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PROBLEM SET #2

1. Consider a two sector growth model in which the representative consumer has the utility function

$$\sum_{t=0}^{\infty} \beta^t \log(c_{1t}^{a_1} c_{2t}^{a_1}) .$$

Here $0 < \beta < 1$, $a_1 \geq 0$, $a_2 \geq 0$, and $a_1 + a_2 = 1$. Investment is produced according to

$$k_{t+1} - (1-\delta)k_t = d x_{1t}^{a_1} x_{2t}^{a_1} .$$

Feasible consumption/investment plans satisfy

$$\begin{aligned} c_{1t} + x_{1t} &= \phi_1(k_{1t}, \ell_{1t}) = k_{1t} \\ c_{2t} + x_{2t} &= \phi_2(k_{2t}, \ell_{2t}) = \ell_{2t} \quad . \end{aligned}$$

where

$$\begin{aligned} k_{1t} + k_{2t} &= k_t \\ \ell_{1t} + \ell_{2t} &= \ell_t \quad . \end{aligned}$$

The initial value of k_t is $\bar{k}_0$. ℓ_t is equal to 1. (In other words, all variables are expressed in per capita terms.)

a) Carefully define a competitive equilibrium for this economy.

b) Reduce the equilibrium conditions for this economy to two difference equations in k_t and c_t, and a transversality condition. Here $c_t = d c_{1t}^{a_1} c_{2t}^{a_1}$ is aggregate consumption.

c) Suppose now that there is a world made up of m different countries all with the same technologies and preferences, but different endowments, $\bar{L}^j \bar{k}_0^j$ and $\bar{L}^j$. (That is, there is a measure $\bar{L}^j$ of consumers, each of whom is endowed with 1 unit of labor in every period and $\bar{k}_0^j$ units of capital in period 0.) Suppose that there is no international borrowing or lending. Define an equilibrium for the world economy. Prove that in this equilibrium the variables $c_{it} = \sum_{j=1}^{m} \bar{L}^j c_{it}^j \Big/ \sum_{j=1}^{m} \bar{L}^j$, $k_t = \sum_{j=1}^{m} \bar{L}^j k_t^j \Big/ \sum_{j=1}^{m} \bar{L}^j$, p_{it}, r_t, and w_t satisfy the equilibrium conditions for the equilibrium in part a where $\bar{k}_0 = \sum_{j=1}^{m} \bar{L}^j \bar{k}_0^j \Big/ \sum_{j=1}^{m} \bar{L}^j$.

d) State and prove versions of the factor price equalization theorem, the Stolper-Samuelson theorem, the Rybczynski theorem, and the Heckscher-Ohlin theorem for this particular world economy.

e) Consider the case where $\delta = 1$. Let $z_0 = c_0 / \left(\beta r_0 k_0\right)$ and $z_t = c_{t-1} / k_t$, $t = 1, 2, \ldots$. Transform the two difference equations from part b into two difference equations in k_t and z_t. Show that

$$\frac{k_t^i - k_t}{k_t} = \frac{z_t}{z_{t-1}} \left(\frac{k_{t-1}^i - k_{t-1}}{k_{t-1}} \right) = \frac{z_t}{z_0} \left(\frac{\bar{k}_0^i - \bar{k}_0}{\bar{k}_0} \right).$$

f) Suppose again that $\delta = 1$. Let $s_t = c_t / y_t$ where $y_t = p_{1t} k_t + p_{2t} = d k_t^{\alpha_1}$. Transform the two difference equations from part b into two difference equations in k_t and s_t. Show that

$$\frac{y_t^i - y_t}{y_t} = \frac{s_t}{s_{t-1}} \left(\frac{y_{t-1}^i - y_{t-1}}{y_{t-1}} \right) = \frac{s_t}{s_0} \left(\frac{y_0^i - y_0}{y_0} \right)$$

where $y_t^i = p_{1t} y_{1t}^i + p_{2t} y_{2t}^i = r_t k_t^i + w_t$.

g) Suppose that $\delta = 1$, but that the utility function is

$$\sum_{t=0}^{\infty} \beta^t \log(a_1 c_{1t}^b + a_2 c_{2t}^b)^{1/b}$$

and that the production function for investment is

$$k_{t+1} = d\,(a_1 x_{1t}^b + x_{2t}^b)^{1/b}.$$

Explain the importance of the results in parts a-g in this world.

