GROWTH AND INTERDEPENDENCE*

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Two of the most interesting facts of the postwar international growth experience are (1) the conditional convergence finding that, after controlling for measures of education and government policies, poor countries tend to grow faster than rich ones; and (2) a small group of export-oriented economies in East Asia have been able to grow at rates that are so high that they defy historical comparisons. This paper shows that it is possible to explain these facts by combining a weak form of the factor-price-equalization theorem of international trade with the Ramsey model of economic growth.

Two of the most interesting facts of the postwar international growth experience are the conditional convergence finding and the East Asian Miracle. The conditional convergence finding consists of the fact that, after controlling for measures of education and government policies, poor countries tend to grow faster than rich ones (See Barro [1991]). The East Asian Miracle consists of the fact that, for more than three decades, a group of small export-oriented economies have been able to grow at rates that are so high that they defy historical comparisons (see World Bank [1993]). This paper shows that it is possible to explain these facts by combining a weak form of the factor-price-equalization theorem of international trade with the Ramsey model of economic growth.

Existing explanations of the conditional convergence finding rely on two premises: (1) holding constant education levels and government policies, rates of return to investment are negatively related to the level of income of a country; and (2) the degree of economic integration among countries is low. Since countries that exhibit high rates of return to investment also have high growth rates, the conditional convergence finding follows from the first premise. Since one has to postulate large rate-of-return differentials to explain observed variation in growth rates, the second premise is needed to ensure that these differentials are not arbitraged away. There is much disagreement, however, regarding

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the source of these rate-of-return differentials. In models that emphasize capital accumulation, poor countries have high rates of return because capital is scarce. In models that emphasize innovation and technology diffusion, poor countries have high rates of return because imitation of existing goods or production processes is assumed to be cheaper than innovation. In any case, both of these explanations consist of first establishing the existence of some sort of law of diminishing returns to investment and then claiming that a low degree of economic integration translates these diminishing returns into rate-of-return differentials.[1] (See Barro and Sala-i-Martin [1995].)

One problem with these explanations is that they rely heavily on the inability of investors to exploit arbitrage opportunities. While existing evidence shows that international capital flows are small, this is not enough to conclude that there can be large rate-of-return differentials across countries. One of the oldest results in the modern theory of international trade is Samuelson's [1948] factor-price-equalization theorem establishing conditions under which foreign trade equalizes factor prices across countries, even in the absence of international factor movements.[2] Although initially developed within the factor proportions theory of trade, the factor-price-equalization theorem also holds in many of the models of imperfect competition and increasing returns that were developed to explain trade among industrialized countries (see Helpman and Krugman [1985]). Moreover, Trefler [1993] has recently shown that a weak form of the factor-price-equalization theorem that allows for factor-augmenting international productivity differences is empirically consistent with

1. Do these rate-of-return differentials exist? A number of studies, surveyed by Frankel [1991], have compared realized returns of two assets that are very similar in two different locations. Assume that these returns differ. Is this evidence of unexploited arbitrage opportunities? Or have realized returns differed from expected ones? Or are these assets not really perfect substitutes, and their price differential reflects just that? To be fair, the issue of whether these rate-of-return differentials exist has not yet been resolved.

2. Using data for the OECD economies during the period 1960–1974, Feldstein and Horioka [1980] showed that net capital flows are small; i.e., there exists an almost perfect correlation between domestic savings and investment. This result is robust to changes in the sample size and period as well as the estimation procedure (see Frankel [1991]). Many have interpreted this finding as evidence of large costs of capital flows. However, it is possible that the Feldstein-Horioka finding merely reflects the fact that international trade equalizes rates of return and eliminates the incentives for capital to flow across countries. According to this view, it is the relative costs of commodity trade and capital movements that determine how countries arbitrage away their rate-of-return differentials. Even if the costs of capital mobility are small, if trading goods is cheaper, we should not observe large capital flows.

observed cross-country variation in factor prices. Using the jargon of growth theory, Trefler's research suggests the empirical validity of a conditional version of the factor-price-equalization theorem.[3]

A further problem with the usual interpretations of the convergence finding is that they leave us little room to explain the East Asian Miracle. If diminishing returns are important, why have rates of return and growth rates not been declining quickly as these economies accumulate capital? Why does the law of diminishing returns not apply to them? Moreover, if the degree of economic integration in the world is so low, what has been the role of foreign trade in the growth process of Hong Kong, Korea, Singapore, and Taiwan? Is the spectacular increase in manufacturing exports that these countries have experienced unrelated to their growth performance?

This paper starts from the premise that international commodity trade plays a key role in the growth process of real economies. To determine this role, I study a simple model of trade and growth that combines Trefler's conditional version of the factor-price-equalization theorem with the Ramsey model of economic growth.[4] The model is closely related to that of Stiglitz [1971]. However, while Stiglitz studies the conditions under which the factor-price-equalization theorem holds in the long run, I impose sufficient structure to ensure that a conditional version of this theorem holds, and instead characterize the behavior of a number of variables both in the steady state and during the transition toward it. The model features a technology that exhibits diminishing returns. Yet countries' ability to trade and eliminate price differentials implies that these diminishing returns are global (only affected by world averages) but not local (unaffected by a small country's actions). By exploiting this property, the model presents a novel picture of the growth process that forces us to reinterpret the source of the conditional convergence finding, and

3. It is interesting to note how similar the evolutions of the empirical research on convergence and factor-price-equalization have been. For many years it was thought that both the convergence hypothesis and the factor-price-equalization theorem were at odds with the data. Recently Barro [1991] and Trefler [1993] have shown that if we allow for factor-augmenting productivity differences across countries, conditional versions of both these results are consistent with existing data.

4. The model developed here emphasizes the role of capital accumulation as a source of economic growth. See Grossman and Helpman [1991] for a detailed exposition of models of trade and growth that emphasize technological progress as a source of growth.

allows us to understand how the East Asian economies have been able to beat diminishing returns through international trade.

In this model of interdependent economies, rate-of-return differentials still explain differences in growth rates over time, but it is savings rates that explain differences in growth rates across countries. In such an environment the existence of diminishing returns does not rule out any configuration for a cross section of growth rates, and one cannot therefore use the conditional convergence finding as evidence of diminishing returns. A prediction of the model is that, holding constant differences in labor productivity, poor countries grow faster than rich ones if and only if factor prices do not change too fast as the world economy grows. Consequently, the model predicts that conditional convergence is associated with aggregate technologies that exhibit a large elasticity of substitution between capital and labor. These are the sorts of technologies that can sustain long-run growth despite diminishing returns (see Jones and Manuelli [1990]).

The model also sheds light on the nature of the East Asian Miracle. Standard growth theory predicts that the rapid process of capital accumulation experienced by the East Asian countries should have led to the use of more capital-intensive techniques in the production of the same set of goods, and a reduction in the marginal product of capital. Even if their saving rates were high, these countries seemed condemned to return to average growth rates. What is it that allows these high-savings economies to beat the curse of diminishing returns? The model's answer is simple: their ability to trade. As the capital stock grows, resources are moved from labor-intensive to capital-intensive industries, raising the demand for capital and sustaining the value of its marginal product. International trade converts an excess production of capital-intensive goods into exports, instead of falling prices. This explanation (which is an application of the Rybczynski theorem of international trade) also accounts for the dramatic increase in the East Asian economies' production and export of manufacturing (capital-intensive) goods.

A caveat is in order. The model presented here has Ricardian elements since it simply postulates cross-country differences in labor productivity, without explaining their origin or evolution. Yet they are an important factor in explaining the postwar international growth experience. Empirically, these productivity differences have been identified as reflecting cross-country variation in levels of human capital and government policies. The work presented here should therefore be seen as complementary to the growing body of research that tries to determine how the nature of government policies and the educational choices of countries relate to the process of economic growth.

