Notes on McCall's Model of Job Search

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$F(v) = \text{prob}(w \le v)$, $w \in [0, B]$.

Choice: accept wage offer w or receive b and search again next period.

An unemployed worker solves

$$\max E \sum_{t=0}^{\infty} \beta^t y_t$$

where

$$y_t = \begin{cases} w & \text{if job offer has been accepted} \\ b & \text{if searching} \end{cases}.$$

The Bellman equation for an unemployed worker:

$$V(w) = \max \left\{ \frac{w}{1-\beta}, b + \beta E V(w') \right\}$$

$$V(w) = \max \left\{ \frac{w}{1-\beta}, b + \beta \int_0^B V(w') dF(w') \right\}.$$

Suppose that we have solved this problem and found $V(w)$. Then

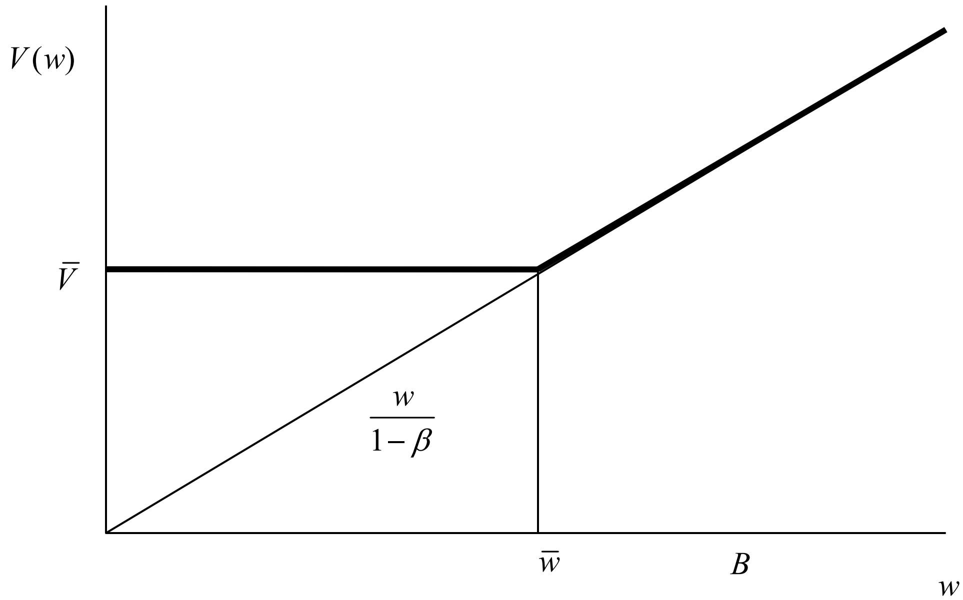
$$\bar{V} = b + \beta \int_0^B V(w') dF(w')$$

is just a constant. Let $\bar{w}$ be such that

$$\frac{\bar{w}}{1-\beta} = \bar{V} = b + \beta \int_0^B V(w') dF(w').$$

Then we can graph the value function and characterize the optimal decision by the unemployed worker as turn down wage offers $w < \bar{w}$ and accept wage offers $w \ge \bar{w}$.

If we can find $\bar{w}$ that satisfies the above relationship, we have found $V(w)$.



$$\frac{\bar{w}}{1-\beta} = b + \beta\int_0^{\bar{w}} \bar{V} dF(w') + \beta\int_{\bar{w}}^{B} \frac{w'}{1-\beta} dF(w')$$
$$\frac{\bar{w}}{1-\beta} = b + \beta\int_0^{\bar{w}} \frac{\bar{w}}{1-\beta} dF(w') + \beta\int_{\bar{w}}^{B} \frac{w'}{1-\beta} dF(w') .$$

Since

$$1 = \int_0^B dF(w') = \int_0^{\bar{w}} dF(w') + \int_{\bar{w}}^{B} dF(w')$$

we can write

$$\frac{\bar{w}}{1-\beta}\int_0^{\bar{w}} dF(w') + \frac{\bar{w}}{1-\beta}\int_{\bar{w}}^{B} dF(w') = b + \beta\int_0^{\bar{w}} \frac{\bar{w}}{1-\beta} dF(w') + \beta\int_{\bar{w}}^{B} \frac{w'}{1-\beta} dF(w')$$
$$\frac{(1-\beta)\bar{w}}{1-\beta}\int_0^{\bar{w}} dF(w') - b = \beta\int_{\bar{w}}^{B} \frac{w'}{1-\beta} dF(w') - \frac{\bar{w}}{1-\beta}\int_{\bar{w}}^{B} dF(w')$$
$$\bar{w}\int_0^{\bar{w}} dF(w') - b = \frac{1}{1-\beta}\int_{\bar{w}}^{B} (\beta w' - \bar{w}) dF(w') .$$

Adding

$$\bar{w}\int_{\bar{w}}^{B} dF(w') = \frac{(1-\beta)\bar{w}}{1-\beta}\int_{\bar{w}}^{B} dF(w')$$

to both sides of this equation, we obtain

$$\overline{w}-b=\frac{\beta}{1-\beta}\int_{\overline{w}}^{B}(w'-\overline{w})dF(w').$$

$\overline{w}-b$ is the cost of turning down a wage offer $\overline{w}$ to continue searching, and $\frac{\beta}{1-\beta}\int_{\overline{w}}^{B}(w'-\overline{w})dF(w')$ is the expected discounted benefit of turning down a wage offer $\overline{w}$ to continue searching.