2. Suppose again that $\delta = 1$, that $c_t = d c_{1t}^{\alpha_1} c_{2t}^{\alpha_2}$ and that $k_{t+1} = d x_{1t}^{\alpha_1} x_{2t}^{\alpha_2}$. Now suppose that

$$\begin{aligned} c_{1t} + x_{1t} &= \phi_1(k_{1t}, \ell_{1t}) = \theta_1 \ell_{1t}^{1-\alpha_1} k_{1t}^{\alpha_1} \\ c_{2t} + x_{2t} &= \phi_2(k_{2t}, \ell_{2t}) = \theta_2 \ell_{2t}^{1-\alpha_2} k_{2t}^{\alpha_2} \end{aligned}.$$

a) Let $F(k,\ell)$ be the maximum value of

$$\begin{aligned} &\max \; d y_{1t}^{a_1} y_{2t}^{a_2} \\ \text{s.t. } & y_1 = \theta_1 \ell_1^{1-\alpha_1} k_1^{\alpha_1} \\ & y_2 = \theta_2 \ell_2^{1-\alpha_2} k_2^{\alpha_2} \\ & k_1 + k_2 = k \\ & \ell_1 + \ell_2 = \ell \\ & k_j, \ell_j \geq 0. \end{aligned}$$

Show that $F(k,\ell)$ has the form $Dk^A \ell^{1-A}$.

b) Suppose now that there is a world made up of m different countries all with the same technologies and preferences, but different endowments, $\bar{L}^j \bar{k}_0^j$ and $\bar{L}^j$. Suppose that there is no international borrowing or lending and there are no international capital flows. Define an equilibrium for the world economy.

c) Using the answers to parts a and b, show that necessary and sufficient conditions for the integrated equilibrium approach to work for all $t = T, T+1, \ldots,$ is that

$$\kappa_1 k_t \geq k_t^i \geq \kappa_2 k_t \text{ for all } i = 1, \ldots, m \text{ and all } t = T, T+1, \ldots.$$

For some $\kappa_1, \kappa_2 > 0$.

d) Suppose that, in some period T,

$$\kappa_1 k_T \geq k_T^i \geq \kappa_2 k_T \text{ for all } i = 1, \ldots, m.$$

Use the answers to parts a, b, and c and the answer to part f of question 3 to calculate analytical expressions for the equilibrium values of the variables in part b for all $t = T, T+1, \ldots$. [Hint: You can show that $\kappa_1 k_t \geq k_t^i \geq \kappa_2 k_t$ for all $i = 1, \ldots, m$ and all $t = T, T+1, \ldots$.]

3. Find data to calculate the bilateral real exchange rate between two countries who have a bilateral trade relation that is important to at least one of the countries. Find data on the prices of traded goods in these two countries. Calculate a decomposition of the bilateral real exchange rate of the form

$$rer_t = rer_t^T + rer_t^N,$$

where rer_t is the natural logarithm of the bilateral real exchange rate and rer_t^T is the logarithm of the bilateral real exchange rate for traded goods. Calculate the correlation between rer_t and rer_t^N in levels, in 1 year differences, and in 4 year differences. Calculate ratio of the standard deviations of rer_t and rer_t^N in levels, in 1 year differences, and in 4 year differences. Calculate a variance decomposition of rer_t in terms of rer_t^T and rer_t^N in levels, in 1 year differences, and in 4 year differences.