and services produced by a country (GO)

2. Consumer Price Index for entire basket of consumption goods and services (CPI)

3. Personal Consumption Deflators for all personal consumption expenditures (PCD).

(Another possibility: Deflator for Gross Domestic Product for all goods and services produced by a country (GDP).)

Construct measures of $rer_{i,us}^{N}$:

Need traded goods price measures

1. Deflator for GO of agriculture, mining, and manufacturing
2. Producer price index for entire basket of producer goods (PPI).
3. CPI for "traded" components of consumption basket - all goods less food.
4. PCD for "traded" components of personal consumption expenditures - commodities.

(Another possibility: Deflator for GDP of agriculture, mining, and manufacturing.)

WHAT WE LEARN

- The frequency of the data does not matter.

- Detrending matters (theory should guide for the choice and the explanation for why it matters).

- The size of bilateral trade relationship is crucial.

  The larger is the trade relationship, the more closely related are $rer_{i,us}$ and $rer_{i,us}^{N}$.

- The data series used to measure the relative price of nontraded goods $rer_{i,us}^{N}$ matters a lot.

good conceptually
sectoral GO deflators
PPIs

not good conceptually
CPI components
PCD components
(sectoral GDP deflators)

highly correlated
CPI components-PCD components
sectoral GOP deflators-PPIs (-sectoral GDP deflators)

widely available
PPIs

less widely available
CPI components
PCD components
(sectoral GDP deflators)

difficult to obtain
sectoral GO deflators

SUGGESTION

MODIFY THE TRADITIONAL THEORY SO THAT GOODS DIFFER BY DEGREE OF TRADABILITY

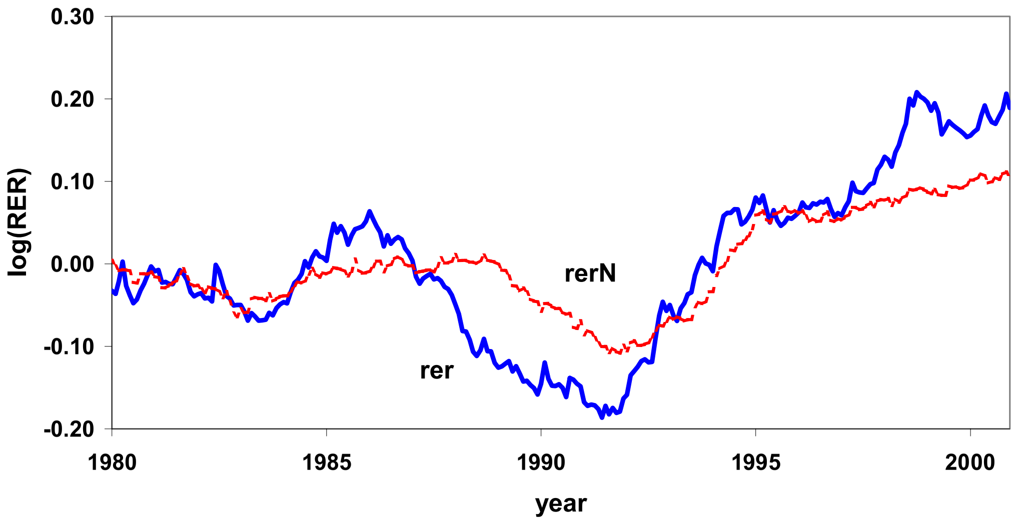
TABLE 1

COMPARISON OF FREQUENCIES: CANADA-U.S. REAL EXCHANGE RATE PPI-CPI data 1980-2000

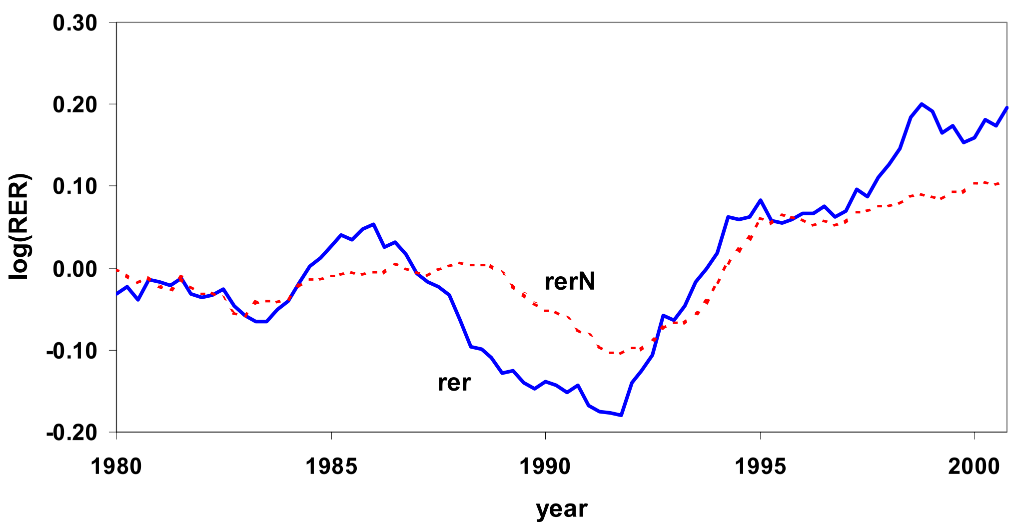
	annual	annual	quarterly	quarterly	quarterly	monthly	monthly	monthly	monthly
Levels									
corr(rer,rer^N)	0.88		0.88			0.88			
std(rer^N)/std(rer)	0.70		0.69			0.69			
vardec(rer,rer^N)	0.66		0.65			0.65			
Detrended levels									
corr(rer,rer^N)	0.88		0.88			0.87			
std(rer^N)/std(rer)	0.51		0.51			0.51			
vardec(rer,rer^N)	0.41		0.41			0.41			
Changes	1 lag (1 year)	4 lags (4 years)	1 lag (1 quarter)	4 lags (1 year)	16 lags (4 years)	1 lag (1 month)	3 lags (1 quarter)	12 lags (1 year)	48 lags (4 years)
corr(rer,rer^N)	0.70	0.82	0.56	0.70	0.82	0.48	0.48	0.67	0.82
std(rer^N)/std(rer)	0.55	0.55	0.51	0.55	0.55	0.55	0.51	0.55	0.55
vardec(rer,rer^N)	0.40	0.51	0.28	0.39	0.51	0.29	0.26	0.37	0.50

CANADA-U.S. REAL EXCHANGE RATE

Monthly CPI/PPI



Quarterly CPI/PPI



Annual CPI/PPI

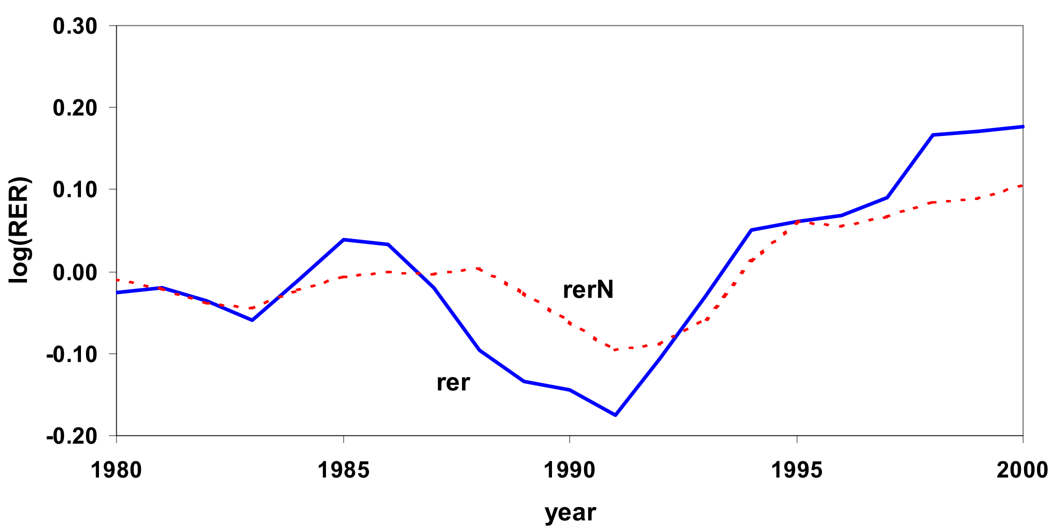


TABLE 2A

CANADA-U.S. REAL EXCHANGE RATE
Annual Data

	GO Deflators 1980-1998	PPI-CPI 1980-2000	Components of CPI 1980-2000	Components of PCD 1980-2000
Levels				
corr(rer,rer^N)	0.81	0.88	0.46	0.42
std(rer^N)/std(rer)	0.51	0.70	0.63	0.57
vardec(rer,rer^N)	0.38	0.66	0.33	0.27
Detrended levels				
corr(rer,rer^N)	0.78	0.88	-0.43	-0.32
std(rer^N)/std(rer)	0.45	0.51	0.17	0.14
vardec(rer,rer^N)	0.29	0.41	0.02	0.02
1 year changes				
corr(rer,rer^N)	0.54	0.70	-0.07	-0.11
std(rer^N)/std(rer)	0.40	0.55	0.20	0.13
vardec(rer,rer^N)	0.20	0.40	0.09	0.06
4 year changes				
corr(rer,rer^N)	0.74	0.82	-0.19	-0.08
std(rer^N)/std(rer)	0.47	0.55	0.16	0.13
vardec(rer,rer^N)	0.33	0.51	0.12	0.09