The paper is organized as follows: Section I develops the model. Section II discusses the implications of the conditional convergence finding. Section III is devoted to the East Asian Miracle.

I. A Model of Trade and Growth

This section presents a simple dynamic general equilibrium model of international trade and growth. The key feature of the model is that, despite the absence of international factor movements, rates of return to capital do not depend on domestic factor endowments but only on world average factor endowments. For simplicity, I choose to model a competitive economy. As mentioned already, there exist many alternative sets of assumptions regarding technology and market structure consistent with factor prices being independent of domestic factor endowments.

A. Description

Consider a world economy with J countries: $j = 1, \ldots, J$; one final good that can be used for consumption and investment; two intermediate goods used in the production of the final good, $i = 1,2$; and two factors of production, capital and labor. Throughout, quantity variables are expressed in per capita terms, and world averages are denoted by omitting the country index. Let $\pi_j \in \mathfrak{R}_+$ denote the share of world's population located in country j, and $\tilde{\pi} = (\pi_1, \ldots, \pi_J) \in \mathfrak{R}_+^J$ be the world vector of population shares. These shares are constant since all countries' populations grow at the same rate n. The final good is nontraded, and we use it as a numeraire in each country. There is free (and costless) trade in both intermediate goods, and consequently, firms in all countries share the same intermediate prices, $p_1(t) \in \mathfrak{R}_+$ and $p_2(t) \in \mathfrak{R}_+$, where $t \in [0,\infty)$ is the time index. International factor movements are not permitted. Let $w_j(t) \in \mathfrak{R}_+$ and $r_j(t) \in \mathfrak{R}_+$ be the wage and rental rate in country j, respectively; and define $\tilde{w}(t) = (w_1(t), \ldots, w_J(t)) \in \mathfrak{R}_+^J$ and $\tilde{r}(t) = (r_1(t), \ldots, r_J(t)) \in \mathfrak{R}_+^J$.

Each country admits a representative consumer. Define $c_j(t) \in \mathfrak{R}_+$ and $k_j(t) \in \mathfrak{R}_+$ as the consumption rate and capital stock of country j at date t, and let $\tilde{c}(t) = (c_1(t), \ldots, c_J(t)) \in \mathfrak{R}_+^J$ and $\tilde{k}(t) =$

$(k_1(t), \ldots, k_J(t)) \in \mathcal{R}_+^J$.[5] Capital does not depreciate. The representative consumer has logarithmic utility:

$$\int_0^\infty \ln c_j \cdot e^{-(\rho-n)\cdot t} \cdot dt, \tag{1}$$

where $1 > \rho > n$ is assumed. The first inequality generates long-run growth under certain parameter conditions (to be discussed later), while the second one ensures that utility is bounded. Since capital movements are not allowed, trade must be balanced, and the budget constraint of country j's representative consumer is

$$c_j + \dot{k}_j + n \cdot k_j = r_j \cdot k_j + w_j. \tag{2}$$

Equation (2) states that total expenditure must equal total income. The former is given by consumption plus investment (which includes increases in the capital stock per person plus the provision of capital to new workers). The latter is the sum of capital and labor income.

There are many competitive firms in the final goods sector with free access to a C.E.S. technology that combines the two intermediate goods, i.e., $(x_{1j}^b + x_{2j}^b)^{1/(1-b)}$ $(b < 1)$, where $x_{ij}(t) \in \mathcal{R}_+$ denotes the purchases of good i by the representative firm of country j at date t, and $\tilde{x}_i(t) = (x_{i1}(t), \ldots, x_{iJ}(t)) \in \mathcal{R}_+^J$. For future reference, define $\sigma = (1 - b)^{-1}$ as the elasticity of substitution between inputs.

Each country also contains many competitive firms that produce intermediates. These firms use labor and capital to produce commodities 1 and 2. Consistent with observed differences in education levels or government policies, I allow for cross-country variation in labor productivity. Let $A_j \in \mathcal{R}_+$ be a measure of labor productivity in country j, and $\tilde{A} = (A_1, \ldots, A_J) \in \mathcal{R}_+^J$. Existing technology is as follows: one worker produces A_j units of good 1, while one unit of capital produces one unit of good 2. This specification of technological possibilities is chosen because it drastically simplifies the mathematics. It is also useful because it boldly illustrates the main insight of the factor proportions theory of international trade, namely that commodity trade is a concealed way of trading factor services. The Appendix interprets this technology as a limiting case of a more general description

5. I do not explicitly impose the constraint that capital stocks be strictly positive for each country and date. This constraint is satisfied by assuming that the smallest element of $\tilde{k}(0)$ is large enough. Later, I will give a precise statement of this condition.

of technology in which each intermediate industry uses both labor and capital and shows that the main results of the paper also hold for the generalized model.

This completes the description of the model. To sum up, the data required to study this model are $\bar{\pi}$, $\bar{k}(0)$, $\bar{A}$, ρ, n, b.

B. Competitive Equilibrium

The competitive equilibrium of the world economy consists of a sequence of prices and quantities such that consumers and firms optimize and markets clear. The assumptions made ensure that such an equilibrium exists and is unique.

The representative consumer in country j supplies labor and capital inelastically and chooses the path for c_j and k_j that maximizes (1) subject to (2). The first-order conditions for this problem are (after eliminating the multiplier):

$$r_j = \rho + \dot{c}_j / c_j \tag{3}$$

$$\dot{k}_j = \left(r_j - n\right) \cdot k_j + w_j - c_j \tag{4}$$

$$\lim_{t \to \infty} k_j \cdot c_j^{-1} \cdot e^{-(\rho - n) \cdot t} = 0. \tag{5}$$

Equation (3) guarantees that, in the margin, the benefit of savings, i.e., the rental rate, equals the cost of forgone consumption, which consists of a pure time preference term plus a correction factor that depends on how steep the consumption path is. Equation (4) is a restatement of the constraint of the problem. Finally, equation (5) is the familiar transversality condition.

The representative firm in the final goods sector of country j takes prices as given and chooses x_{1j} and x_{2j} so as to maximize profits. The demands of intermediates as a function of prices and the demand of final goods (which is only domestic) are given by

$$x_{ij} = p_i^{1/(b-1)} \cdot \left(r_j \cdot k_j + w_j\right) \tag{6}$$

for $i = 1,2$. Note that, since unit costs of production must equal the price of the final good, our choice of numeraire implies that

$$p_1^{b/(b-1)} + p_2^{b/(b-1)} = 1, \tag{7}$$

and the price of the final good is the same in all countries. Therefore, the Law of One Price also applies to the final good despite being nontraded.

Free trade and perfect competition ensure that firms producing intermediates in country j are willing to employ any quantity of factors if their prices satisfy these relationships:

$$w_j = A_j \cdot p_1 \tag{8}$$

$$r_j = p_2. \tag{9}$$

If a factor price exceeds (falls short of) this value, the firm's demand for this factor will be zero (infinity).

In each country, full employment of factors with prices as given in (8) and (9) is ensured by employing all labor in industry 1 and all capital in sector 2. Therefore, the model satisfies Trefler's conditional version of the factor-price-equalization theorem which states that *if two countries have the same factor productivities*, then they also have the same factor prices. Since we have assumed cross-country differences only in labor productivities, rental rates are equalized across countries despite the absence of international factor movements and for all feasible vectors $\tilde{k}(t)$ and $\tilde{A}$. That there exists a large set of vector pairs $(\tilde{k}(t),\tilde{A})$ for which rental rates are independent of domestic factor endowments should not be surprising and is a robust result. What is special about the model here is that rental rates be independent of domestic conditions for *any* feasible pair $(\tilde{k}(t),\tilde{A})$.[6] This result is not robust and follows directly from the particular specification of technology that has been adopted. See the Appendix for further discussion of this issue.

