## INTERNATIONAL FINANCE GPEM - UNIVERSITAT POMPEU FABRA

## **PROBLEM SET #3**

1. Consider a world with two countries. In each country i, i = 1, 2, the consumer has the utility function

$$\sum_{t=0}^{\infty} \beta^{t} [z_{t}^{i} \log(a_{1}c_{1t}^{i\rho} + a_{2}c_{2t}^{i\rho})^{1/\rho} + (1-z_{t}^{i}) \log(\overline{\ell}^{i} - \ell_{t}^{i})].$$

Here  $z_t^i$  is a preference parameter that can vary over time,  $1 > \rho > 0$ ,  $a_i > 0$ , and  $a_1 + a_2 = 1$ . Consumer *i* is endowed with  $\overline{\ell}^i$  units of labor and  $\overline{k}^i = 1$  units of capital in each period. Good *i*, *i* = 1,2, is produced only in country *i* using the production technology

$$y_{it} = \theta_{it} k_{it}^{\mu_i} \ell_{it}^{1-\mu_i}.$$

Here  $\theta_{it}$  is a technology parameter that can vary over time. Although both  $z_t^i$  and  $\theta_{it}$  can vary over time, they do so deterministically. There is no uncertainty.

a) Define an Arrow-Debreu equilibrium for this economy.

b) Define a sequential markets equilibrium for this economy.

c) Define a social planner's problem that the equilibrium allocations in both parts a and b solve.

In parts d and e consider a symmetric model in which  $a_1 = a_2 = 0.5$ ,  $\bar{\ell}^1 = \bar{\ell}^2$ , and  $\mu_1 = \mu_2$ . You can go a long way in answering these questions analytically. You can learn a lot, and have some real fun, however, writing a simple computer program that solves for equilibria for different values of the parameters.

d) Suppose that  $z_t^1 = z_t^2 = z > 0$ , but that the technology parameters  $\theta_{it}$  follow the cyclical pattern

$$(\theta_{10}, \theta_{11}, \theta_{12}, \theta_{13}, ...) = (\theta, \underline{\theta}, \theta, \underline{\theta}, ...)$$
$$(\theta_{20}, \theta_{21}, \theta_{22}, \theta_{23}, ...) = (\underline{\theta}, \overline{\theta}, \underline{\theta}, \overline{\theta}, ...)$$

where  $\overline{\theta} > \underline{\theta} > 0$ . Characterize the solution to the symmetric social planner's problem where welfare weights are equal. Calculate real GDP, the trade balance, and the real exchange rate. (You need to define a base year and choose a price index.)

e) Suppose now that  $\theta_{1t} = \theta_{2t} = \theta > 0$ , but that the preference parameters  $z_t^i$  follow the cyclical pattern

$$(z_0^1, z_1^1, z_2^1, z_3^1, ...) = (\overline{z}, \overline{z}, \overline{z}, \overline{z}, ...)$$
$$(z_0^2, z_1^2, z_2^2, z_3^2, ...) = (\overline{z}, \overline{z}, \overline{z}, \overline{z}, ...)$$

where  $\overline{z} > \underline{z} > 0$ . Repeat the analysis of part d.

2. Choose two countries that are important (to each other) trading partners, one with higher output per worker than the other. Use Summers-Heston data to try to answer Lucas's question: To what extent can differences in output per worker be explained by differences in capital per worker? Use both Summers-Heston data on capital per worker and IMF real interest rate data to try to make inferences about differences in capital per worker be worker across countries. Discuss what you have learned from this exercise and what more you think would be relevant in explaining differences in output per worker across countries.

3. Find annual time series data on real output, real investment, hours worked, and working age population for some country. If you have sufficient data for other variables, calibrate an annual deprecation rate  $\delta$  and a capital share  $\alpha$ . Otherwise, use the values  $\delta = 0.05$  and  $\alpha = 0.30$  in what follows.

a) Use the data for real investment to construct a series for the capital stock following the rule

$$\begin{split} K_{t+1} &= (1-\delta)K_t + I_t \\ K_{T_0} &= \overline{K}_{T_0} \;. \end{split}$$

where  $T_0$  is the first year for which you have data on output and investment. Choose  $\overline{K}_{T_0}$  so that

$$K_{T_0+1} / K_{T_0} = (K_{T_0+10} / K_{T_0})^{1/10}.$$

b) Repeat part a, but choose  $\overline{K}_{T_0}$  so that

$$K_{T_0} / Y_{T_0} = \left( \sum_{t=T_0}^{T_0+9} K_t / Y_t \right) / 10.$$

c) Compare the two series constructed in parts a and b.

d) Perform a growth accounting exercise for this economy. That is, decompose the growth and fluctuation in real GDP per working-age person into three factors, one of which depends on total factor productivity, one of which depends on the capital/output ratio, and the third of which depends on hours worked per working-age person. Discuss what happens during different time periods.