FIGURE 3A

CANADA-U.S. REAL EXCHANGE RATE

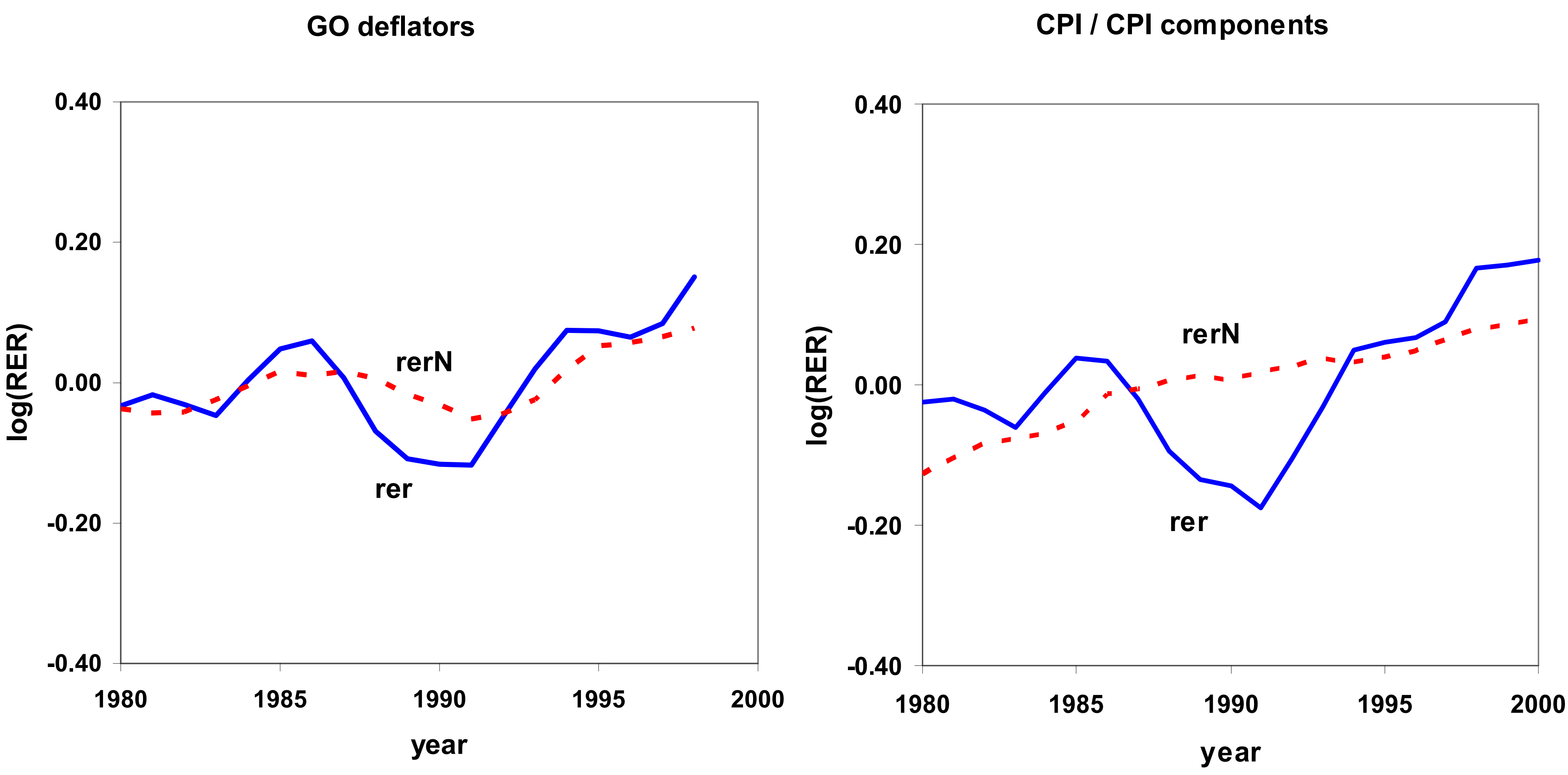


TABLE 2B

GERMANY-U.S. REAL EXCHANGE RATE
Annual Data

	GO Deflators 1980-2000	PPI-CPI 1980-2000	Components of CPI 1980-2000	Components of PCD 1980-2000
Levels				
corr(rer,rer^N)	-0.55	-0.33	-0.05	-0.24
std(rer^N)/std(rer)	0.25	0.73	0.15	0.25
vardec(rer,rer^N)	0.04	0.21	0.02	0.05
Detrended levels				
corr(rer,rer^N)	0.18	-0.15	0.24	0.37
std(rer^N)/std(rer)	0.20	0.13	0.12	0.10
vardec(rer,rer^N)	0.04	0.02	0.01	0.01
1 year changes				
corr(rer,rer^N)	0.16	-0.24	0.18	-0.02
std(rer^N)/std(rer)	0.13	0.14	0.10	0.07
vardec(rer,rer^N)	0.03	0.04	0.01	0.01
4 year changes				
corr(rer,rer^N)	0.24	0.02	0.31	0.49
std(rer^N)/std(rer)	0.21	0.12	0.10	0.09
vardec(rer,rer^N)	0.07	0.10	0.01	0.02

FIGURE 3B

GERMANY-U.S. REAL EXCHANGE RATE

GO deflators **CPI / CPI components**

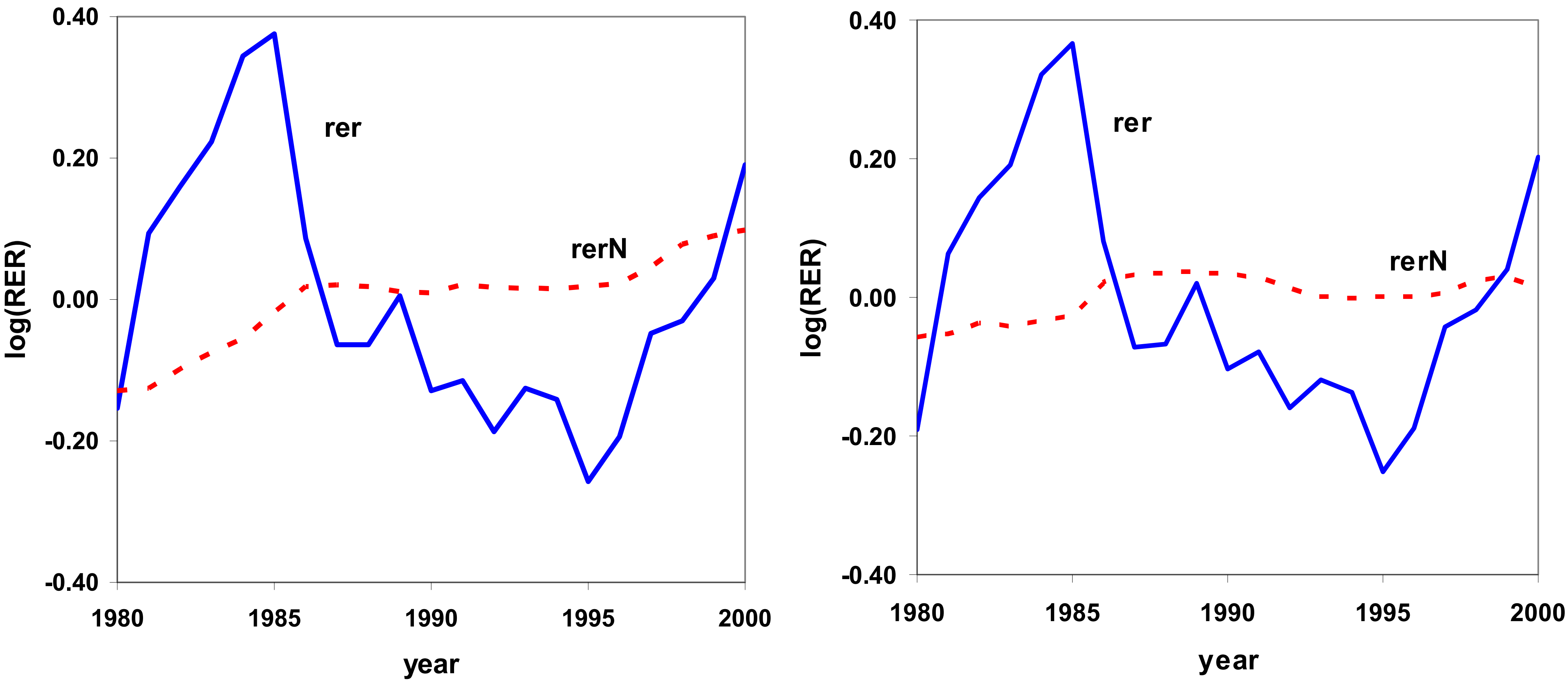


TABLE 2C

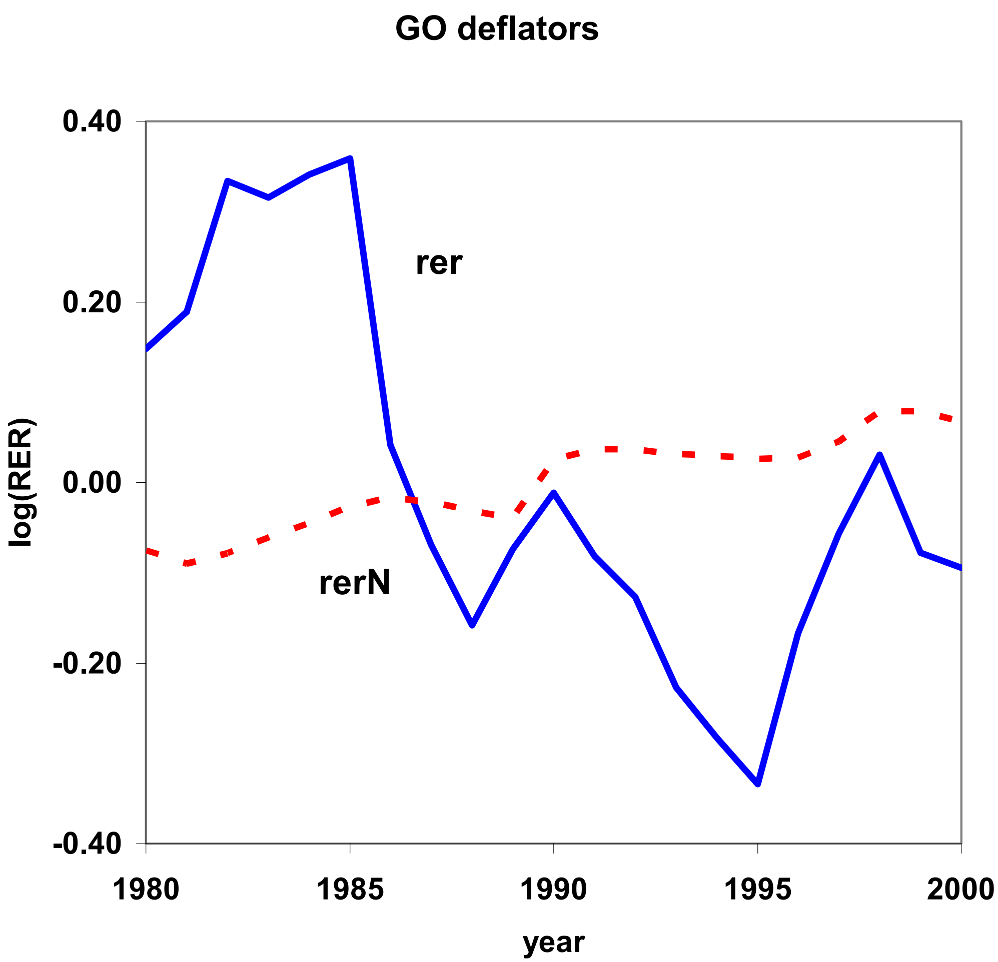
JAPAN-U.S. REAL EXCHANGE RATE

Annual Data

	GO Deflators 1980-2000	PPI-CPI 1980-2000	Components of CPI 1980-2000	Components of PCD 1990-2000
Levels				
corr(rer,rer^N)	-0.33	0.92	-0.74	-0.60
std(rer^N)/std(rer)	0.14	0.27	0.17	0.13
vardec(rer,rer^N)	0.02	0.11	0.02	0.01
Detrended levels				
corr(rer,rer^N)	0.47	0.95	-0.27	0.35
std(rer^N)/std(rer)	0.12	0.16	0.07	0.07
vardec(rer,rer^N)	0.02	0.03	0.00	0.00
1 year changes				
corr(rer,rer^N)	0.30	0.87	-0.32	0.13
std(rer^N)/std(rer)	0.12	0.17	0.09	0.07
vardec(rer,rer^N)	0.02	0.05	0.01	0.01
4 year changes				
corr(rer,rer^N)	0.52	0.95	-0.36	0.43
std(rer^N)/std(rer)	0.12	0.16	0.06	0.07
vardec(rer,rer^N)	0.02	0.05	0.01	0.01