Let

$$h(w)=\frac{\beta}{1-\beta}\int_{w}^{B}(w'-w)dF(w').$$

Then

$$\overline{w}-b=h(\overline{w}).$$

Notice that

$$h(0)=\frac{\beta Ew}{1-\beta}$$
$$h(B)=0$$
$$h'(w)=-\frac{\beta}{1-\beta}\left(1-F(w)\right)<0$$
$$h''(w)=\frac{\beta}{1-\beta}F'(w)>0.$$

Note: To find the expression for $h'(w)$, we apply Leibnitz's rule for differentiating functions with integrals,

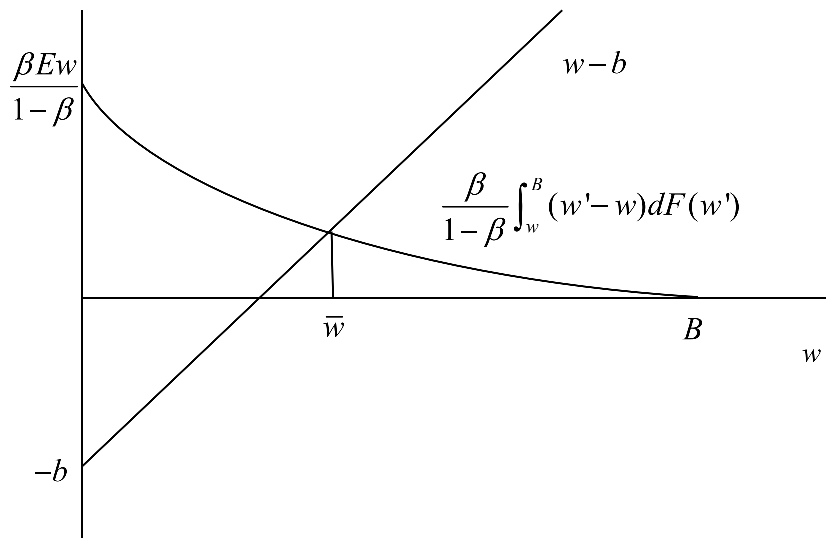
$$g(x)=\int_{a(x)}^{b(x)}f(x,y)dy$$
$$g'(x)=f(x,b(x))b'(x)-f(x,a(x))a'(x)+\int_{a(x)}^{b(x)}\frac{\partial f(x,y)}{\partial x}dy.$$

to $h(w)$:

$$h(w)=\frac{\beta}{1-\beta}\int_{w}^{B}(w'-w)dF(w')$$
$$h'(w)=-\frac{\beta}{1-\beta}(w-w)-\frac{\beta}{1-\beta}\int_{w}^{B}dF(w')$$

$$h'(w) = -\frac{\beta}{1-\beta}\left(1 - F(w)\right).$$

Given these characteristics of $h(w)$, we can draw a graph illustrate how $\overline{w}$ is determined:



Increase in unemployment benefits

Comparative statics: An increase in b leads to an increase in $\overline{w}$. We can see this by shifting the line $\overline{w}-b$ in the graph downward.

We can also do this algebraically by applying the implicit function theorem:

$$\overline{w} - b = \frac{\beta}{1-\beta}\int_{\overline{w}}^{B} (w' - \overline{w})dF(w')$$
$$\overline{w}(b) - b = \frac{\beta}{1-\beta}\int_{\overline{w}(b)}^{B} (w' - \overline{w}(b))dF(w')$$
$$\overline{w}(b) - b = h(\overline{w}(b))$$
$$\overline{w}'(b) - 1 = h'(\overline{w}(b))\overline{w}'(b)$$
$$\left(1 - h'(\overline{w}(b))\right)\overline{w}'(b) = 1$$
$$\overline{w}'(b) = \frac{1}{1 - h'(\overline{w}(b))} > 0.$$

Example

Uniform distributions on $[r, B-r]$ with $B/2 > r > 0$.

$$F(w,r) = \begin{cases} 0 & \text{if } w \le r \\ \dfrac{w-r}{B-2r} & \text{if } B-r \ge w \ge r \\ 1 & \text{if } w \ge B-r \end{cases}.$$

For $\overline{w} < r$,

$$h(w) = \frac{\beta}{1-\beta}\left(\int_w^B w' dF(w') - \int_w^B w dF(w')\right)$$
$$h(w) = \frac{\beta}{1-\beta}\left(\int_r^{B-r} \frac{w'}{B-2r} dF(w') - w\int_w^B dF(w')\right)$$
$$h(w) = \frac{\beta}{1-\beta}\left(\frac{B}{2} - w\right)$$
$$h(w) = \frac{\beta(B-2w)}{2(1-\beta)}.$$

For $B-r \ge w \ge r$,

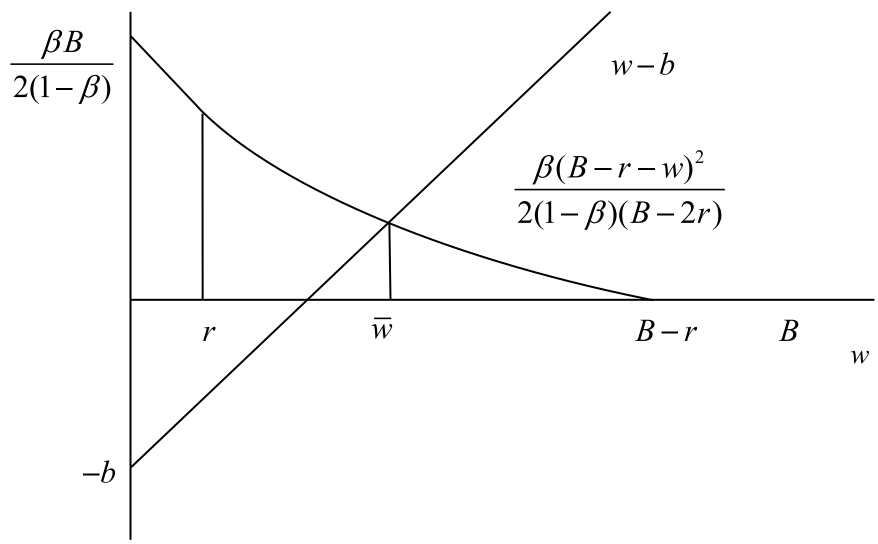
$$h(w) = \frac{\beta}{1-\beta}\int_w^{B-r} \frac{1}{B-2r}(w'-w)dF(w')$$
$$h(w) = \frac{\beta}{(1-\beta)(B-2r)}\left(\frac{(w')^2}{2} - ww'\right)\Bigg|_w^{B-r}$$
$$h(w) = \frac{\beta}{(1-\beta)(B-2r)}\left(\frac{(B-r)^2}{2} - w(B-r) - \frac{w^2}{2} + w^2\right)$$
$$h(w) = \frac{\beta}{2(1-\beta)(B-2r)}\left((B-r)^2 - 2w(B-r) + w^2\right)$$
$$h(w) = \frac{\beta(B-r-w)^2}{2(1-\beta)(B-2r)}.$$