Since all countries face the same commodity prices, equation (6) implies that firms in the final goods sector of each country use the same proportions of both intermediates; i.e., $x_{1j}/x_{2j} = x_1/x_2$ for all j. Since the world average productions of goods 1 and 2 are A and k, respectively; market-clearing in world commodity markets requires that $x_1/x_2 = A/k$ or, alternatively,

$$p_1/p_2 = (k/A)^{1-b}. \tag{10}$$

It is interesting to note that intermediate prices depend only on the mean values for the stock of capital and labor productivity parameter, but are independent of other characteristics of the distributions of these variables. This is a very convenient aggregation property of this model.

6. Note that the factor-price-equalization set is the entire Edgeworth box when the axes are capital and productivity-adjusted labor.

C. World Averages

Even a world of open economies is a closed economy. It should come as no surprise to find that world averages behave as if they had been generated by a closed-economy model. More precisely, the dynamics of c and k are[7]

$$\dot{c}/c = \left(A^b + k^b\right)^{(1-b)/b} \cdot k^{b-1} - \rho \tag{11}$$

$$\dot{k} = \left(A^b + k^b\right)^{1/b} - n \cdot k - c. \tag{12}$$

These are the equations of a Ramsey model with a C.E.S. aggregate production function, with elasticity of substitution equal to $\sigma = (1 - b)^{-1}$. Also, the aggregate initial and transversality conditions are implied by the corresponding country ones. The international trade theorist will immediately recognize this result as the integrated-economy parable extended into a dynamic setting.

The nature of the growth process depends on the properties of the aggregate technology. As the world economy grows, the average product of capital falls, making further capital accumulation less productive. Regardless of the behavior of the savings rate, the model unambiguously predicts a decline in the growth rates of c and k. Whether the world economy exhibits positive long-run (asymptotic) growth or stagnates, depends upon whether the aggregate technology is capable of sustaining the marginal product of capital above the rate of time preference or not. If $\sigma > 1$, the marginal product of capital is bounded below at one, and the world economy is an endogenous growth model (i.e., it has a saddlepath-stable steady state with positive growth). Both c and k grow without bound at rates that are positive and decreasing, and asymptotically approach $1 - \rho$. If $\sigma < 1$, the marginal product of capital eventually equals ρ, and the world economy is an exogenous growth model (i.e., it has a saddlepath-stable steady state without growth, unless exogenous technological progress is added). If $k(0)$ is below its steady-state value, both c and k grow at rates that are positive and decreasing, and asymptotically approach zero.[8]

7. To obtain (11) and (12), differentiate $c = 1/J \cdot \sum_j c_j \cdot \pi_j$ and $k = 1/J \cdot \sum_j k_j \cdot \pi_j$, and then use (3)–(4) and (7)–(10) to eliminate prices and country-specific variables.

8. See Jones and Manuelli [1990] and Barro and Sala-i-Martin [1995] for proofs.

II. Convergence and Long-Run Growth

Barro [1991] constructed a database for 98 countries containing measures of real GDP in 1960 and 1985, and a vector of control variables that includes measures of education, political instability, and government policies. This vector of controls was meant to capture cross-country differences in labor productivity. Using this information, he constructed a graph with that part of a country's growth rate that cannot be explained by the vector of controls on the Y-axis and the initial GDP level on the X-axis. I refer to this graph as the cross section of growth rates. Barro found that this cross section is downward sloping. This finding, confirmed by a number of other studies,[9] has been termed *conditional* convergence since it means that, *if two countries have the same vector of controls*, the poor country tends to grow faster than the rich one and their per capita incomes exhibit a tendency to converge. As Barro himself emphasized, conditional convergence does not imply that per capita incomes tend to converge across countries, since those countries that have a high initial GDP also tend to have values for the control variables that lead to high growth rates.

Barro's empirical research poses two immediate problems for those trying to use his findings to discriminate among existing growth theories. The first problem consists of the fact that growth models tend to have predictions for the growth rates of the capital stock and not for the growth rates of real GDP. These two growth rates will not be very different if and only if factor shares in income remain stable over time. Following standard practice, I will finesse this problem away by assuming just that.[10]

The second and most important problem consists of the fact that existing growth theories were not designed to answer the question of *why some countries grow faster than others,* but in-

9. See, for instance, Barro and Sala-i-Martin [1992] and Mankiw, Romer, and Weil [1992].

10. To see why this assumption works in our model (as it does in most models), remember that GDP is given by $q_j = r_j \cdot k_j + w_j$. Equations (8) and (9) imply that

$$\frac{q_j}{q} = \frac{A_j}{A} + \left(\frac{r \cdot k}{q}\right) \cdot \left(\frac{k_j}{k} - \frac{A_j}{A}\right).$$

If the world's share of capital in income does not change much and we control for differences in A_j, a country's GDP growth rate exceeds the world's average if and only if its capital-stock growth rate exceeds the corresponding world average. Note that I am not assuming that factor shares be similar across countries. The latter is true neither in the model nor in the data.

stead, they aimed at the alternative question of *why the growth rate of a country varies over time.* These are different questions, and I refer to them as the cross-sectional and time-series questions, respectively. While growth empirics compares growth rates across countries in a given period, growth theory compares growth rates over time in a given country. To connect these two strands of the literature, one has to make some assumptions.

A. Autarky

Explicitly or implicitly, it has become commonplace when interpreting cross-country growth regressions to take the view that international linkages are either nonexistent or unimportant for the problem at hand. It follows that countries with similar parameter values follow the same development path, unaffected by other countries' position in this common path. If countries have different parameter values, one has to control for these (as Barro did) to determine the position of a country in the development path. In this conceptualization of the growth process, explaining why a single country exhibits different growth rates on two different dates is equivalent to explaining why two different countries have different growth rates on the same date. This view is convenient since it implies that the cross-sectional and time-series questions have the same answer.

To illustrate this style of analysis, consider a world economy that is identical in all respects to the model in the previous section, except for the fact that international trade is not possible due to large transportation costs or protectionist economic policies. In this world of autarkic economies, each country's consumption and capital stock have laws of motion as described by equations (11)–(12), but different initial capital stocks, population size, and labor productivity parameters. The existence of diminishing returns implies that a graph that plots a country's growth rate of k_j against time is downward sloping. The reason is simple: as capital accumulation proceeds, the marginal product of capital falls making further capital accumulation less productive. Moreover, if we are willing to assume that $\sigma < 1$, it is possible to show that the larger is σ the flatter is this time-series graph. To see this, approximate the growth rate of the capital stock as follows:

$$\frac{1}{t} \cdot \ln\left(\frac{k_j(t)}{k_j(0)}\right) = \frac{1-e^{-\lambda \cdot t}}{t} \cdot \ln\left(\frac{k_j^*}{k_j(0)}\right), \tag{13}$$

where k_j^* is the steady-state value of k_j and λ, the speed at which each country converges to its steady state, is given by[11]

$$\lambda = -\frac{\rho - n}{2} + \frac{1}{2}\cdot\sqrt{(\rho - n)^2 + \frac{4}{\sigma}\cdot(\rho - \rho^{1/\sigma})\cdot(\rho^{\sigma} - n)}.$$

λ serves as a natural measure of the slope of the time-series graph. Ceteris paribus, the larger is σ, the smaller is λ, and the flatter is the time-series graph. Therefore, a further implication of this model is that, for given ρ and n, the flatter the observed time series of growth rates, the larger is the elasticity of substitution that we should infer from the data. A large elasticity of substitution implies that growth rates decline slowly over time and that their steady-state value might even be positive. The assumption that the world is a collection of autarkic economies allows us to extend the properties of the time-series graph to the cross-sectional graph. This is why existing literature has concluded that the convergence finding is strong evidence of diminishing returns, although the low (with respect to some priors) estimate of λ means that diminishing returns are slow to set in.