FIGURE 3C

JAPAN-U.S. REAL EXCHANGE RATE



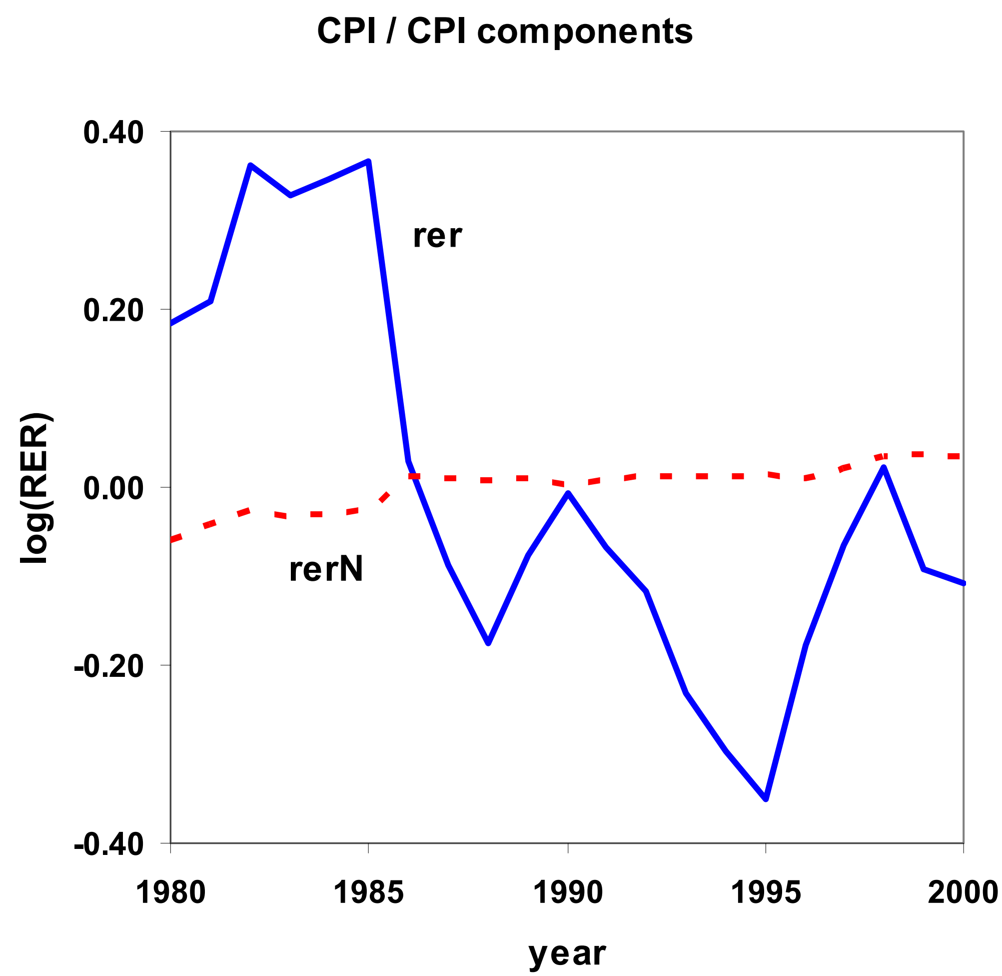


TABLE 2D

KOREA-U.S. REAL EXCHANGE RATE
Annual Data

	GO Deflators 1980-2000	PPI-CPI 1980-2000	Components of CPI 1980-2000	Components of PCD 1980-2000
Levels				
corr(rer,rer^N)	0.65	0.43	0.64	0.72
std(rer^N)/std(rer)	0.21	0.48	0.23	0.36
vardec(rer,rer^N)	0.05	0.22	0.06	0.18
Detrended levels				
corr(rer,rer^N)	0.82	0.94	0.63	0.72
std(rer^N)/std(rer)	0.18	0.30	0.21	0.24
vardec(rer,rer^N)	0.04	0.14	0.05	0.08
1 year changes				
corr(rer,rer^N)	0.81	0.88	0.57	0.49
std(rer^N)/std(rer)	0.23	0.26	0.22	0.16
vardec(rer,rer^N)	0.08	0.10	0.06	0.03
4 year changes				
corr(rer,rer^N)	0.80	0.94	0.62	0.72
std(rer^N)/std(rer)	0.18	0.30	0.21	0.24
vardec(rer,rer^N)	0.05	0.13	0.06	0.08

FIGURE 3D

KOREA-U.S. REAL EXCHANGE RATE

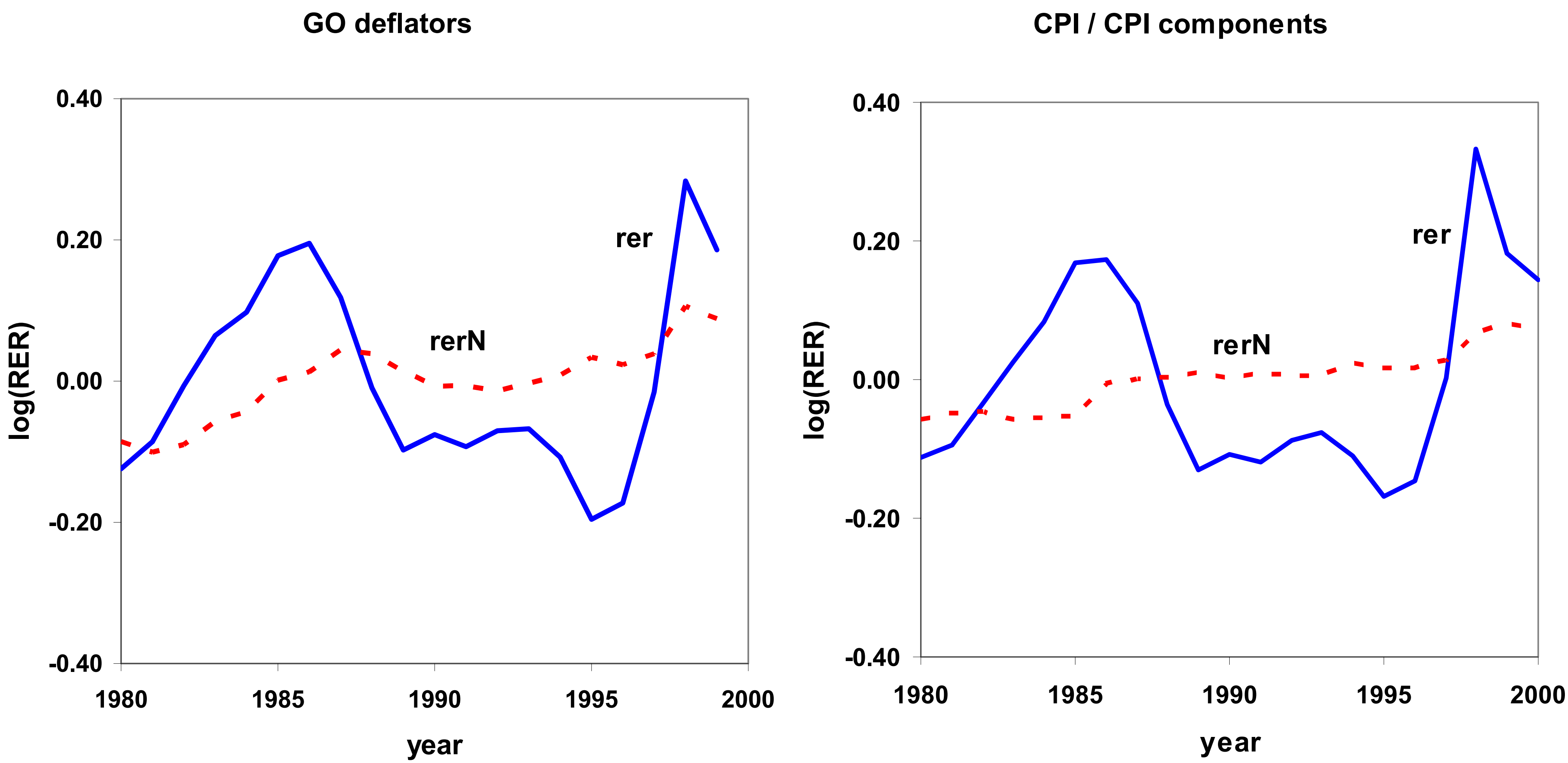


TABLE 2E

MEXICO-U.S. REAL EXCHANGE RATE
Annual Data

	GO Deflators 1980-2000	PPI-CPI 1981-2000	Components of CPI 1980-2000	Components of PCD 1980-2000
Levels				
corr(rer,rer^N)	0.75	0.74	0.64	0.81
std(rer^N)/std(rer)	0.36	0.21	0.55	0.23
vardec(rer,rer^N)	0.18	0.06	0.33	0.08
Detrended levels				
corr(rer,rer^N)	0.84	0.73	0.67	0.84
std(rer^N)/std(rer)	0.36	0.22	0.46	0.24
vardec(rer,rer^N)	0.20	0.06	0.26	0.08
1 year changes				
corr(rer,rer^N)	0.52	0.54	0.26	0.51
std(rer^N)/std(rer)	0.25	0.19	0.28	0.16
vardec(rer,rer^N)	0.07	0.04	0.08	0.03
4 year changes				
corr(rer,rer^N)	0.91	0.78	0.73	0.92
std(rer^N)/std(rer)	0.38	0.24	0.51	0.27
vardec(rer,rer^N)	0.25	0.08	0.34	0.12