For $w \ge B-r$,

$$h(w) = 0.$$

Consequently,

$$h(w) = \begin{cases} \dfrac{\beta(B-2w)}{2(1-\beta)} & \text{if } w \le r \\ \dfrac{\beta(B-r-w)^2}{2(1-\beta)(B-2r)} & \text{if } B-r \ge w \ge r \\ 0 & \text{if } w \ge B-r \end{cases}$$



Check properties of $h(w)$:

$$h(0) = \frac{\beta B}{2(1-\beta)}$$

$$h(B) = h(B-r) = 0$$

$$h'(w) = \begin{cases} -\dfrac{\beta}{(1-\beta)} & \text{if } w \le r \\ -\dfrac{\beta(B-r-w)}{(1-\beta)(B-2r)} & \text{if } B-r \ge w \ge r \\ 0 & \text{if } w \ge B-r \end{cases}$$

$$h''(w) = \begin{cases} 0 & \text{if } w \ge r \\ \dfrac{\beta}{(1-\beta)(B-2r)} & \text{if } B-r \ge w \ge r\,. \\ 0 & \text{if } w \ge B-r \end{cases}$$

Notice that $h(w)$ and $h'(w)$ — but not $h''(w)$ — are continuous at $w = r$ and at $w = B - r$.

Digression on mean preserving spreads

Another expression for the mean of a random variable:

$$Ew = \int_0^B w dF(w)$$

Integration by parts:

$$\int_a^b v du = uv\Big|_a^b - \int_a^b u dv$$
$$\int_0^B (1 - F(w)) dw = w(1 - F(w))\Big|_0^B + \int_0^B w dF(w)$$
$$\int_0^B w dF(w) = \int_0^B (1 - F(w)) dw$$
$$Ew = B - \int_0^B F(w) dw.$$

Class of distributions that depend on a parameter r:

$$Ew = B - \int_0^B F(w, r) dw.$$

Requirement that $F(w, r_1)$ and $F(w, r_2)$ have the same mean:

(1) $$\int_0^B \left(F(w, r_1) - F(w, r_2) \right) dw = 0.$$

Single crossing property:

Let W be the subset of $[0, B]$ that is the union of the supports of $F(w, r_1)$ and $F(w, r_2)$. Then $F(w, r_1)$ and $F(w, r_2)$ have the single crossing property if there exists $\bar{w}$ such that

(2) $$F(w, r_2) - F(w, r_1) < 0 \text{ if } w \geq \bar{w} \text{ and } w \in W$$
$$F(w, r_2) - F(w, r_1) > 0 \text{ if } w \leq \bar{w} \text{ and } w \in W.$$

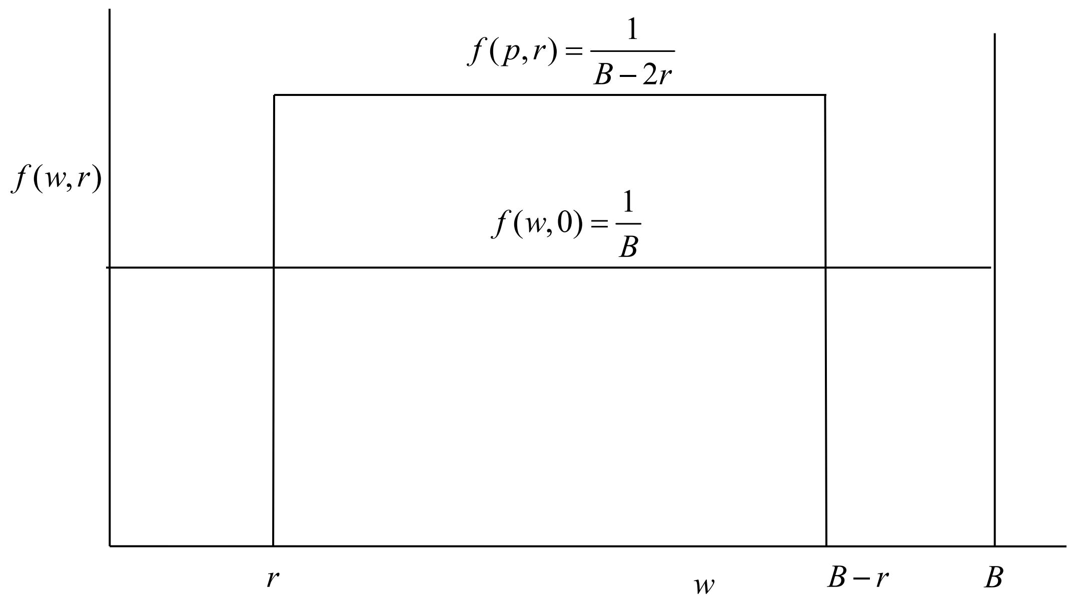
Example

The class of uniform distributions on $[r, B - r]$ with $B/2 > r \geq 0$.

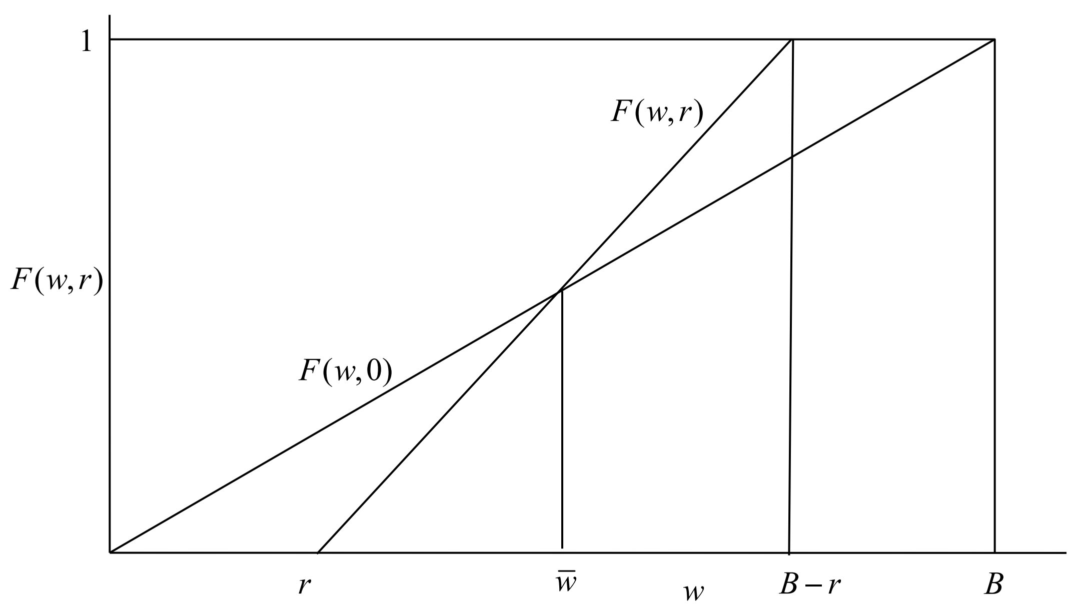
These distributions have the same mean, $Ew = B/2$. They also satisfy the single crossing property.

