

EXAMINATION

Please answer **two** of the following three questions.

1. Consider a world with two countries. There is a representative consumer in each country who has preferences over the interval of goods $X = [0,1]$ given by the utility function

$$\int_x \log c(x) dx$$

In each country there is a single factor, labor. Endowments are $\bar{\ell}_1 = \bar{\ell}_2 = \bar{\ell}$. Production functions are linear but differ across countries:

$$y_j(x) = \ell_j(x) / a_j(x)$$

where

$$a_1(x) = e^x$$
$$a_2(x) = e^{1-x}.$$

Initially, there are no transportation costs or tariffs.

- a) Define an equilibrium of this model.
- b) Characterize as much as possible the patterns of specialization and trade in the equilibrium.
- c) Suppose now that the countries engage in a trade war in which each imposes a 10 percent tariff on imports from the other. Explain how your definition of equilibrium is altered and characterize as much as possible how the new equilibrium differs from that in parts a and b.
- d) Suppose now that country 1 imposes a 10 percent tariff on imports from 2, but country 2 does not impose a tariff. Explain carefully how you would calculate the equilibrium of this world economy. Can you say anything about what happens in this equilibrium compared to that in part c?

2. Consider a two sector growth model in which the representative consumer has the utility function

$$\sum_{t=0}^{\infty} \beta^t \log(c_{1t}^{a_1} c_{2t}^{a_2}).$$

Here $0 < \beta < 1$, $a_1 \geq 0$, $a_2 \geq 0$, and $a_1 + a_2 = 1$. Investment is produced according to

$$k_{t+1} = dx_{1t}^{a_1} x_{2t}^{a_2}$$

Feasible consumption investment plans satisfy

$$\begin{aligned} c_{1t} + x_{1t} &= k_{1t} \\ c_{2t} + x_{2t} &= \ell_{2t} \end{aligned}$$

where $k_{1t} = k_t$ and $\ell_{2t} = \bar{\ell}$. The initial value of k_t is \bar{k}_0 , and $\bar{\ell}$ is equal to 1. (In other words, all variables are expressed in per capita terms.)

- a) Carefully define a competitive equilibrium for this economy.
- b) Reduce the equilibrium conditions for this economy to two difference equations in k_t and c_t and a transversality condition. Here $c_t = dc_{1t}^{a_1} c_{2t}^{a_2}$ is aggregate consumption.
- c) Suppose now that there is a world made up of m different countries all with the same technologies and preferences, but different endowments, $\bar{L}^j \bar{k}_0^j$ and \bar{L}^j . (That is, there is a measure \bar{L}^j of consumers, each of whom is endowed with 1 unit of labor in every period and \bar{k}_0^j units of capital in period 0.) Suppose that there is no international borrowing or lending and no trade in the investment good. Define an equilibrium of this world economy.
- d) Explain carefully why the world economy in part c is a dynamic Heckscher-Ohlin model.
- e) Explain how you could use the world economy in part c to demonstrate that the convergence properties of a world of closed economies differ radically from the convergence properties of a world of open economies.

3. Consider two countries, one with a higher output per worker than the other. In the more developed country, the level of output per worker in 2000 is 40,000 (measured in Summers-Heston 1996 international dollars). The level of capital per worker is 90,000. Over the five year period 1998-2000 the average *ex post* real interest rate on bank loans to prime level business borrowers was 5 per cent per year. In this country the share of GDP earned by capital is 0.30 and capital depreciates at a rate of 5 percent per year. In the less developed country output per worker is 20,000. The level of capital per worker is 30,000. Over the period 1998-2000 the real interest rate averaged 15 percent per year.

a) Consider a simple, one sector model in which output in each country is produced using a Cobb-Douglas production function. Argue that differences in output per worker cannot be explained by differences in capital per worker alone. To do this, use two different sets of calculations, one in which you use the information on capital per worker directly, and the other in which you use differences in real interest rates to infer differences in capital per worker.

b) Calibrate production Cobb-Douglas functions for each country that have different total factor productivity parameters but are otherwise identical and that are consistent with the observed data on output per worker and interest rates.

c) Suppose that the two countries, which have been closed to investment flows, now open themselves. Both the developed and the less developed country have 50 million workers. Calculate the impact of this opening on both countries or — if you do not have a calculator — explain carefully how to do so.

d) Discuss some of the limitations of the analysis in parts a, b, and c. Indicate how you could alter the analysis to eliminate or lessen these limitations.