FIGURE 3E

MEX ICO-U.S. REAL EXCHANGE RATE

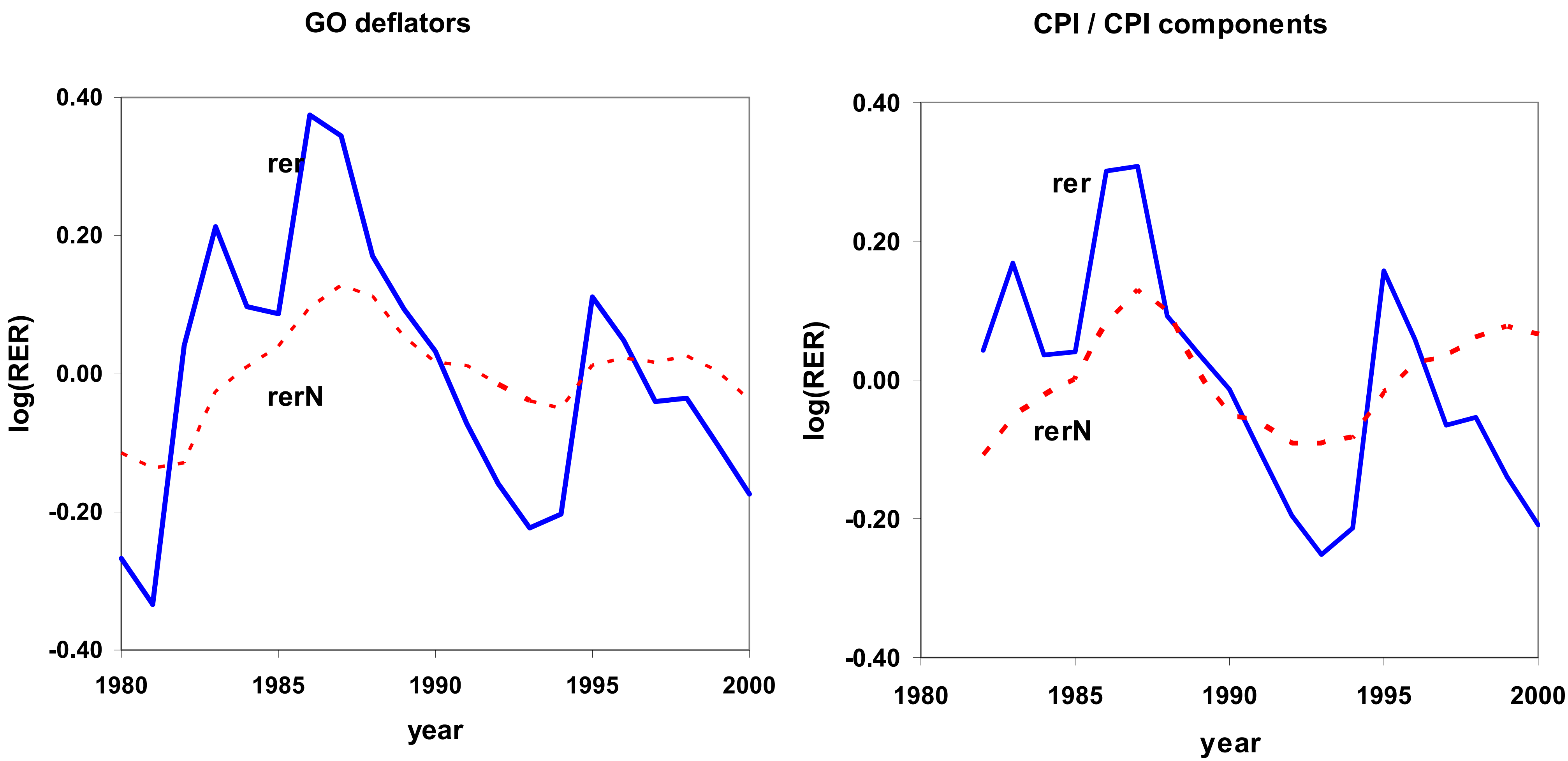


TABLE 3

COMPARISON OF SERIES:
TRADE WEIGHTED AVERAGE
Annual Data

	GO Deflators	PPI-CPI	Components of CPI	Components of PCD
Levels				
corr(rer,rer^N)	0.44	0.73	0.23	0.27
std(rer^N)/std(rer)	0.36	0.48	0.45	0.36
vardec(rer,rer^N)	0.21	0.33	0.22	0.15
Detrended levels				
corr(rer,rer^N)	0.68	0.77	0.00	0.23
std(rer^N)/std(rer)	0.32	0.32	0.22	0.15
vardec(rer,rer^N)	0.18	0.20	0.08	0.03
1 year changes				
corr(rer,rer^N)	0.47	0.63	0.02	0.14
std(rer^N)/std(rer)	0.27	0.33	0.19	0.13
vardec(rer,rer^N)	0.11	0.19	0.06	0.04
4 year changes				
corr(rer,rer^N)	0.70	0.78	0.10	0.37
std(rer^N)/std(rer)	0.34	0.34	0.23	0.16
vardec(rer,rer^N)	0.21	0.25	0.14	0.08

TABLE 4

COMPARISON OF SERIES:
CORRELATIONS OF DIFFERENT MEASURES OF rer^N
Annual Data

	Canada	Germany	Japan	Korea	Mexico	weighted average
Levels						
PPI-CPI-GO deflator	0.97	0.92	-0.61	0.09	0.70	0.52
CPI components-GO deflator	0.52	0.91	0.85	0.76	0.54	0.64
PCD components-GO deflator	0.54	0.99	0.91	0.02	0.95	0.72
PCD components-CPI components	0.996	0.88	0.94	0.19	0.68	0.84
Detrended levels						
CPI-PPI/GO deflator	0.96	0.54	0.37	0.89	0.71	0.74
CPI components/GO deflator	-0.18	0.83	0.61	0.71	0.82	0.37
PCD components/GO deflator	-0.12	0.88	0.81	0.77	0.96	0.47
PCD components/CPI components	0.92	0.90	0.40	0.88	0.86	0.80
1 year changes						
CPI-PPI/GO deflator	0.88	0.48	0.28	0.88	0.56	0.64
CPI components/GO deflator	-0.19	0.73	0.44	0.52	0.73	0.29
PCD components/GO deflator	-0.14	0.65	0.79	0.59	0.86	0.41
PCD components/CPI components	0.79	0.60	0.48	0.53	0.75	0.69
4 year changes						
CPI-PPI/GO deflator	0.98	0.56	0.47	0.91	0.81	0.80
CPI components/GO deflator	0.04	0.89	0.54	0.69	0.86	0.46
PCD components/GO deflator	0.03	0.97	0.79	0.75	0.97	0.54
PCD components/CPI components	0.87	0.92	0.19	0.91	0.89	0.75

TABLE 5

COMPARISON OF COUNTRIES: GROSS OUTPUT DEFLATORS

Annual Data

	Canada	Germany	Japan	Korea	Mexico
Importance of trade to country i					
2000 bilateral trade/GDP	0.58	0.05	0.04	0.14	0.44
2000 bilateral trade/trade	0.82	0.08	0.26	0.20	0.83
Rank of U.S. as partner	1	3	1	1	1
Importance of trade to U.S.					
2000 bilateral trade/U.S. GDP	0.04	0.01	0.02	0.01	0.03
2000 bilateral trade/U.S. trade	0.21	0.04	0.11	0.03	0.13
Rank of country i as partner	1	6	3	7	2
Levels					
corr(rer,rer^N)	0.81	-0.55	-0.33	0.65	0.75
std(rer^N)/std(rer)	0.51	0.25	0.14	0.21	0.36
vardec(rer,rer^N)	0.38	0.04	0.02	0.05	0.18
Detrended levels					
corr(rer,rer^N)	0.78	0.18	0.47	0.82	0.84
std(rer^N)/std(rer)	0.45	0.20	0.12	0.18	0.36
vardec(rer,rer^N)	0.29	0.04	0.02	0.04	0.20
1 year changes					
corr(rer,rer^N)	0.54	0.16	0.30	0.81	0.52
std(rer^N)/std(rer)	0.40	0.13	0.12	0.23	0.25
vardec(rer,rer^N)	0.20	0.03	0.02	0.08	0.07
4 year changes					
corr(rer,rer^N)	0.74	0.24	0.52	0.80	0.91
std(rer^N)/std(rer)	0.47	0.21	0.12	0.18	0.38
vardec(rer,rer^N)	0.33	0.07	0.02	0.05	0.25

TABLE A

GROSS DOMESTIC PRODUCT DEFLATORS
Annual Data

	Canada 1980-1998	Germany 1980-2000	Japan 1980-2000	Korea 1980-2000	Mexico 1980-2000	weighted average
Levels						
corr(rer,rer^N)	0.80	-0.32	-0.69	0.00	0.74	0.34
std(rer^N)/std(rer)	0.90	0.53	0.22	0.26	0.50	0.59
vardec(rer,rer^N)	0.69	0.15	0.03	0.06	0.33	0.38
Detrended levels						
corr(rer,rer^N)	0.63	0.19	-0.24	0.56	0.84	0.47
std(rer^N)/std(rer)	0.75	0.27	0.18	0.18	0.47	0.49
vardec(rer,rer^N)	0.48	0.07	0.03	0.04	0.33	0.29
1 year changes						
corr(rer,rer^N)	0.05	-0.14	-0.29	0.61	0.47	0.11
std(rer^N)/std(rer)	0.81	0.18	0.16	0.26	0.36	0.48
vardec(rer,rer^N)	0.33	0.05	0.03	0.09	0.15	0.18
4 year changes						
corr(rer,rer^N)	0.65	0.18	-0.16	0.56	0.91	0.51
std(rer^N)/std(rer)	0.68	0.27	0.16	0.19	0.47	0.46
vardec(rer,rer^N)	0.54	0.12	0.03	0.06	0.40	0.34

TABLE B

COMPARISON OF GROSS DOMESTIC PRODUCT DEFLATORS AND GROSS OUTPUT DEFLATORS

Annual Data

	Canada 1980-1998	Germany 1980-2000	Japan 1980-2000	Korea 1980-2000	Mexico 1980-2000	weighted average
Levels						
corr(rer(GDP),rer(GO))	0.99	0.99	0.997	0.94	0.98	0.99
corr(rer^N(GDP),rer^N(GO))	0.96	0.97	0.89	0.86	0.97	0.94
std(rer(GDP))/std(rer(GO))	1.27	1.11	1.10	1.34	1.09	1.18
std(rer^N(GDP))/std(rer^N(GO))	2.23	1.68	1.74	1.66	1.52	1.87
Detrended levels						
corr(rer(GDP),rer(GO))	0.98	0.88	0.995	0.99	0.99	0.98
corr(rer^N(GDP),rer^N(GO))	0.88	0.80	0.69	0.86	0.98	0.86
std(rer(GDP))/std(rer(GO))	1.07	0.94	1.06	1.22	1.11	1.08
std(rer^N(GDP))/std(rer^N(GO))	1.77	1.51	1.57	1.22	1.45	1.59
1 year changes						
corr(rer(GDP),rer(GO))	0.99	0.99	0.99	0.99	0.99	0.99
corr(rer^N(GDP),rer^N(GO))	0.87	0.48	0.57	0.76	0.92	0.78
std(rer(GDP))/std(rer(GO))	1.04	1.05	1.05	1.19	1.09	1.06
std(rer^N(GDP))/std(rer^N(GO))	2.13	1.50	1.49	1.35	1.58	1.76
4 year changes						
corr(rer(GDP),rer(GO))	0.99	0.998	0.997	0.99	0.99	0.99
corr(rer^N(GDP),rer^N(GO))	0.96	0.85	0.71	0.90	0.99	0.91
std(rer(GDP))/std(rer(GO))	1.01	1.07	1.06	1.20	1.14	1.07
std(rer^N(GDP))/std(rer^N(GO))	1.47	1.75	1.49	1.28	1.42	1.47

EXTENDED RESULTS

Examine sample of 50 countries and all possible 1225 bilateral real exchange rates.

Use same methodology and summary statistics and CPI-PPI measures of prices.

Examine robustness of results to

1. Presence of U.S. in bilateral trade partner pairs in the sample.
2. Presence of rich-country/poor-country bilateral trade pairs in the sample.
3. Presence of high-inflation/low inflation bilateral trade pairs in the sample.

Find that there is a substantive relation between $rer_{i,us}$ and $rer_{i,us}^{N}$ on average.

The relation does not depend on these three factors (at least in the manner one might expect).

Strength of the relation depends crucially on size of the trade relationship between two trade partners.