Let us compare $F(w,0)$ with $F(w,r)$, $B/2 > r > 0$. Here $r_1 = r$ and $r_2 = 0$ in terms of the equal mean property (1) and the single crossing property (2).

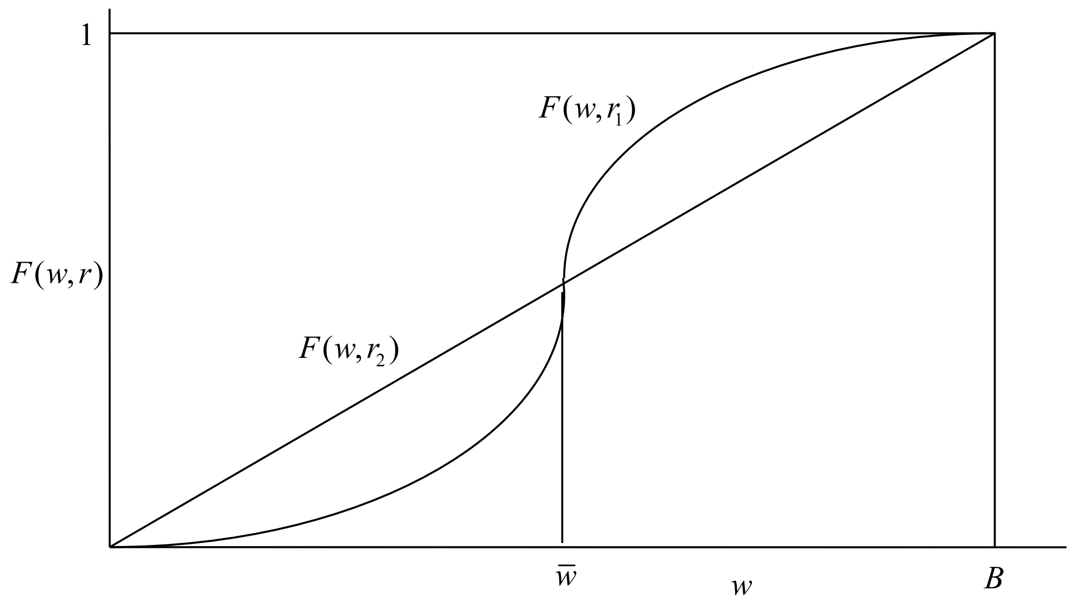
Density function $f(w,r) = dF(w,r)$:



Distribution function $F(w,r)$:



Mean preserving spreads



Properties (1) and (2) imply properties (1) and

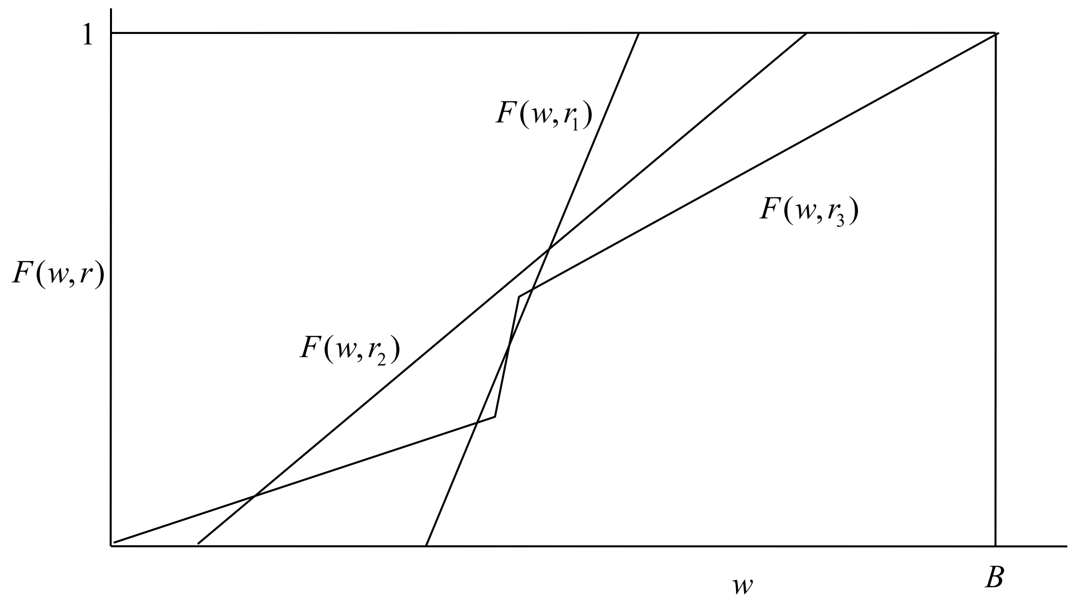
(3) $$\int_0^v \left(F(w,r_2) - F(w,r_1)\right) dw > 0 \text{ for all } v,\ v \in W .$$

Properties (1) and (3) do not imply property (1) and (2), however. That is, when combined with the equal mean property (1), property (3) is weaker than the single crossing property (2).

Rothschild and Stiglitz (1970) say that, if $F(w,r_1)$ and $F(w,r_2)$ satisfy properties (1) and (3), then $F(w,r_2)$ is a mean preserving spread of $F(w,r_1)$. Equivalently, we can also say that $F(w,r_1)$ is a mean preserving contraction of $F(w,r_2)$.