The main advantage of disregarding international linkages is that it makes international growth comparisons very simple. Note that we are only required to study the dynamical system for a single autarkic economy. This allows us to use off-the-shelf macroeconomic models for international growth comparisons, without having to make any investment in adapting them to the new use we put them to. One problem with this approach, however, is that it permits no role for international economic arrangements to explain cross-country differences in growth rates. This view solely emphasizes country characteristics. Yet we know that systemic elements have an important role in the growth process of real economies. For instance, there is substantial evidence suggesting that trade and exchange rate policy have effects on growth and investment. Lee [1993] shows that countries adopting

11. Rewrite equations (11) and (12) in terms of the logs of c and k, and take a first-order Taylor approximation around the steady state, i.e., $d \ln c/dt = d \ln k/dt = 0$:

$$\frac{d\ln c}{dt} = \frac{\rho^{1/\sigma} - \rho}{\sigma}\cdot(\ln k - \ln k^*)$$

$$\frac{d\ln k}{dt} = (\rho - n)\cdot(\ln k - \ln k^*) + (n - \rho^{\sigma})\cdot(\ln c - \ln c^*),$$

where c^* and k^* are the steady-state values for c and k. The solution of this log-linearized system is (13).

protectionist policies and exchange controls tend to grow more slowly than countries that do not adopt these policies. Levine and Renelt [1992] find that there is a robust positive correlation between the investment share and the ratio of international trade to GDP in a cross section of countries. A conceptualization of international growth comparisons that does not allow us to account for these observations seems essentially flawed.

In any case, one should be aware that the convergence test is a joint test of diminishing returns *and* the view that international linkages do not have much bearing on the growth process. We cannot infer from the convergence finding alone that the world exhibits diminishing returns unless we can test separately the view that international linkages do not have a noticeable effect on the growth process. Since any reasonable test of the latter should fail, one still might hope that removing the autarky assumption does not much alter the connection between time series and cross sections of growth rates. Unfortunately, this is not the case. I shall next show that, in a world of interdependence, the existence of conditional convergence is not a proper test of diminishing returns technologies, since the latter do not impose any restriction on the sign of the slope of a cross section of growth rates. Moreover, the connection between the slope of the cross section and time-series graphs for the growth rate changes dramatically as we move from a world of autarky to a world of free trade. It turns out that the model of trade and growth developed above predicts that the smaller the estimate of the slope we obtain from the data, the less likely it is both that growth rates decline slowly and that their steady-state value is positive. This is just the opposite of what we found in the world of autarky.

B. Free Trade

The growth rate of a country depends on how much a country invests and how productive this investment is. In models of autarkic economies the investment rate might increase or decrease with the stock of capital, but the law of diminishing returns ensures that this investment is less productive the more capital the economy has accumulated. In fact, this second effect is so large that it always dominates, and this is why the model predicts that poor countries should grow faster than rich ones, once we control for differences in labor productivity. In models of trading economies the law of diminishing returns applies only to world averages. In a given period, investment is equally productive in each country, and as a result, differences in growth rates can be attributed only to differences in investment rates. Depending on parameter values, these rates might increase or decrease with the stock of capital. This is why in a world of trading economies the existence of diminishing returns does not have to be associated with conditional convergence.

It happens often that extremes allow radical simplifications. If instead of assuming that the world is a collection of autarkic economies we take the alternative view that countries are so interdependent that differences in rates of return are arbitraged away, then performing international growth comparisons becomes a tractable problem again. Define $k_j^R = k_j/k$ and $A_j^R = A_j/A$, and let $\tilde{k}^R = (k_1^R, \ldots, k_J^R)$ and $\tilde{A}^R = (A_1^R, \ldots, A_J^R)$. We refer to the vectors $\tilde{k}^R$ and $\tilde{A}^R$ as the cross sections of capital stocks and labor productivities. The jth element of $\tilde{k}^R$ has the following law of motion:

$$\dot{k}_j^R = \phi \cdot \left(k_j^R - A_j^R\right), \tag{14}$$

where ϕ is defined as follows:[12]

$$\phi = \frac{A}{k} \cdot \left[(\rho - n) \cdot \int_t^{\infty} p_1 \cdot e^{-\int_t^{\tau}(p_2 - n)\cdot dv} \cdot d\tau - p_1\right].$$

Integrating (14), we obtain

$$k_j^R(t) = A_j^R + e^{\int_0^t \phi \cdot dv} \cdot \left(k_j^R(0) - A_j^R\right). \tag{15}$$

Equation (14) or its integral version (15) provides a full characterization of the cross section of growth rates as a function of prices, average quantities, and the initial distribution of capital stocks and labor productivities.[13] Remember that p_2 and $p_1 \cdot A$ are

12. To obtain (14), integrate (3) and (4), combine them, and use (5) (or look at Barro and Sala-i-Martin [1995, pp. 66–67]) to find that

$$c_j = (\rho - n) \cdot \left(k_j + \int_t^{\infty} w_j \cdot e^{-\int_t^{\tau}(r_j - n)\cdot dv} \cdot d\tau\right).$$

Then, subtract (12) from (4), and use the expression above and the price equations (7)–(10) to simplify until (14).

13. We can now state a necessary and sufficient condition for all countries to have strictly positive capital stocks at all dates:

$$\min_{j \in \{1, \ldots, J\}} k_j^R(0)/A_j^R > \max_{t \in [0, \infty)} 1 - e^{-\int_0^t \phi \cdot dv}.$$

In a world in which $\phi < 0$ for all t, it is sufficient to assume that each and every element of $\tilde{k}(0)$ is strictly positive.

the rental rate and the average wage rate, respectively. Ceteris paribus, if the growth rate of wages is low, ϕ is negative, and countries that have a low stock of capital relative to their labor productivity parameter, accumulate capital at a higher rate. The intuition for this result follows from the assumption that the world is populated by countries that act as permanent-income consumers, calculating the net present value of their income and choosing their optimal consumption path. Since all countries have the same spending shares (a property of homothetic preferences), they all spend the same fraction of their wealth in each date and therefore exhibit identical rates of wealth accumulation.[14] But the growth rate of wealth is a weighted average of the growth rates of its two components: the stock of capital and the net present value of wages. The growth rate of the latter is the same for all countries, since wage growth (but not levels) is independent of domestic conditions. If this growth is low, consumers must be accumulating capital at a rate that exceeds that of total wealth. But how much? It depends on how large is the share of capital in a consumer's wealth. The lower the share of capital is, the higher is the rate of capital accumulation that is required to sustain the optimal consumption path. This is why countries that have a low stock of capital relative to their labor productivity parameter tend to accumulate capital faster when the growth rate of wages is low. A symmetric argument works for the case in which the net present value of wages grows at a high rate.