Table I

COUNTRIES IN THE SAMPLE
Percent World Trade in 2000

Argentina	0.39	Hong Kong (P.R.C.)	3.02	Peru	0.10
Australia	1.01	India	0.70	Philippines	0.66
Austria	1.04	Indonesia	0.75	Saudi Arabia	0.84
Belgium	2.75	Ireland	1.00	South Africa	0.36
Brazil	0.90	Israel	0.50	Singapore	2.08
Canada	3.98	Italy	3.65	Spain	2.08
Chile	0.27	Japan	6.45	Sri Lanka	0.10
Colombia	0.18	Jordan	0.04	Sweden	1.23
Costa Rica	0.10	Korea	2.50	Switzerland	1.39
Cyprus	0.06	Luxembourg	0.16	Thailand	0.96
Denmark	0.73	Malaysia	1.42	Trinidad and Tobago	0.04
Egypt	0.19	Mexico	2.44	Turkey	0.63
El Salvador	0.06	Netherlands	3.61	Uruguay	0.05
Finland	0.64	New Zealand	0.20	United Kingdom	4.88
France	5.04	Norway	0.70	United States	15.37
Germany	8.10	Pakistan	0.15	Venezuela	0.38
Greece	0.35	Panama	0.13		

Table II

U.S. BILATERAL REAL EXCHANGE RATES
Weighted Means

		income level		inflation		trade intensity		std(rer)	
	all	high	low	high	low	high	low	high	low
levels									
corr(rer, rer^N)	0.63	0.62	0.65	0.69	0.61	0.74	0.28	0.58	0.68
std(rer^N)/std(rer)	0.44	0.45	0.42	0.37	0.46	0.49	0.29	0.27	0.63
vardec(rer, rer^N)	0.26	0.28	0.21	0.18	0.28	0.32	0.10	0.11	0.43
1 year lags									
corr(rer, rer^N)	0.50	0.47	0.54	0.51	0.49	0.53	0.39	0.42	0.58
std(rer^N)/std(rer)	0.35	0.36	0.34	0.29	0.37	0.40	0.20	0.19	0.52
vardec(rer, rer^N)	0.17	0.19	0.11	0.11	0.19	0.22	0.08	0.06	0.28
4 year lags									
corr(rer, rer^N)	0.66	0.61	0.75	0.77	0.62	0.73	0.44	0.60	0.71
std(rer^N)/std(rer)	0.36	0.35	0.39	0.34	0.37	0.41	0.21	0.22	0.52
vardec(rer, rer^N)	0.22	0.23	0.18	0.18	0.23	0.31	0.10	0.10	0.35
countries	49	24	25	18	31	25	24	31	18
percent of U.S. trade	88.13	59.80	28.33	21.32	66.81	66.22	21.91	45.92	42.21

Table III

INCOME LEVELS
ALL BILATERAL REAL EXCHANGE RATES
Weighted Means

	all	high-high	high-low	low-low
levels				
corr(rer, rer^N)	0.53	0.50	0.60	0.63
std(rer^N)/std(rer)	0.66	0.74	0.47	0.64
vardec(rer, rer^N)	0.32	0.33	0.26	0.43
1 year lags				
corr(rer, rer^N)	0.45	0.42	0.52	0.58
std(rer^N)/std(rer)	0.46	0.50	0.36	0.43
vardec(rer, rer^N)	0.19	0.21	0.13	0.21
4 year lags				
corr(rer, rer^N)	0.60	0.57	0.65	0.70
std(rer^N)/std(rer)	0.54	0.59	0.40	0.51
vardec(rer, rer^N)	0.26	0.27	0.19	0.33
bilateral pairs	1225	300	625	300
percent of world trade	71.88	49.80	19.78	2.30

Table IV

INFLATION LEVELS
ALL BILATERAL REAL EXCHANGE RATES
Weighted Means

	all	high-high	high-low	low-low
levels				
corr(rer, rer^N)	0.53	0.68	0.63	0.51
std(rer^N)/std(rer)	0.66	0.63	0.46	0.71
vardec(rer, rer^N)	0.32	0.47	0.24	0.33
1 year lags				
corr(rer, rer^N)	0.46	0.65	0.51	0.44
std(rer^N)/std(rer)	0.46	0.43	0.34	0.49
vardec(rer, rer^N)	0.19	0.23	0.13	0.20
4 year lags				
corr(rer, rer^N)	0.60	0.76	0.69	0.58
std(rer^N)/std(rer)	0.54	0.53	0.40	0.57
vardec(rer, rer^N)	0.25	0.39	0.20	0.26
bilateral pairs	1225	153	576	496
percent of world trade	71.88	0.81	13.11	57.96

Table V

TRADE INTENSITY
ALL BILATERAL REAL EXCHANGE RATES
Weighted Means

		trade intensity	
	all	high	low
levels			
corr(rer, rer^N)	0.53	0.62	0.46
std(rer^N)/std(rer)	0.66	0.71	0.62
vardec(rer, rer^N)	0.32	0.36	0.28
1 year lags			
corr(rer, rer^N)	0.46	0.49	0.42
std(rer^N)/std(rer)	0.46	0.56	0.37
vardec(rer, rer^N)	0.19	0.24	0.14
4 year lags			
corr(rer, rer^N)	0.60	0.66	0.54
std(rer^N)/std(rer)	0.54	0.61	0.48
vardec(rer, rer^N)	0.25	0.30	0.21
bilateral pairs	1225	51	1174
percent of world trade	71.88	33.51	38.37

Table VI

REAL EXCHANGE RATE VARIABILITY
ALL BILATERAL REAL EXCHANGE RATES
Weighted Means

		std(rer)	
	all	high	low
levels			
corr(rer, rer^N)	0.53	0.59	0.50
std(rer^N)/std(rer)	0.66	0.36	0.87
vardec(rer, rer^N)	0.32	0.20	0.40
1 year lags			
corr(rer, rer^N)	0.46	0.46	0.45
std(rer^N)/std(rer)	0.46	0.26	0.60
vardec(rer, rer^N)	0.19	0.09	0.26
4 year lags			
corr(rer, rer^N)	0.60	0.60	0.60
std(rer^N)/std(rer)	0.54	0.29	0.71
vardec(rer, rer^N)	0.25	0.13	0.33
bilateral pairs	1225	863	362
percent of world trade	71.88	29.55	42.33

Table VII

U.S. BILATERAL REAL EXCHANGE RATES
TRADING BLOC TRADE PARTNERS
Weighted Means

	all	EU	nonEU	NAFTA	non-NAFTA	nonNAFTA-nonEU
levels						
corr(rer, rer^N)	0.63	0.23	0.74	0.82	0.50	0.65
std(rer^N)/std(rer)	0.44	0.27	0.49	0.45	0.44	0.53
vardec(rer, rer^N)	0.26	0.08	0.31	0.35	0.20	0.27
1 year lags						
corr(rer, rer^N)	0.50	0.41	0.52	0.55	0.46	0.49
std(rer^N)/std(rer)	0.35	0.17	0.40	0.42	0.30	0.38
vardec(rer, rer^N)	0.17	0.04	0.20	0.22	0.13	0.18
4 year lags						
corr(rer, rer^N)	0.66	0.43	0.72	0.77	0.58	0.66
std(rer^N)/std(rer)	0.36	0.18	0.41	0.46	0.30	0.37
vardec(rer, rer^N)	0.22	0.05	0.26	0.30	0.16	0.23
countries	49	14	35	2	47	33
percent of U.S. trade	88.13	19.19	68.94	34.31	53.82	34.63

Table VIII

TRADING BLOC TRADE PARTNERS
ALL BILATERAL REAL EXCHANGE RATES
Weighted Means

	all	EU& EU	EU& NAFTA	EU& nonEU/ nonNAFTA	NAFTA& NAFTA	NAFTA& nonEU/ nonNAFTA	nonEU/ nonNAFTA& nonEU/ nonNAFTA
levels							
corr(rer, rer^N)	0.53	0.40	0.25	0.47	0.82	0.65	0.61
std(rer^N)/std(rer)	0.66	1.05	0.28	0.59	0.44	0.53	0.56
vardec(rer, rer^N)	0.32	0.40	0.09	0.29	0.34	0.27	0.36
1 year lags							
corr(rer, rer^N)	0.46	0.43	0.42	0.37	0.55	0.50	0.47
std(rer^N)/std(rer)	0.46	0.70	0.17	0.34	0.42	0.39	0.40
vardec(rer, rer^N)	0.19	0.27	0.04	0.13	0.22	0.19	0.17
4 year lags							
corr(rer, rer^N)	0.60	0.63	0.43	0.46	0.77	0.66	0.46
std(rer^N)/std(rer)	0.54	0.83	0.21	0.49	0.45	0.37	0.26
vardec(rer, rer^N)	0.25	0.35	0.06	0.20	0.30	0.23	0.22
bilateral pairs	1225	91	42	462	3	99	528
percent of world trade	71.88	21.44	6.77	11.27	10.62	11.64	10.25

6. **Growth theory needs to be reconsidered in the light of trade theory. In particular, a growth model that includes trade can have the opposite convergence properties from a model of closed economies.**

C. Bajona and T. J. Kehoe, “On Dynamic Heckscher-Ohlin Models II: Infinitely-Lived Consumers,” Federal Reserve Bank of Minneapolis, 2004.

Trade and Growth

In 2004 Mexico has income per capita of 6400 U.S. dollars. In 1926 the United Stated had income per capita of about 6400 U.S. dollars (real 2004 U.S. dollars).

To study what will happened in Mexico over the next 80 years, should we study what happened to the United States since 1926?

…or should we take into account that the United States was the country with the highest income in the world in 1926, while Mexico has a very large trade relation with the United States — a country with a level of income per capita approximately 6 times larger in 2004?

We study this question using the Heckscher-Ohlin model of international trade: Countries differ in their levels of capital per worker.

The General Dynamic Heckscher-Ohlin Model

n countries

countries differ in initial capital-labor ratios $\bar{k}_0^i$

and in size of population L^i.

two traded goods — a capital intensive good and a labor intensive good

$$y_j = \phi_j(k_j, \ell_j)$$

$$\frac{\phi_{1L}(k/\ell,1)}{\phi_{1K}(k/\ell,1)} < \frac{\phi_{2L}(k/\ell,1)}{\phi_{2K}(k/\ell,1)}$$

nontraded investment good

$$x = f(x_1, x_2)$$

Feasibility:

$$\sum_{i=1}^{n} L^i (c^i_{jt} + x^i_{jt}) = \sum_{i=1}^{n} L^i y^i_{jt} = \sum_{i=1}^{n} L^i \phi_j (k^i_{jt}, \ell^i_{jt}).$$

$$k^i_{1t} + k^i_{2t} = k^i_t$$

$$\ell^i_{1t} + \ell^i_{2t} = 1$$

$$k^i_{t+1} - (1-\delta) k^i_t = x^i_t = f(x^i_{1t}, x^i_{2t})$$

Infinitely-Lived Consumers

consumer in country i, $i = 1,...,n$:

$$\max \sum_{t=0}^{\infty} \beta^t u(c_{1t}^i, c_{2t}^i)$$

$$\text{s.t. } p_{1t} c_{1t}^i + p_{2t} c_{2t}^t + q_t^i k_{t+1}^i + b_{t+1}^i = w_t^i + (1 + r_t^{bi}) b_t^i + [q_t^i (1 - \delta) + r_t^i] k_t^i$$

$$c_{jt}^i \geq 0,\ k_t^i \geq 0,\ b_t^i \geq -B$$

$$k_0^i = \bar{k}_0^i,\ b_0^i = 0.$$

international borrowing and lending:

$$\sum_{i=1}^{n} L^i b_t^i = 0.$$

no international borrowing and lending:

$$b_t^i = 0.$$

Integrated Economy

Characterization and computation of equilibrium is relatively easy when we can solve for equilibrium of an artificial world economy in which we ignore restrictions on factor mobility and then disaggregate the consumption, production, and investment decisions.

This is a guess-and-verify approach: We first solve for the world equilibrium and then we see if we can disaggregate the consumption, production, and investment decisions.