Rothschild and Stiglitz (1970) use properties (1) and (3) — rather than the stronger properties (1) and (2) — because properties (1) and (3) generate a transitive partial ordering of distributions $F(w,r)$: If $F(w,r_2)$ is a mean preserving spread of $F(w,r_1)$ and $F(w,r_3)$ is a mean preserving spread of $F(w,r_2)$, then $F(w,r_3)$ is a mean preserving spread of $F(w,r_1)$. This is not true if we use properties (1) and (2). (The ordering is partial because, for two arbitrary distributions. $F(w,r_2)$ is not necessarily either a mean preserving spread or a mean preserving contraction of $F(w,r_1)$.)

To see that (1) and (2) do not define a transitive partial order, we draw a graph in which $F(w,r_3)$ crosses $F(w,r_2)$ once, and $F(w,r_2)$ crosses $F(w,r_1)$ once, but $F(w,r_3)$ crosses $F(w,r_1)$ more than once.



The concept of mean preserving spread is attractive because, as Rothschild and Stiglitz (1971) show, the following three definitions of an increase in riskiness of a draw are equivalent:

1. Any risk averse worker prefers to take a draw from $F(w,r_1)$ to taking a draw from $F(w,r_2)$.

2. $F(w,r_2)$ assigns more probability to its tails than does $F(w,r_1)$.

3. $F(w,r_2)$ equals $F(w,r_1)$ plus noise.

If we were to define $F(w,r_2)$ as riskier than $F(w,r_1)$ if $F(w,r_2)$ had the same mean as $F(w,r_1)$ but a higher variance, then we would have a transitive partial ordering, but it would not have these three attractive properties.

Increase in risk in the labor market

Rewrite

$$\bar{w}-b=\frac{\beta}{1-\beta}\int_{\bar{w}}^{B}(w'-\bar{w})dF(w')$$

as

$$\bar{w}-b=\frac{\beta}{1-\beta}\int_{\bar{w}}^{B}(w'-\bar{w})dF(w')+\frac{\beta}{1-\beta}\int_{0}^{\bar{w}}(w'-\bar{w})dF(w')-\frac{\beta}{1-\beta}\int_{0}^{\bar{w}}(w'-\bar{w})dF(w')$$

$$\bar{w}-b=\frac{\beta}{1-\beta}\int_{0}^{B}(w'-\bar{w})dF(w')-\frac{\beta}{1-\beta}\int_{0}^{\bar{w}}(w'-\bar{w})dF(w')$$

$$\bar{w}-b=\frac{\beta Ew}{1-\beta}-\frac{\beta\bar{w}}{1-\beta}-\frac{\beta}{1-\beta}\int_{0}^{\bar{w}}(w'-\bar{w})dF(w')$$

$$\bar{w}-(1-\beta)b=\beta Ew-\beta\int_{0}^{\bar{w}}(w'-\bar{w})dF(w')\,.$$

Integration by parts:

$$\int_a^b vdu=uv\Big|_a^b-\int_a^b udv$$

$$\beta\int_0^{\bar{w}}(w'-\bar{w})dF(w')=\beta(w'-\bar{w})F(w')\Big|_0^{\bar{w}}-\beta\int_0^{\bar{w}}F(w')dw'$$

$$\beta\int_0^{\bar{w}}(w'-\bar{w})dF(w')=-\beta\int_0^{\bar{w}}F(w')dw'\,.$$

Therefore

$$\bar{w}-(1-\beta)b=\beta Ew+\beta\int_0^{\bar{w}}F(w')dw'$$

$$\bar{w}-b=\beta(Ew-b)+\beta\int_0^{\bar{w}}F(w')dw'\,.$$

Let

$$g(w)=\int_0^{w}F(w')dw'\,.$$

Then

$$\bar{w}-b=\beta(Ew-b)+\beta g(w)\,.$$

Notice that

$$g(0)=0$$
$$g(w)\geq 0$$
$$1\geq g'(w)=F(w)\geq 0$$
$$g''(w)=F'(w)\geq 0\,.$$

For distribution functions in the class $F(w,r)$,

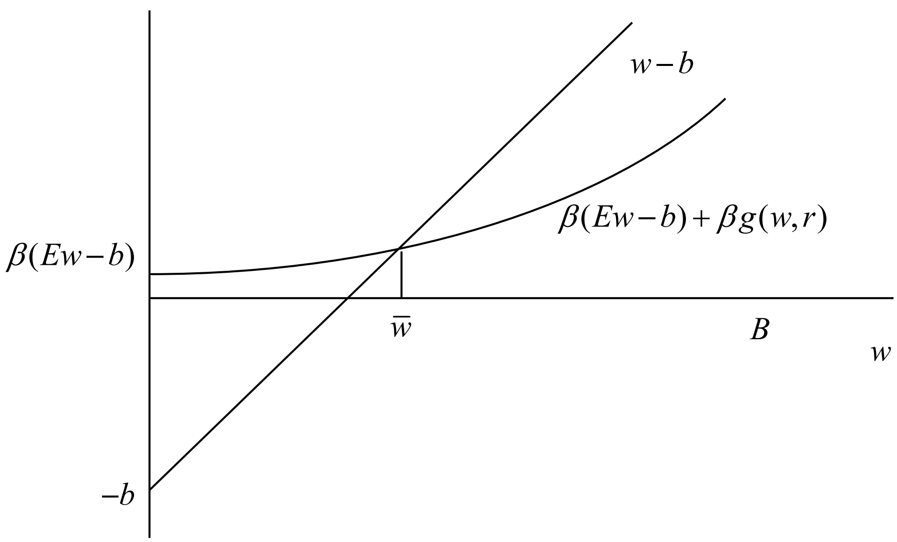
$$g(w,r)=\int_0^w F(w',r)dw'.$$

Notice that, if $F(w,r_2)$ is a mean preserving spread of $F(w,r_1)$, then

$$\int_0^w \left(F(w',r_2)-F(w',r_1)\right)dw'>0$$
$$\int_0^w F(w',r_2)dw'>\int_0^w F(w',r_1)dw'$$
$$g(w,r_2)>g(w,r_1) \text{ for all } w\in W.$$

To determine $\overline{w}$ we solve

$$\overline{w}-b=\beta(Ew-b)+\beta\int_0^{\overline{w}} F(w',r)dw'$$
$$\overline{w}-b=\beta(Ew-b)+\beta g(\overline{w},r)$$



Comparative statics: An increase in risk leads to an increase in $\overline{w}$. We can see this by shifting the curve $\beta(Ew-b)+\beta g(\overline{w},r)$ in the graph upward.