Define $z = c/k$, and note that $\phi = \dot{z}/z$.[15] That is, the distribution of capital stocks approaches (moves away) from the distribu-

14. In this model, a country's consumption growth rate is independent of its income level, since this growth rate depends only on the (common) rates of return and time preference. Yet if we run the standard growth regressions using consumption as the dependent variable, one immediately finds that there is also conditional convergence in consumption growth rates. One way to reconcile the model with the data is to allow for a more general description of preferences, such as the Stone-Geary utility function studied by Caselli and Ventura [1996]. Another alternative is to assume that agents have finite horizons, as in Blanchard [1985]. In his model, although all consumers have the same rate of consumption growth, the aggregate rate of consumption growth depends negatively on wealth (and therefore initial income).

15. To see this, substitute the consumption function derived in footnote 9 into (12) and note that

$$\frac{\dot{z}}{z} = \frac{\dot{c}}{c} - \frac{\dot{k}}{k} = \frac{A}{k} \cdot \left[(\rho - n) \cdot \int_t^{\infty} p_{\tau} \cdot e^{-\int_t^{\tau}(r_u - n) du} \cdot d\tau - p_t\right],$$

which is the definition of ϕ.

tion of labor productivities if and only if the average consumption-capital ratio decreases (increases). Rewrite equations (11) and (12) as follows:

$$\dot{z}/z = z - \rho + n - \left(A^b + k^b\right)^{(1-b)/b} \cdot A^b \cdot k^{-1} \tag{16}$$

$$\dot{k} = \left(A^b + k^b\right)^{1/b} - (n + z) \cdot k. \tag{17}$$

To characterize the dynamics of the cross section of capital stocks, we can use the phase diagram of this system. In the steady state z is constant, and the distribution of capital stocks remains unchanged, regardless of whether the world economy sustains long-run growth or not. During the transition the behavior of z depends crucially on the assumed elasticity of substitution between capital and labor. Figure I shows the phase diagram of a world economy that sustains long-run growth; i.e., $\sigma > 1$. As the economy travels along the stable arm, z declines monotonically toward its steady-state value. As a result, the cross section of capital stocks approaches that of the labor productivity parameters.

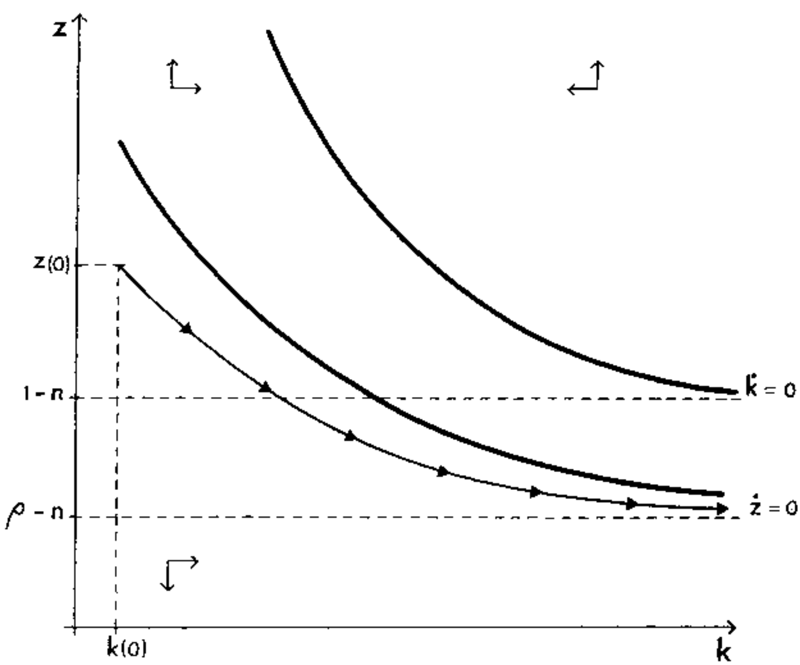

FIGURE I

Two different cases arise when we assume that $\sigma < 1$. Figure II shows the case in which σ is less than one but still high enough for the steady state to be located in the downward-sloping region of the $\dot{z} = 0$ line. If the economy starts with a low level of capital, z increases and then declines. At low levels of development the cross section of capital stocks moves away from the cross section of labor productivities, but this trend is reversed as the world economy reaches a certain level of development. Figure III shows the case in which σ is low enough for the steady state to be located in the upward-sloping region of the $\dot{z} = 0$ line. Since z is increasing during the transition, the distribution of capital stocks always moves away from the distribution of labor productivity parameters.

Basically, one can read the discussion above as saying that

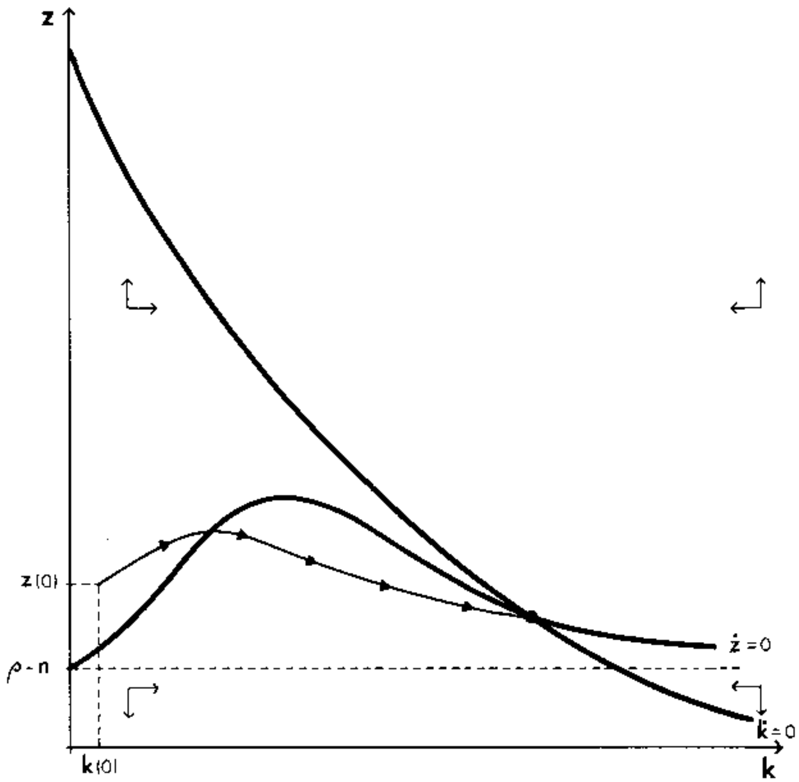

FIGURE II

the distribution of capital stocks will converge toward the distribution of labor productivities if the elasticity of substitution between capital and labor is large enough. This makes sense given our theory of savings: the higher the substitutability among factors, the lower the growth of wages as the world economy accumulates capital. This forces countries whose wealth has a large labor component to accumulate capital faster in order to sustain their optimal consumption path.