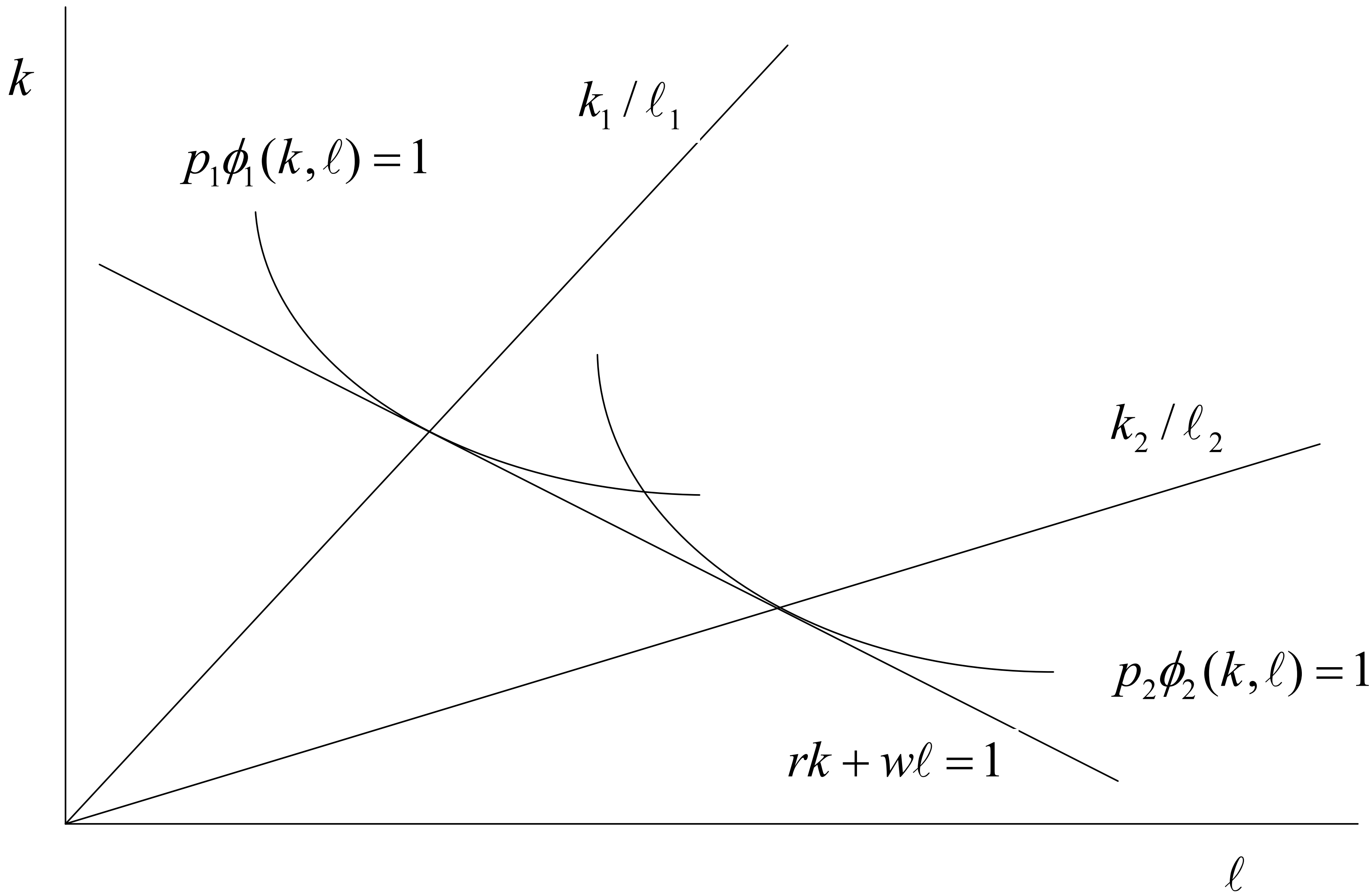
Potential problem: We cannot assign each country nonnegative production plans for each of the two goods while maintaining factor prices equal to those in the world equilibrium.

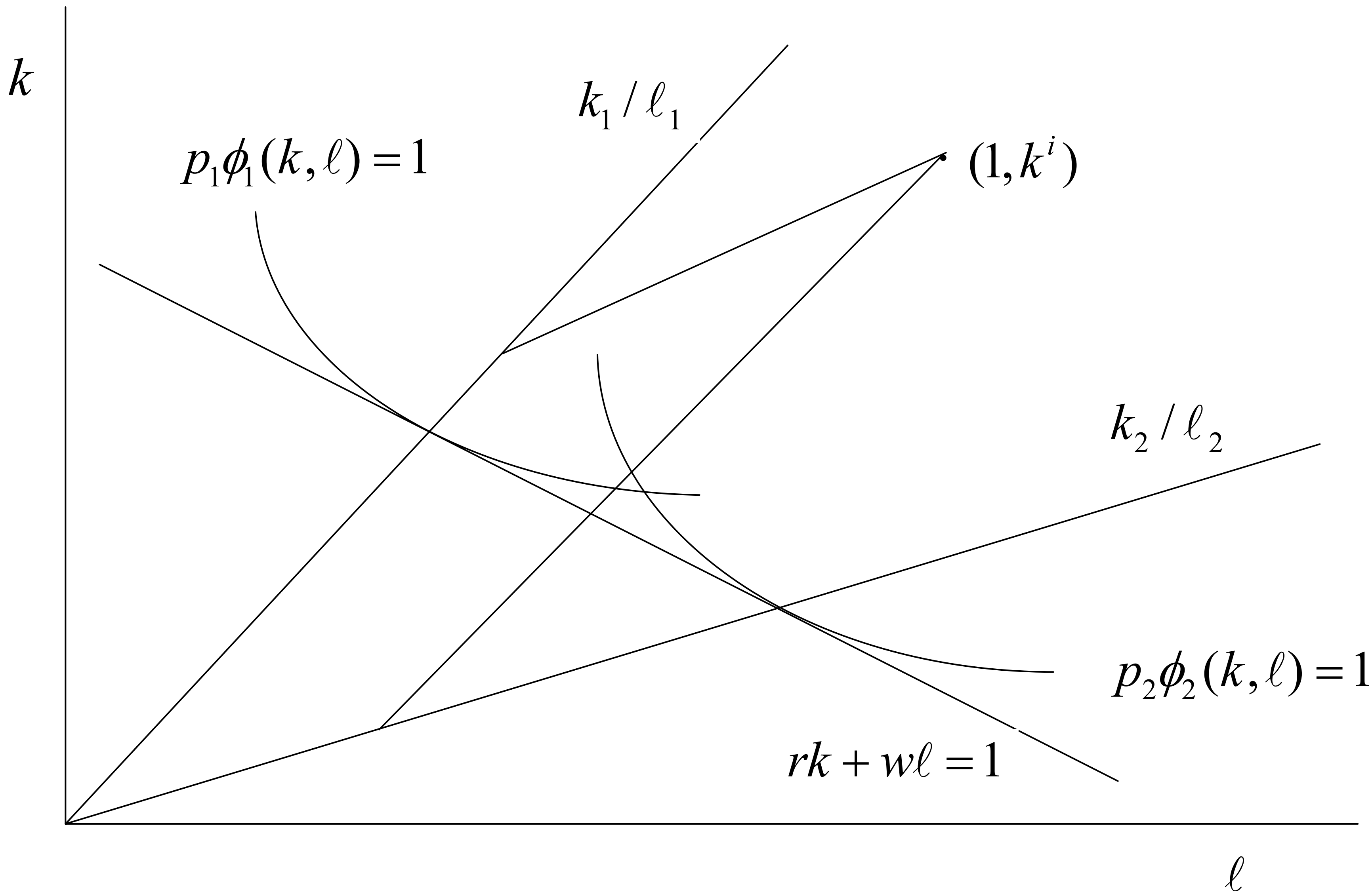
factor price equalization/cone of diversification

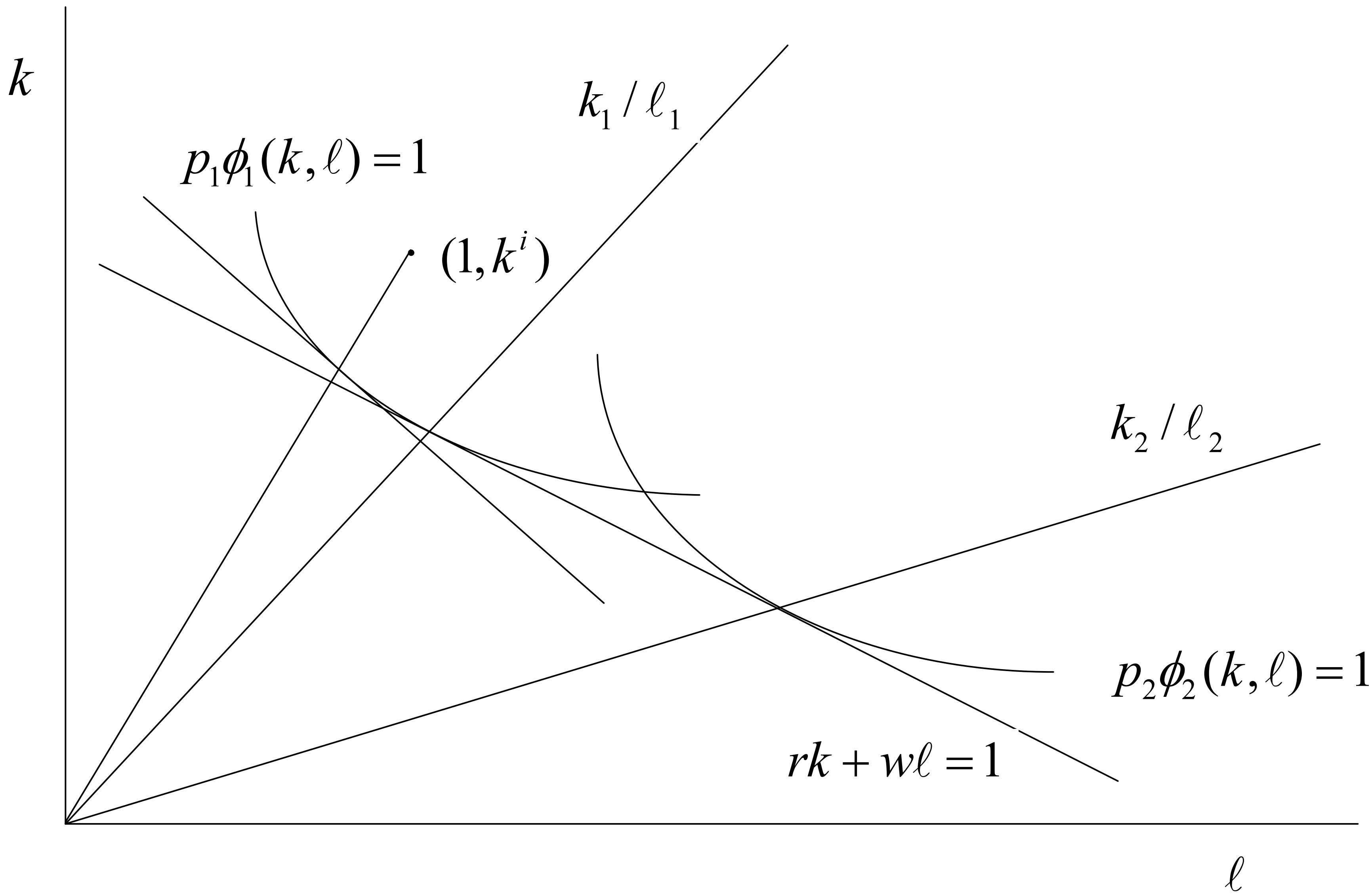
(Another potential problem: We cannot assign each country nonnegative investment.)