Suppose that a decrease in r is a mean preserving spread. The differential versions of properties (1) and (3) are

(4) $$\int_0^B \frac{\partial F(w,r)}{\partial r}dw=0$$

(5) $$\int_0^v \frac{\partial F(w,r)}{\partial r}dw<0 \text{ for all } v\in W.$$

Be careful: We are following the opposite convention as Ljungqvist and Sargent where an increase in r is a mean preserving spread. We are doing so because it matches our example with uniform distributions.

We can also do this algebraically by applying the implicit function theorem:

$$\begin{aligned}
\overline{w} - b &= \beta(Ew - b) + \beta\int_0^{\overline{w}} F(w', r)dw' \\
\overline{w}(r) - b &= \beta(Ew - b) + \beta\int_0^{\overline{w}(b)} F(w', r)dw' \\
\overline{w}'(r) &= \beta F(\overline{w}(r), r)\overline{w}'(r) + \beta\int_0^{\overline{w}(r)} \frac{\partial F(w', r)}{\partial r}dw' \\
\left(1 - \beta F(\overline{w}(r), r)\right)\overline{w}'(r) &= \beta\int_0^{\overline{w}(r)} \frac{\partial F(w', r)}{\partial r}dw' .
\end{aligned}$$

Notice that $\beta F(\overline{w}(r), r) \le \beta < 1$, which implies that

$$\overline{w}'(r) = \frac{\beta\int_0^{\overline{w}(r)} \frac{\partial F(w', r)}{\partial r}dw'}{1 - \beta F(\overline{w}(r), r)} \le 0 .$$

An increase in risk increases the reservation wage and increases the expected utility of an unemployed worker.

Example

Uniform distributions on $[r, B - r]$ with $B/2 > r > 0$.

For $\overline{w} < r$,

$$g(w) = 0 .$$

For $B - r \ge w \ge r$,

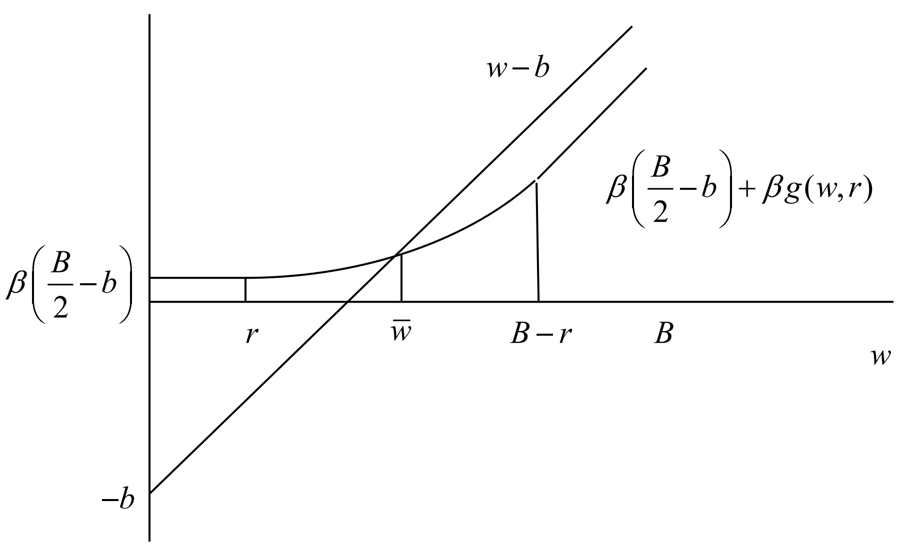
$$\begin{aligned}
g(w) &= \int_r^w \frac{w'}{B - 2r}dw' \\
g(w) &= \left.\frac{(w')^2}{2(B - 2r)}\right|_r^w \\
g(w) &= \frac{w^2 - r^2}{2(B - 2r)} .
\end{aligned}$$

For $w \geq B-r$,

$$g(w)=\int_r^{B-r}\frac{w'}{B-2r}dw'+\int_{B-r}^{w}dw'$$
$$g(w)=\frac{(B-r)^2-r^2}{2(B-2r)}+w-(B-r)$$
$$g(w)=w+\frac{B^2+r^2-2Br-r^2-2B^2-4r^2+6Br}{2(B-2r)}$$
$$g(w)=w-\frac{B-2r}{2}.$$

Consequently,

$$g(w)=\begin{cases}0 & \text{if } w\leq r\\ \dfrac{w^2-r^2}{2(B-2r)} & \text{if } B-r\geq w\geq r\\ w-\dfrac{B-2r}{2} & \text{if } w\geq B-r\end{cases}$$



Check properties of $g(w)$:

$$g(0)=0$$
$$g(w)>0 \text{ for all } w\in[r,B-r].$$

$$g'(w) = \begin{cases} 0 & \text{if } w \le r \\ \dfrac{w}{B-2r} & \text{if } B-r \ge w \ge r \\ 1 & \text{if } w \ge B-r \end{cases}$$

$$g''(w) = \begin{cases} 0 & \text{if } w \ge r \\ \dfrac{1}{B-2r} & \text{if } B-r \ge w \ge r\,. \\ 0 & \text{if } w \ge B-r \end{cases}$$

Notice that $g(w)$ and $g'(w)$ — but not $g''(w)$ — are continuous at $w = r$ and at $w = B - r$.

Mean waiting time

Let $\bar{n}$ be the average number of periods that a worker is unemployed. Since F is constant and the policy rule that the unemployed worker follows is stationary, $\bar{n}$ is the expected waiting time for the worker to find a job no matter how may periods the worker has already been searching.

Let

$$\pi = \int_0^{\bar{w}} dF(w)$$

be the probability that a job offer is rejected.

Then

$$\bar{n} = (1-\pi) + \pi(1+\bar{n})\,.$$

That is, in period t the expected waiting time $\bar{n}$ can be decomposed into the probability $(1-\pi)$ of accepting the job offer in period $t+1$, in which case the waiting time will have been 1, and the probability π of rejecting the job offer in period $t+1$, in which case the worker will have waited one period and now again have the expected waiting time $\bar{n}$.

$$\bar{n} = \frac{1}{1-\pi}\,.$$

Notice that the higher is $\bar{w}$, the higher is π and the higher is $\bar{n}$.