C. Cross Sections and Time Series

A surprising implication of this model is the relationship that emerges between the cross-section and time-series graphs for the growth rate. If the aggregate technology exhibits a high elasticity of substitution, factor prices do not change much as factor proportions vary. As a result, one should expect the following: (i) since the wage rate does not increase quickly, neither does the net present value of wages, and for the reasons already discussed, we

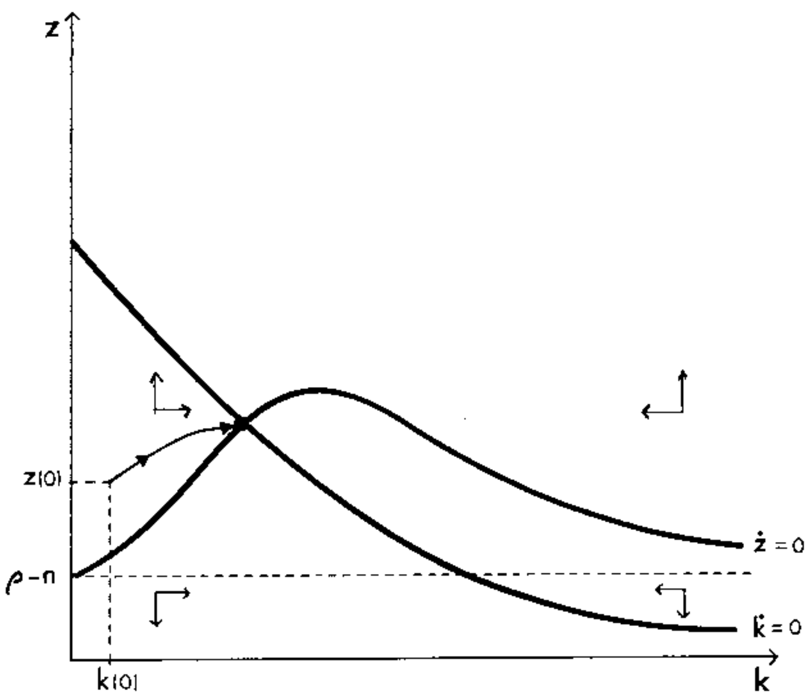

FIGURE III

would observe a downward-sloping cross section of growth rates; and (ii) since the rental rate decreases slowly, one would expect the growth rate not to fall rapidly, and as a result, we would observe a (downward-sloping, but) almost flat time-series graph for the growth rate. A similar argument shows that if there is a low elasticity of substitution, we should expect both an upward-sloping cross section and a (downward-sloping and) steep time series for the growth rate. Since the existence of diminishing returns does not rule out any configuration for a cross section of growth rates, one cannot use the conditional convergence finding as evidence of diminishing returns. However, if one is willing to keep as a maintained hypothesis the existence of diminishing returns, then the conditional convergence finding can be used as evidence that the elasticity of substitution is high, and that the world economy might be closer to the endogenous growth model ($\sigma > 1$) than the exogenous growth model ($\sigma < 1$).

This result stands in stark contrast to the intuition that arises from closed-economy models and that dominates current research. In a world of autarkic economies, the cross-section and time-series graphs for the growth rate are identical, since it is the same mechanism that is at work determining their slope: rate-of-return differentials. A large elasticity of substitution implies both that the marginal product of capital declines slowly and that rate-of-return differentials across countries are small. As a result, a downward-sloping cross section of growth rates is usually interpreted as indicating a small elasticity of substitution and therefore supporting the exogenous growth model. In a world of free trade just the opposite is true, and one should interpret a downward-sloping cross section of growth rates as indicating that the elasticity of substitution is high and therefore supporting the endogenous growth model.

III. Making Miracles

Lucas [1993] has argued that if we are to understand the process of economic growth, we should have models that are able to replicate the East Asian Miracle. This consists of the fact that, for more than three decades, a few export-oriented small economies in East Asia have been growing at rates that are extremely high by historical standards. These countries' outstanding growth performance has been accompanied by a spectacular increase in their volume of manufacturing exports. Moreover, Young [1995] has shown that these miracles can be explained, in the traditional growth-accounting sense, as the sole result of factor accumulation and not of factor productivity growth. Even if their savings rates are high, standard growth theory predicts that the growth rates of these countries should have returned to average. How have these countries been able to defy the law of diminishing returns for such a long period? In a world of trading economies, this type of a growth miracle is possible. The secret is to open the economy and be patient. The reward is a continuous process of capital accumulation and structural transformation.

A. Growth in Miracle Economies

Up to now we have assumed that all countries are identical except for their initial capital stock and labor productivity. Next, I consider the effects of differences in rates of time preference, population growth, and rates of return to capital. With respect to the latter I assume that the rental rate in a miracle economy is $p_2 \cdot (1 + \theta_j)$, where θ_j is the discrepancy (in percentage terms) between the domestic and foreign rental rate. This discrepancy could reflect different taxation systems, subsidies to investment, capital-augmenting differences in technology, and other. For our purposes, the source of this discrepancy is not important.[16]

Consider the case of a small open economy, i.e., $\pi_j \cdot k_j^R \approx 0$, which satisfies the following parameter restrictions: $n - n_j \geq p_2^* \cdot \theta_j + \rho - \rho_j > 0$, where p_2^* is the world's steady-state rental rate. We know that if $\sigma > 1$, then $p_2^* = 1$; and if $\sigma \leq 1$, then $p_2^* = \rho$. The first inequality ensures that the small economy remains small forever. The second inequality ensures that we are about to observe a miracle.

Since the "smallness" assumption allows us to approximate the laws of motion of world averages by equations (11)–(12), the relative capital stock of economy j has these dynamics:

$$\dot{k}_j^R = \left(\phi + p_2 \cdot \theta_j + \rho - \rho_j\right) \cdot k_j^R - \phi_j \cdot A_j^R, \tag{18}$$

16. Since we have assumed that international capital flows are not possible, any discrepancy between domestic and foreign rates of return can be sustained. If the costs of international capital flows were small, whether such discrepancy can be sustained or not depends on its source. For instance, if $\theta_j > 0$ because the government subsidizes investment by domestic firms, foreign firms would not have any incentive to invest despite the discrepancy in rates of return. On the other hand, if $\theta_j > 0$ because the government invests in infrastructure that is complementary to capital, foreign firms would want to invest in the country and take advantage of this infrastructure.

where ϕ_j is defined as follows:[17]

$$\phi_j = \frac{A}{k} \cdot \left[\left(\rho_j - n_j\right) \cdot \int_t^{\infty} p_1 \cdot e^{-\int_t^{\tau}\left(p_2\left(1+\theta_j\right)-n_j\right)dv} \cdot d\tau - p_1 \right].$$

Note that, if we set $\theta_j = 0$, $\rho_j = \rho$, and $n_j = n$, equation (18) is identical to (14).

It is easy to see why a group of countries in which $p_2 \cdot \theta_j + \rho - \rho_j > 0$ would look like a miracle in a cross section of countries. As equation (18) shows, holding constant k_j^R, A_j^R and n_j, the smaller the rate of time preference of a country, the faster its growth rate. The reason is simple: the more patient a country is, the lower is the propensity to consume out of its wealth (which includes the capital stock and the net present value of wages), and the faster a country grows. Also, the larger is θ_j, the higher the growth rate. The reason is that a high interest rate leads to a low net present value of wages and, as a result, a lower level of consumption. This is the usual income effect of high interest rates. The substitution effect is ruled out by our choice of logarithmic preferences.

Population growth affects a country's growth rate in a more complicated way. First, a high population growth rate means that a large fraction of savings is devoted to endow new workers with the average capital per person and less is available to increase this average. Second, a high population growth rate makes the net present value of wages large (since many workers in the future means many wages to be earned) and, for a given marginal propensity to consume out of wealth, induces high consumption. Third, a high rate of population growth leads to a low marginal propensity to consume out of wealth. The first two effects tend to reduce the growth rate, while the third effect tends to raise it.

An interesting result is that, regardless of the nature of technology, the long-run growth rate of k_j^R is[18]

$$\lim_{t\to\infty} \frac{\dot{k}_j^R}{k_j^R} = p_2^* \cdot \theta_j + \rho - \rho_j. \tag{19}$$

17. To find (18), follow the same steps used to derive (14), but allowing for $\theta_j \neq 0$, $\rho_j \neq \rho$, and $n_j \neq n$.