If the integrated economy approach does not work, it could be very difficult to calculate an equilibrium.

We would have to determine the pattern of specialization over an infinite time horizon.


k
k_1 / ℓ_1
$p_1\phi_1(k, \ell) = 1$
k_2 / ℓ_2
$p_2\phi_2(k, \ell) = 1$
$rk + w\ell = 1$
ℓ


k
k_1 / ℓ_1
$p_1\phi_1(k,\ell)=1$
$(1,k^i)$
k_2 / ℓ_2
$p_2\phi_2(k,\ell)=1$
$rk + w\ell = 1$
ℓ



k
k_1 / ℓ_1
$p_1\phi_1(k,\ell)=1$
$(1,k^i)$
k_2 / ℓ_2
$p_2\phi_2(k,\ell)=1$
$rk + w\ell = 1$
ℓ

Results for General Model

International borrowing and lending implies factor price equalization in period $t = 1, 2, \ldots$ Production plans and international trade patterns are indeterminate.

Any steady state has factor price equalization.

If there exists a steady state in which the total capital stock is positive, then there exists a continuum of such steady states, indexed by the distribution of world capital $\hat{k}^1, \ldots, \hat{k}^n$.

International trade occurs in every steady state of the model in which $\hat{k}^i \neq \hat{k}$ for some i.

We focus on models with no international borrowing and lending.

Ventura Model

$$u(c_1,c_2)=v\left(f(c_1,c_2)\right)=\log\left(f(c_1,c_2)\right)$$

$$\phi_1(k_1,\ell_1)=k_1$$

$$\phi_2(k_2,\ell_2)=\ell_2$$

$$f(x_1,x_2)=\begin{cases} d\left(a_1x_1^b+a_2x_2^b\right)^{1/b} & \text{if } b\neq 0 \\ dx_1^{a_1}x_2^{a_2} & \text{if } b=0 \end{cases}.$$

Ventura (1997) actually examines the continuous-time version of this model.

In the Ventura model, we can solve for the equilibrium of the world economy by solving a 1 sector growth model in which

$$c_t = f(c_{1t}, c_{2t})$$

$$c_t + k_{t+1} - (1-\delta)k_t = f(k_t, 1).$$

If $b < 0$ and $1/\beta - 1 + \delta > da_1^{1/b}$, the equilibrium converges to $\hat{k} = 0$.

If $b > 0$ and $1/\beta - 1 + \delta < da_1^{1/b}$, the economy grows without bound, and the equilibrium converges to a balanced growth path.

In every other case, the equilibrium converges to a steady state in which $f_K(\hat{k}, 1) = 1/\beta - 1 + \delta$.

The 2 sectors matter a lot for disaggregating the integrated equilibrium!

In particular, we cannot solve for the equilibrium values of the variables for one of the countries by solving an optimal growth problem for that country in isolation.

Instead, the equilibrium path for k_t^i and the steady state value of $\hat{k}^i$ depends on $\bar{k}_0^i$ as well as on the path for k_t and the steady state value of $\hat{k}$.

Proposition:

$$\frac{y_{t+1}^i - y_{t+1}}{y_{t+1}} = \frac{r_{t+1} c_t / y_{t+1}}{r_t c_{t-1} / y_t} \left(\frac{y_t^i - y_t}{y_t} \right)$$

If $\delta = 1$,

$$\frac{y_{t+1}^i - y_{t+1}}{y_{t+1}} = \frac{s_{t+1}}{s_t} \left(\frac{y_t^i - y_t}{y_t} \right)$$

where $s_t = c_t / y_t$.

Proof: The first-order conditions from the consumers' problems are

$$\frac{c_t^i}{c_{t-1}^i} = \beta(1 + r_t - \delta).$$

The demand functions are

$$c_t^i = (1-\beta)\left[\sum_{s=t}^{\infty}\left(\prod_{\tau=t+1}^{s}\frac{1}{1+r_\tau-\delta}\right)w_s + (1+r_t-\delta)k_t^i\right]$$

$$c_t^i - c_t = (1-\beta)(1+r_t-\delta)(k_t^i - k_t).$$

Combining this with the feasibility condition, we obtain

$$k_{t+1}^i - k_{t+1} = \frac{c_t}{c_{t-1}}(k_t^i - k_t).$$

The difference between a country's income per worker and the world's income per worker can be written as:

$$y_{t+1}^{i} - y_{t+1} = r_{t+1}(k_{t+1}^{i} - k_{t+1})$$

Using the expression for $k_{t+1}^{i} - k_{t+1}$ found above and operating, we obtain:

$$\frac{y_{t+1}^{i} - y_{t+1}}{y_{t+1}} = \frac{r_{t+1}c_t / y_{t+1}}{r_t c_{t-1} / y_t}\left(\frac{y_t^{i} - y_t}{y_t}\right)$$

In the case $\delta = 1$ this becomes (using $c_{t+1} / c_t = \beta r_{t+1}$),

$$\frac{y_{t+1}^{i} - y_{t+1}}{y_{t+1}} = \frac{s_{t+1}}{s_t}\left(\frac{y_t^{i} - y_t}{y_t}\right)$$

where $s_t = c_t / y_t$.

$$\frac{y_t^i - y_t}{y_t} = \frac{s_t}{s_0}\left(\frac{y_0^i - y_0}{y_0}\right)$$

Proposition. In the Ventura model with $\delta = 1$ and $0 < k_0 < \hat{k}$,

if $b > 0$, differences in relative income levels decrease over time;

if $b = 0$, differences in relative income levels stay constant over time; and

if $b < 0$, differences in relative income levels increase over time.

$$\frac{y_t^i - y_t}{y_t} = \frac{s_t}{s_0}\left(\frac{y_0^i - y_0}{y_0}\right)$$

Proposition. In the Ventura model with $\delta = 1$ and $0 < k_0 < \hat{k}$,

if $b > 0$, differences in relative income levels decrease over time;

if $b = 0$, differences in relative income levels stay constant over time; and

if $b < 0$, differences in relative income levels increase over time.

Notice contrast with convergence results for world of closed economies!

Generalized Ventura Model

$u(c_1,c_2)=v\big(f(c_1,c_2)\big)=\log\big(f(c_1,c_2)\big)$, and f, ϕ_1, and ϕ_2 are general constant-elasticity-of-substitution functions

Define

$$
\begin{aligned}
F(k,\ell)=\max\ & f(y_1,y_2)\\
\text{s.t. } & y_1=\phi_1(k_1,\ell_1)\\
& y_2=\phi_2(k_2,\ell_2)\\
& k_1+k_2=k\\
& \ell_1+\ell_2=\ell\\
& k_j,\ell_j\geq 0.
\end{aligned}
$$

The cone of diversification has the form $\kappa_1 k_t \geq k_t^i \geq \kappa_2 k_t$.

(In Ventura model $F(k,\ell)=f(k,\ell)$.)

C. E. S. Model

$$y_1 = \phi_1(k_1, \ell_1) = \theta_1 \left(\alpha_1 k_1^b + (1-\alpha_1)\ell_1^b \right)^{1/b}$$

$$y_2 = \phi_2(k_2, \ell_2) = \theta_2 \left(\alpha_2 k_2^b + (1-\alpha_2)\ell_2^b \right)^{1/b}$$

$$f(y_1, y_2) = d \left(a_1 y_1^b + a_2 y_1^b \right)^{1/b}$$

(All elasticities of substitution are equal.)

In this case

$$F(k, \ell) = D\left(Ak^b + (1-A)\ell^b \right)^{1/b}$$

where

$$A=\frac{\left[\left(a_1\alpha_1\theta_1^b\right)^{\frac{1}{1-b}}+\left(a_2\alpha_2\theta_2^b\right)^{\frac{1}{1-b}}\right]^{1-b}}{\left[\left(a_1\alpha_1\theta_1^b\right)^{\frac{1}{1-b}}+\left(a_2\alpha_2\theta_2^b\right)^{\frac{1}{1-b}}\right]^{1-b}+\left[\left(a_1(1-\alpha_1)\theta_1^b\right)^{\frac{1}{1-b}}+\left(a_2(1-\alpha_2)\theta_2^b\right)^{\frac{1}{1-b}}\right]^{1-b}}$$

$$D=d\left\{\left[\left(a_1\alpha_1\theta_1^b\right)^{\frac{1}{1-b}}+\left(a_2\alpha_2\theta_2^b\right)^{\frac{1}{1-b}}\right]^{1-b}+\left[\left(a_1(1-\alpha_1)\theta_1^b\right)^{\frac{1}{1-b}}+\left(a_2(1-\alpha_2)\theta_2^b\right)^{\frac{1}{1-b}}\right]^{1-b}\right\}^{\frac{1-b}{b}}$$

$$\kappa_i=\left(\frac{\alpha_i}{1-\alpha_i}\right)^{\frac{1}{1-b}}\frac{\left(a_1(1-\alpha_1)\theta_1^b\right)^{\frac{1}{1-b}}+\left(a_2(1-\alpha_2)\theta_2^b\right)^{\frac{1}{1-b}}}{\left(a_1\alpha_1\theta_1^b\right)^{\frac{1}{1-b}}+\left(a_2\alpha_2\theta_2^b\right)^{\frac{1}{1-b}}}.$$

Cobb-Douglas Model

$$y_1 = \phi_1(k_1, \ell_1) = \theta_1 k_1^{\alpha_1} \ell_1^{1-\alpha_1}$$

$$y_2 = \phi_2(k_2, l_2) = \theta_2 k_2^{\alpha_2} \ell_2^{1-\alpha_2}$$

$$f(y_1, y_2) = d y_1^{a_1} y_2^{a_2}$$

(This is the special case of the C.E.S. model where $b = 0$.)

In this case

$$F(k, \ell) = D k^{A} \ell^{1-A}$$

where

$$A = a_1\alpha_1 + a_2\alpha_2$$

$$D = \frac{d\left[\theta_1 a_1 \alpha_1^{\alpha_1}(1-\alpha_1)^{1-\alpha_1}\right]^{a_1}\left[\theta_2 a_2 \alpha_2^{\alpha_2}(1-\alpha_2)^{1-\alpha_2}\right]^{a_2}}{A^A(1-A)^{1-A}}$$

$$\kappa_i = \left(\frac{\alpha_i}{1-\alpha_i}\right)\frac{1-A}{A}.$$

Proposition: In the Cobb-Douglas model with $\delta = 1$, suppose that factor price equalization occurs at period T. Then factor price equalization occurs at all $t \geq T$.

Furthermore, the equilibrium capital stocks can be solved for as

$$k_t^i = \gamma^i k_t$$

where $\gamma^i = k_T^i / k_T$ and $k_{t+1} = \beta A D k_t^A$ for $t \geq T$.

more generally,

Proposition: In the C.E.S. model with $b \geq 0$ and $\delta = 1$, suppose that factor price equalization occurs at time T. Then factor price equalization occurs at all $t \geq T$.

Proposition: In the C.E.S. model with $b < 0$ and $\delta = 1$, suppose that factor price equalization occurs at time T and that $0 < k_t < \hat{k}$. Then there exists an $\varepsilon_T > 0$ such that, if

$$\left|k_t^i - \kappa_1 k_t\right| < \varepsilon_T \text{ or } \left|k_t^i - \kappa_2 k_t\right| < \varepsilon_T$$

for some i, then factor price equalization cannot occur for all $t \geq T$.