Quits

Suppose a worker has a job that pays $w > \bar{w}$ and quits. The next period, her expected utility is

$$\bar{V} = b + \beta\int_0^B V(w')dF(w') = \frac{\bar{w}}{1-\beta} < \frac{w}{1-\beta}.$$

That is, quitting lowers utility.

Fires

Suppose that once a worker has accepted a job offer, she faces a constant probability δ, $1 > \delta \geq 0$, of being fired.

The Bellman equation for an unemployed worker:

$$V(w) = \max\left\{w + \beta(1-\delta)V(w) + \beta\delta\left(b + \int_0^B V(w')dF(w')\right), b + \beta\int_0^B V(w')dF(w')\right\}.$$

Let $C([0,B])$ be the set of continuous bounded functions on $[0,B]$. Define the operator $T : C([0,B]) \to C([0,B])$ by the rule

$$T(V)(w) = \max\left\{w + \beta(1-\delta)V(w) + \beta\delta\left(b + \int_0^B V(w')dF(w')\right), b + \beta\int_0^B V(w')dF(w')\right\}.$$

Notice that, $w + \beta(1-\delta)V(w) + \beta\delta\left(b + \int_0^B V(w')dF(w')\right)$ is increasing in w if $V(w)$ is increasing in w, while $b + \beta\int_0^B V(w')dF(w')$ does not depend on w. Consequently, T maps increasing functions into increasing functions. This implies that the optimal $V(w)$ is increasing.

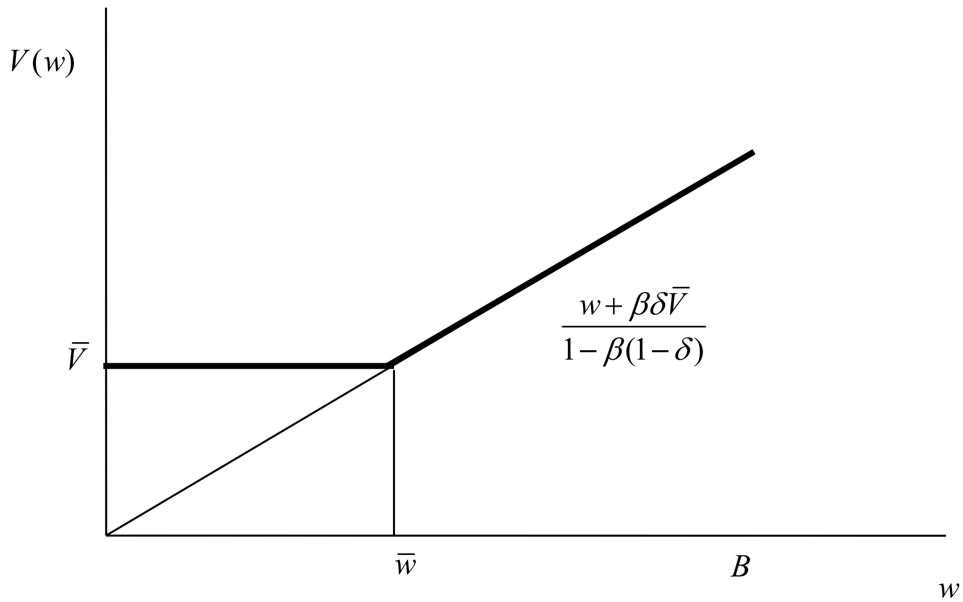
Once again, suppose that we have solved for V and let $\bar{V}$ be the constant value of being unemployed:

$$\bar{V} = b + \beta\int_0^B V(w')dF(w').$$

Since $V(w)$ is increasing, we can characterize the value function as before:

Here

$$V(w) = \begin{cases} \dfrac{w + \beta\delta\left(b + \beta\int_0^B V(w')dF(w')\right)}{1-\beta(1-\delta)} & \text{if } w \ge \overline{w} \\ b + \beta\int_0^B V(w')dF(w') & \text{if } w \le \overline{w} \end{cases} .$$



Let

$$\frac{\overline{w} + \beta\delta\overline{V}}{1-\beta(1-\delta)} = \overline{V}$$

$$\overline{w} + \beta\delta\overline{V} = \left(1-\beta(1-\delta)\right)\overline{V}$$

$$\frac{\overline{w}}{1-\beta} = \overline{V} .$$

Once again, the optimal decision of the unemployed worker is to turn down wage offers $w < \overline{w}$ and to accept wage offers $w \ge \overline{w}$.

Following the same steps as in the model with $\delta = 0$, we find

$$\frac{\overline{w}}{1-\beta} = b + \beta\int_0^{\overline{w}} \frac{\overline{w}}{1-\beta} dF(w') + \beta\int_{\overline{w}}^B \frac{w' + \beta\delta\left(\dfrac{\overline{w}}{1-\beta}\right)}{1-\beta(1-\delta)} dF(w')$$

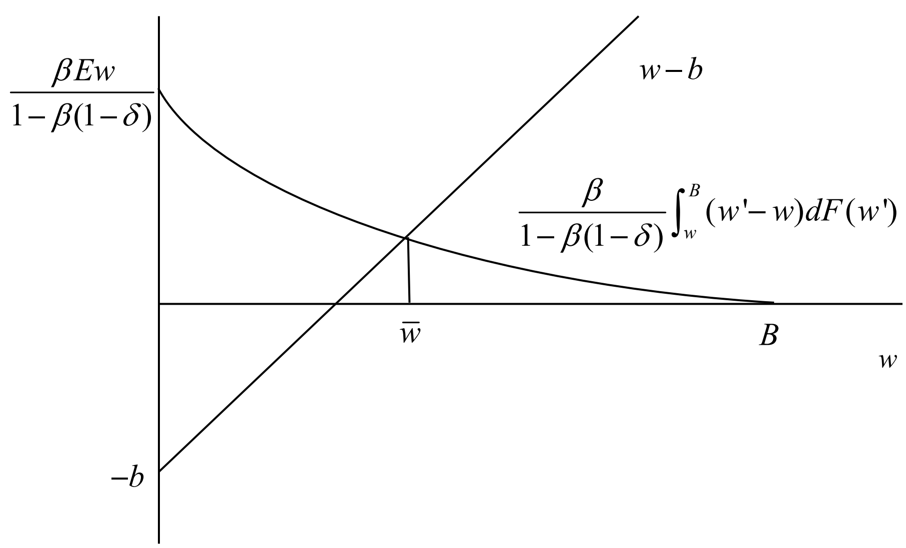
$$\frac{\overline{w}}{1-\beta}\int_0^{\overline{w}} dF(w') + \frac{\overline{w}}{1-\beta}\int_{\overline{w}}^{B} dF(w') = b + \beta\int_0^{\overline{w}} \frac{\overline{w}}{1-\beta} dF(w') + \beta\int_{\overline{w}}^{B} \frac{w'+\beta\delta\left(\frac{\overline{w}}{1-\beta}\right)}{1-\beta(1-\delta)} dF(w')$$