18. Integrate (18) to obtain

$$k_j^R(t) = e^{\int_0^t \left(\phi + p_2^* \theta_j + \rho - \rho_j\right) dv} \cdot \left(k_j^R(0) - A_j^R \cdot \int_0^t \phi_j \cdot e^{\int_0^{\tau} \left(\phi + p_2^* \theta_j + \rho - \rho_j\right) dv} \cdot d\tau \right).$$

Since the capital stock is positive at all dates and $\lim_{t\to\infty} \phi = 0$, it follows that $\lim_{t\to\infty} k_j^R = \infty$. Since

For this result to hold, the small economy must remain small asymptotically; i.e., $\lim_{t\to\infty} \pi_j \cdot k_j^R \approx 0$. It follows from (19) that a necessary and sufficient condition for this to be the case is that $n - n_j \geq p_2^* \cdot \theta_j + \rho - \rho_j$, which is a parameter restriction we have already assumed.[19]

To understand the implications of (19), consider first the case in which the world economy is an endogenous growth model. The asymptotic growth rate of country j is not equal to that of the world economy, which is given by $1 - \rho$. One might have thought that growth rates of countries that exhibit a high degree of economic integration would converge to some kind of average. However, this is not the case. In fact, the asymptotic growth rate of country j is the same that it would have been in autarky, $1 + \theta_j - \rho_j$. In other words, obtaining the static efficiency gains associated with specialization in production does not require a small country to sacrifice its high long-run rate of economic growth. The international economic system is flexible enough to accommodate large variation in growth rates, even if domestic prices are closely tied to international ones.

It is clear from equation (19) that policies that affect the rental rate or the rate of time preference (the willingness to save) have both a transitory and permanent impact on the growth rate of a country. This effect of savings on the long-run rate of economic growth is also characteristic of autarky models that feature endogenous growth. What is truly surprising here is that even if existing technology cannot support long-run growth in autarky, our small open economy might. To see this, assume that $\sigma \leq 1$. Asymptotically, our small economy still grows at a positive rate; i.e., $\rho \cdot (1 + \theta_j) - \rho_j$. Therefore, a small economy that com-

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$$\lim_{t\to\infty} \frac{\dot{k}_j^R}{k_j^R} = \lim_{t\to\infty} \phi + p_2^* \cdot \theta_j + \rho - \rho_j - \lim_{t\to\infty} \frac{A_j^R \cdot \phi_j}{k_j^R},$$

the proof of (19) consists of showing that $\lim_{t\to\infty} \phi_j$ is finite. But asymptotically prices approach the constants p_1^* and p_2^*, and consequently, ϕ_j approaches a constant:

$$\lim_{t\to\infty} \phi_j = A \cdot p_1^* \cdot \frac{\rho_j - p_j}{p_2^* \cdot \left(1 + \theta_j\right) - n_j}.$$

19. Our assumptions ensure that the miracle economy does not leave the factor-price-equalization set, i.e., the range of capital per effective worker ratios that ensure that the factor-price-equalization theorem holds. See the Appendix for a discussion of this issue.

bines a high savings rate (i.e., low time preference or high rental rate) with a low rate of population growth can beat diminishing returns by adopting an open trade regime, even when its underlying technology would not sustain long-run growth in autarky.

How is this possible? It is well understood among growth theorists that a country can experience a positive long-run rate of economic growth if and only if it has an aggregate technology that is asymptotically linear on the factor that can be accumulated which, in our case, is capital. Therefore, if we want to understand (19) and its implications, one needs to explain why this economy is behaving "as if" it had a linear technology. To see this, it is useful to compare the value of GDP of an autarkic and a trading economy, after the different sectors have been integrated vertically:

$$GDP(\text{autarky}) = \left(k_j^b + A_j^b\right)^{1/b} \quad GDP(\text{free trade}) = p_1 \cdot A_j + p_2 \cdot k_j .$$

In autarky, increases in k lead to changes in prices. In the trading economy this is not the case, since intermediate prices depend only on world averages and those are basically unaffected by changes in domestic conditions. This is why opening to international trade for a small economy is like choosing a linear technology in the aggregate. This is also why it makes sense to talk about terms of trade shocks (i.e., changes in p_1 and p_2) as if they were productivity shocks.

B. Structural Transformation versus Capital Deepening

Admittedly, the model has too simple a production structure to seriously address the question of how the economic structure of a fast growing economy evolves over time. However, a couple of interesting observations can be made even at this level of abstraction. In particular, the model can help us understand how the growth miracles discussed above have been accompanied by large changes in the composition of output and a dramatic increase in manufacturing exports.

Young's [1995] findings strongly suggest that the rapid growth of the East Asian economies is not due to extraordinary productivity growth, but the result of rapid capital accumulation. The picture that comes out of autarkic growth models that emphasize capital accumulation is one in which economic growth does not fundamentally alter the structure of the economy. In such models, increases in the capital-labor ratio make labor scarce and capital abundant. Incipient excess demand for labor and excess supply of capital are eliminated by increased wages and reduced rates of return to capital. These price changes provide firms with incentives to use labor-saving techniques and thus absorb the increased capital-labor ratio. As capital is accumulated, the economy keeps producing the same set of goods by using more capital-intensive techniques in their production. In this class of models economic growth is basically equivalent to capital deepening.

The picture of the growth miracles that comes out of the model here is quite different. As before, increases in the capital-labor ratio make labor scarce and capital abundant. However, the incipient excess demand for labor and excess supply of capital are not eliminated through price changes but through changes in the structure of production. Instead of using more capital-intensive techniques in each sector, the miracle economies absorb the extra capital by expanding capital-intensive sectors and contracting labor-intensive ones. This reallocation of economic activity raises the demand for capital and reduces the demand for labor. This is how a trading economy can absorb the higher capital-labor ratio at existing prices. In this class of models, economic growth leads to structural transformation and not capital deepening.

The model developed here makes this observation embarrassingly obvious. Assume that the world is in the steady state so that factor prices are constant (alternatively, one can think in deviations from world aggregates). As the miracle economy keeps accumulating capital, industry 1 expands while industry 2 remains the same (remember that $x_{1j} = k_j$ and $x_{2j} = A_j$). One might think that, by choosing such an extreme model, I have loaded the cards to find this result. On the contrary, by assuming such a large difference (literally, infinite) in capital per effective worker across industries, I have made the structural transformation that this economy experiences as small as it can be. The Rybczynski theorem of international trade says that an increase in a country's capital stock leads to a more than proportional expansion in the capital-intensive industry and a contraction in the labor-intensive one.[20] In our model, we are in the limiting case in which the expansion of the capital-intensive industry is just propor-

20. Assume that both industries use capital and labor. Full employment of factors requires that the shares of employment in industries 1 and 2, l_{1j} and l_{2j}, satisfy

$$k_j = l_{1j} \cdot k_{1j} + l_{2j} \cdot k_{2j},$$

tional and the contraction is zero. In the generalized model presented in the Appendix, the degree of structural transformation a miracle economy experiences would never be smaller than in the special model studied here.