Proof: Recall that the cone of diversification has the form

$$\kappa_1 k_t \geq k_t^i \geq \kappa_2 k_t .$$

We can rewrite these conditions as

$$\kappa_1 - 1 \geq \frac{k_t^i - k_t}{k_t} \geq \kappa_2 - 1.$$

Suppose that all countries have their capital-labor ratios in the cone of diversification at all periods $t \geq T$. Then

$$\frac{k_{t+1}^i - k_{t+1}}{k_{t+1}} = \frac{c_t / k_{t+1}}{c_{t-1} / k_t} \left(\frac{k_t^i - k_t}{k_t} \right) = \frac{z_{t+1}}{z_t} \left(\frac{k_t^i - k_t}{k_t} \right)$$

for all $t \geq T$ where $z_t = c_{t-1} / k_t$ and $z_0 = c_0 / \left(\beta r_0 k_0 \right)$.

If $b < 0$, z_t increases as k_t increases.

If $b = 0$, z_t is constant.

If $b > 0$, z_t decreases as k_t increases.

Continuous-Time Ventura Model

$$\int_0^\infty e^{-\rho t} \log\left(f(c_1, c_2) \right) \, dt$$

$$y_1 = k_1$$

$$y_2 = \ell_2$$

$$\dot{k} = f(x_1, x_2)$$

$$f(y_1, y_2) = \begin{cases} d\left(a_1 y_1^b + a_2 y_2^b \right)^{1/b} & \text{if } b \neq 0 \\ d x_1^{a_1} x_2^{a_2} & \text{if } b = 0 \end{cases}$$

We can find the integrated equilibrium by solving

$$\max \int_0^\infty e^{-\rho t} \log c \, dt$$

$$\text{s.t. } c + \dot{k} = f(k, 1) = g(k)$$

$$c, \dot{k} \geq 0$$

$$k(0) = \bar{k}_0.$$

Ventura (1997) shows that

$$\frac{k^i(t)-k(t)}{k(t)}=\frac{c(t)/k(t)}{c(0)/k(0)}\left(\frac{k^i(0)-k(0)}{k(0)}\right)=\frac{z(t)}{z(0)}\left(\frac{k^i(0)-k(0)}{k(0)}\right)$$

and draws phase diagrams in (k, z) space to analyze convergence/divergence of k^i and k.

Notice that this is not the same as convergence/divergence of y^i and y, where

$$y^i = w + rk^i = f(y_1^i, y_2^i).$$

Instead, let us study the behavior of

$$\frac{y^i(t)-y(t)}{y(t)}=\frac{s(t)}{s(0)}\left(\frac{y^i(0)-y(0)}{y(0)}\right)$$

where

$$s(t)=\frac{r(t)c(t)}{y(t)}=\frac{f_K(k,1)c(t)}{f(k,1)}=\frac{g'(k)c(t)}{g(k)},$$

by analyzing phase diagrams in (k, s) space.

Here, of course,

$$g(k)=f(k,1)$$

We use the first-order conditions

$$\frac{\dot{c}}{c} = g(k) - \rho$$

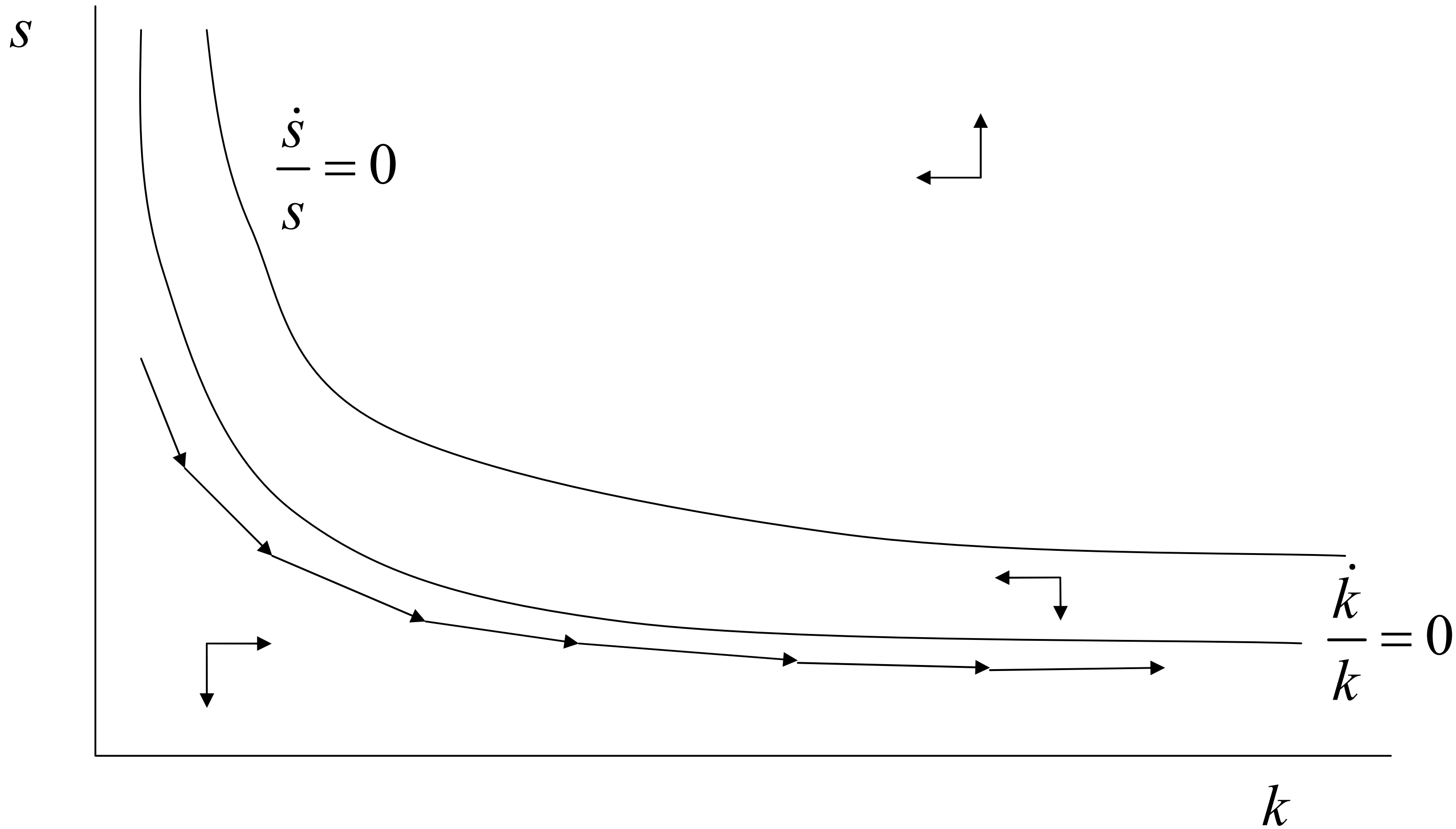
$$\frac{\dot{k}}{k} = \frac{g(k)}{k} - \frac{c}{k}$$

to obtain

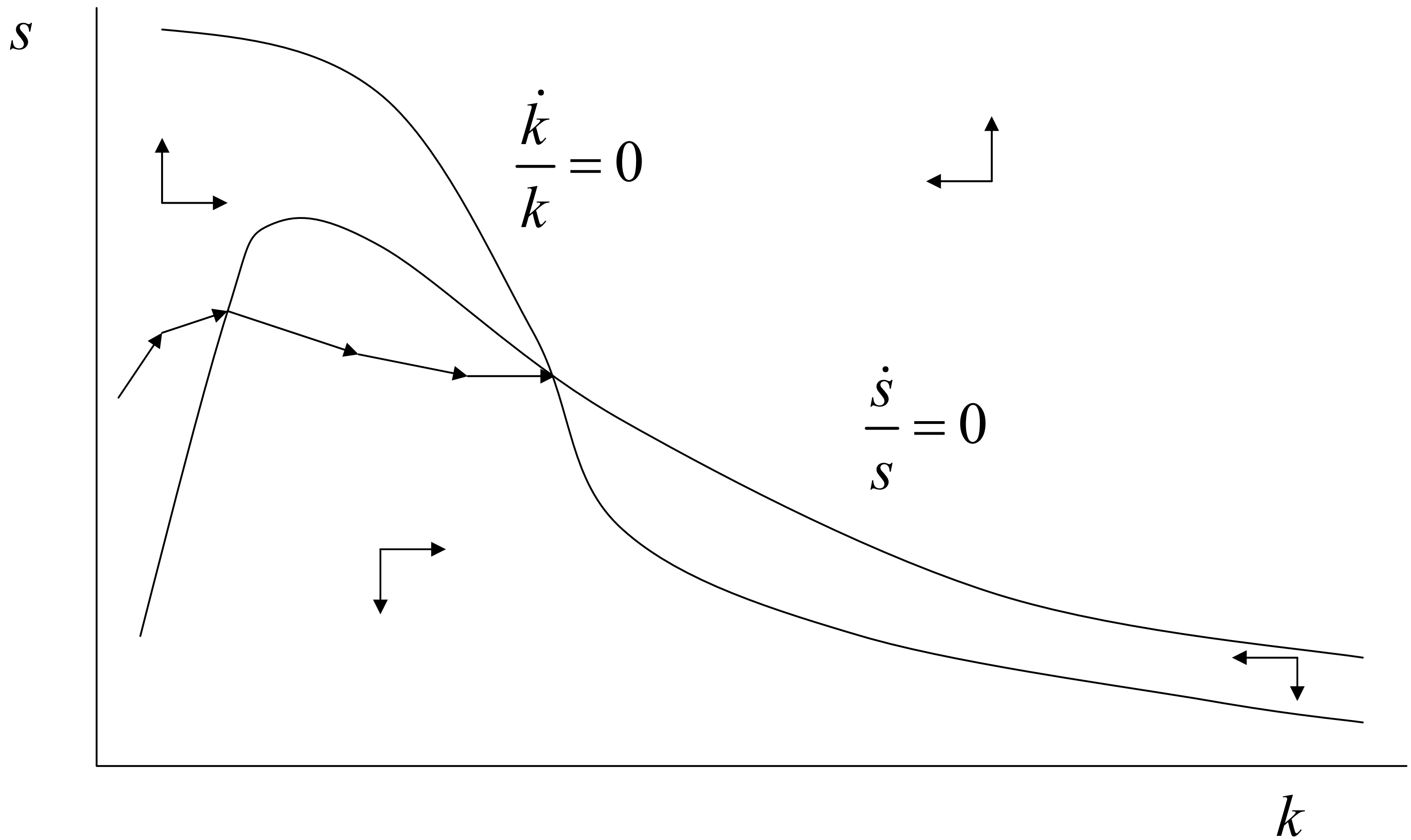
$$\frac{\dot{s}}{s} = g'(k) - \rho - \left(\frac{g'(k)^2 - g(k)g''(k)}{g'(k)^2} \right) \left(g'(k) - s \right)$$

$$\frac{\dot{k}}{k} = \frac{g(k)}{g'(k)k} \left(g'(k) - s \right)$$

$$b > 0$$



$$b < 0$$



$b << 0$ and $\delta > 0$