$$\overline{w}\int_0^{\overline{w}} dF(w') = b + \beta\int_{\overline{w}}^{B}\left(\frac{w'+\beta\delta\left(\frac{\overline{w}}{1-\beta}\right)}{1-\beta(1-\delta)} - \frac{\overline{w}}{\beta(1-\beta)}\right) dF(w')$$

$$\overline{w}\int_0^{\overline{w}} dF(w') + \overline{w}\int_{\overline{w}}^{B} dF(w') = b + \beta\int_{\overline{w}}^{B}\left(\frac{w'+\beta\delta\left(\frac{\overline{w}}{1-\beta}\right)}{1-\beta(1-\delta)} - \frac{\overline{w}}{\beta(1-\beta)} + \frac{\overline{w}}{\beta}\right) dF(w')$$

$$\overline{w} - b = \frac{\beta}{1-\beta(1-\delta)}\int_{\overline{w}}^{B} (w'-\overline{w}) dF(w')$$

$\overline{w}-b$ is the cost of turning down a wage offer $\overline{w}$ to continue searching, and $\frac{\beta}{1-\beta(1-\delta)}\int_{\overline{w}}^{B} (w'-\overline{w}) dF(w')$ is the expected discounted benefit of turning down a wage offer $\overline{w}$ to continue searching.



We can analyze the impact of an increase in unemployment benefits b or of a mean preserving spread in F as in the model with $\delta = 0$ and obtain similar results.

Comparative statics: An increase in δ leads to an decrease in $\overline{w}$. We can see this by shifting the curve $\beta/(1-\beta(1-\delta))\int_{w}^{B}(w'-w)dF(w')$ in the graph downward.

Algebraically, we can apply the implicit function theorem:

$$\overline{w}-b=\frac{\beta}{1-\beta(1-\delta)}\int_{\overline{w}}^{B}(w'-\overline{w})dF(w')$$
$$\overline{w}(\delta)-b=\frac{\beta}{1-\beta(1-\delta)}\int_{\overline{w}(\delta)}^{B}(w'-\overline{w}(\delta))dF(w')$$

$$\overline{w}'(\delta)=-\frac{\beta^2}{(1-\beta(1-\delta))^2}\int_{\overline{w}(\delta)}^{B}(w'-\overline{w}(\delta))dF(w')-\frac{\beta}{1-\beta(1-\delta)}\left(1-F(\overline{w}(\delta)\right)\overline{w}'(\delta)$$
$$\left(1+\frac{\beta}{1-\beta(1-\delta)}\left(1-F(\overline{w}(\delta)\right)\right)\overline{w}'(\delta)=-\frac{\beta^2}{(1-\beta(1-\delta))^2}\int_{\overline{w}(\delta)}^{B}(w'-\overline{w}(\delta))dF(w')$$
$$\overline{w}'(\delta)=-\frac{\beta^2\int_{\overline{w}(\delta)}^{B}(w'-\overline{w}(\delta))dF(w')}{(1-\beta(1-\delta))^2+\beta(1-\beta(1-\delta))\left(1-F(\overline{w}(\delta)\right)}<0\,.$$

An increase in δ decreases the expected utility of both unemployed workers and employed workers.

Unemployment rate

Suppose that there is a continuum of ex ante identical workers who move between periods of employment and unemployment:

$$u_{t+1}=\delta(1-u_t)+F(\overline{w})u_t$$

In the stationary solution

$$u_{t+1}=u_t=\hat{u}$$
$$\hat{u}=\delta(1-\hat{u})+F(\overline{w})\hat{u}$$
$$\hat{u}=\frac{\delta}{1+\delta-F(\overline{w})}.$$

We have already calculated the mean duration of unemployment as

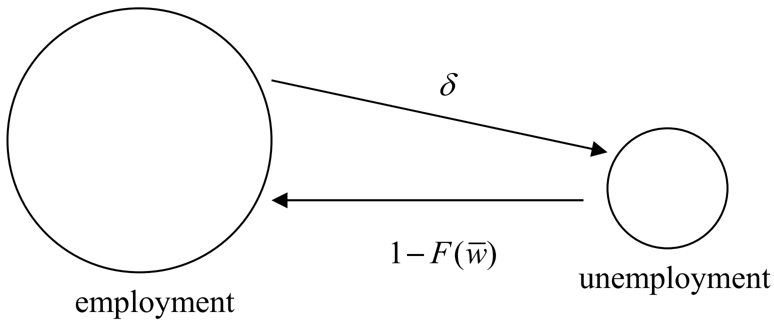
$$n_u=\frac{1}{1-F(\overline{w})}.$$

The mean duration of employment can be found by solving

$$n_e=(1-\delta)(1+n_e)+\delta$$

$$n_e = \frac{1}{\delta}.$$

Notice that flows in and out of employment are governed by a stationary Markov chain:



$$\Pi = \begin{bmatrix} 1-\delta & \delta \\ 1-F(\overline{w}) & F(\overline{w}) \end{bmatrix}.$$

The invariant distribution is

$$\hat{\pi} = \begin{bmatrix} 1-\hat{u} \\ \hat{u} \end{bmatrix} = \begin{bmatrix} \dfrac{1-F(\overline{w})}{1+\delta-F(\overline{w})} \\ \dfrac{\delta}{1+\delta-F(\overline{w})} \end{bmatrix}.$$

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