This picture of the growth process is consistent with a key feature of the East Asian countries' experience: the dramatic increase in their volume of manufactured exports as a share of GDP. To sketch how an explanation for this remarkable growth of manufacturing exports can be constructed, identify manufactured sectors with the capital-intensive intermediate and traditional or agricultural sectors with the labor-intensive intermediate. Since demands are homothetic, all countries spend the same share on manufactures, which must equal the world's share, μ. Simple algebra establishes that the share of manufacturing exports in GDP is[21]

$$(1 - \mu) \cdot \frac{\mu \cdot \left(k_j^R - A_j^R\right)}{\mu \cdot \left(k_j^R - A_j^R\right) + A_j^R}. \tag{20}$$

As expected, capital-abundant countries are net exporters of manufactures, while labor-abundant countries are net importers. This is the pattern of geographical specialization in production that supports the equalization in (productivity-adjusted) factor prices across countries. Accordingly, one would expect that countries that grow above average move from net importers to net

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where k_{1j} and k_{2j} are the capital per worker ratios of industries 1 and 2, respectively. Since industry 1 is the labor-intensive industry, $k_{1j} < k_{2j}$. Assume that there is an increase in k_j. The factor-price-equalization theorem ensures that factor prices do not change, and neither will the techniques used in production, i.e., k_{1j} and k_{2j}. Differentiating the full-employment condition and using the fact that $l_{1j} + l_{2j} = 1$,

$$\frac{dl_{1j}}{l_{1j}} = \frac{k}{k - k_{2j}} \cdot \frac{dk}{k} \quad \frac{dl_{2j}}{l_{2j}} = \frac{k}{k - k_{1j}} \cdot \frac{dk}{k}.$$

Since $k_{1j} \leq k \leq k_{2j}$, the statement in the text follows.

21. The value of manufacturing exports is $p_2 \cdot (k_j - k)$, and the value of GDP is $p_1 \cdot A_j + p_2 \cdot k_j$. To obtain (20), divide these two expressions and use the fact that $\mu = p_2 \cdot k/(p_1 \cdot A + p_2 \cdot k)$. By computing the share of exports in this way, I am assuming that $\theta_j \cdot k_j$ should not be counted as GDP. This is the right procedure if θ_j reflects differences in taxes or subsidies. If θ_j reflects capital-augmenting productivity differences, then we have that the share of manufacturing exports in GDP is

$$(1 - \mu) \cdot \frac{\mu \cdot \left(\theta_j \cdot k_j^R - A_j^R\right)}{\mu \cdot \left(\theta_j \cdot k_j^R - A_j^R\right) + A_j^R}.$$

exporters of manufacturing goods as their relative level of income increases. This is what the East Asian economies have been doing for more than three decades.

APPENDIX

Let $\hat{w}_j = w_j/A_j$, and consider the family of continuous and twice differentiable unit cost functions $m(r_j,\hat{w}_j;\alpha)$ indexed by $\alpha \in [\underline{\alpha},\overline{\alpha}]$, with nonnegative first derivatives and nonpositive second derivatives. If evaluated at the same (productivity-adjusted) factor prices, r_j and $\hat{w}_j$,

$$\text{(A1)} \qquad \frac{m_r(r_j,\hat{w}_j;\alpha_1)}{m_{\hat{w}}(r_j,\hat{w}_j;\alpha_1)} \leq \frac{m_r(r_j,\hat{w}_j;\alpha_2)}{m_{\hat{w}}(r_j,\hat{w}_j;\alpha_2)} \quad \text{if } \alpha_1 \leq \alpha_2$$

$$\text{(A2)} \qquad \lim_{\alpha\to\underline{\alpha}} \frac{m_r(r_j,\hat{w}_j;\alpha)}{m_{\hat{w}}(r_j,\hat{w}_j;\alpha)} = 0 \quad \text{and} \quad \lim_{\alpha\to\overline{\alpha}} \frac{m_r(r_j,\hat{w}_j;\alpha)}{m_{\hat{w}}(r_j,\hat{w}_j;\alpha)} = \infty.$$

Let α_1 and α_2 be the indexes of industry 1 and 2, respectively, and assume that $\alpha_1 \leq \alpha_2$. It follows from (A1) that industry 1 is the labor-intensive industry since, for all values of r_j and $\hat{w}_j$, this industry chooses a lower ratio of capital per effective worker than industry 2. The model in the text is the special case in which $\alpha_1 = \underline{\alpha}$ and $\alpha_2 = \overline{\alpha}$. The canonical example of a family of unit cost functions of this sort is the Cobb-Douglas family; i.e., $m(r_j,\hat{w}_j;\alpha) = r_j^{\alpha} \cdot \hat{w}_j^{1-\alpha}$ with $\underline{\alpha} = 0$ and $\overline{\alpha} = 1$.

Equations (3)–(7) remain valid but we have to generalize the pricing equations (8) and (9) as follows:

$$\text{(A3)} \qquad m(r_j,\hat{w}_j;\alpha_i) = p_i$$

for $i = 1, 2$; and also equation (10) as follows:

$$\text{(A4)} \qquad p_1/p_2 = (x_2/x_1)^{1-b},$$

where x_1 and x_2 are the world production of the corresponding intermediates.

We next derive conditions under which the conditional factor-price-equalization theorem holds in this generalized model. Assume that all countries share the same rental and productivity-adjusted wage (that is, (A3) holds). Then, we have that

$$\text{(A5)} \qquad \frac{x_2}{x_1} = \frac{f\left(m_r(r,\hat{w};\alpha_2), m_{\hat{w}}(r,\hat{w};\alpha_2)\cdot A;\alpha_2\right)}{f\left(m_r(r,\hat{w};\alpha_1), m_{\hat{w}}(r,\hat{w};\alpha_1)\cdot A;\alpha_1\right)},$$

where $f(.,.;\alpha)$ is the family of production functions associated with the family of unit cost functions $m(.,.;\alpha)$. Then, (7), (A3), (A4), and (A5) determine commodity and factor prices. Full employment of factors at these prices in all countries requires that the average capital per effective worker ratio in each country lies between the industry ratios at these common prices:

$$\text{(A6)} \qquad \frac{m_r(r,\hat{w};\alpha_1)}{m_{\hat{w}}(r,\hat{w};\alpha_1)} \leq \frac{k_j}{A_j} \leq \frac{m_r(r,\hat{w};\alpha_2)}{m_{\hat{w}}(r,\hat{w};\alpha_2)}.$$

If the pair $(\tilde{k}(0),\tilde{A}(0))$ is such that this condition is satisfied in each country and date, then an equilibrium exists in which (A3), (A4), and (A5) hold. In the text we did not have to worry about (A6) since (A2) ensures that any pair $(\tilde{k}(0),\tilde{A}(0))$ satisfies this condition when $\alpha_1 = \underline{\alpha}$ and $\alpha_2 = \overline{\alpha}$.

If (A6) holds, the predictions of the generalized model are virtually the same as those of the special model studied in Sections II and III. One difference is that, in general, it is no longer possible to obtain a closed-form solution for the aggregate production function, although one can show that this technology will be convex in the sense of Jones and Manuelli [1990]. Another difference is that the generalized model has nontrivial predictions for the sectoral distributions of employment and labor.

The generalized model raises an issue for the argument in Section IV that does not arise in the special model. If $\sigma \leq 1$, one can show that the miracle economy will violate the upper bound of (A6) in finite time for any technology in which $\alpha_2 \neq \overline{\alpha}$ (note that k_j/A_j keeps growing while the upper bound does not). If $\sigma > 1$, this is not necessary (since the upper bound grows in this case), but we still cannot rule out the possibility. If the upper bound of (A6) is violated, there are two possibilities. If the costs of international capital flows are small in absolute value (the only assumption we have really used in this paper is that the costs of capital flows were at least epsilon, since commodity trade already eliminates the incentives for capital to move), capital will start to flow out of the miracle economy. In this case, the predictions of the model remain valid for GNPs but not for GDPs, and the miracle continues, but in another form. If the costs of international capital flows are large in absolute value, the economy's factor prices will depart from world values since no more reallocation of activity is possible and diminishing returns set in. Factor prices will then be those of an autarkic economy that has an aggregate technology defined by α_2. In this case, the miracle can continue as long as the economywide capital per effective worker is less than the capital per effective worker that the most capital-intensive industry would choose at world factor prices.

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