Ph.D. Preliminary Examination

MICROECONOMIC THEORY

MINORS

Fall 2009

The time limit for this exam is \(3\frac{1}{4}\) hours. Notation:

\[\text{IR} \] is the set of real numbers
\[\text{IR}^n_+ = \{x \in \text{IR}^n : x_1 \geq 0 & \ldots & x_n \geq 0\}\]
\[\text{IR}^n_{++} = \{x \in \text{IR}^n : x_1 > 0 & \ldots & x_n > 0\}\]

For vectors \(x = (x_1, \ldots, x_n)\) and \(y = (y_1, \ldots, y_n)\) in \(\text{IR}^n\):

\[x \geq y \text{ means } x_1 \geq y_1 & \ldots & x_n \geq y_n.\]

Answer Question I.1 (required);

AND

Answer three additional questions, one from each of Parts II, III, and IV. (So the total number of questions to be answered for the exam is four).

You may use calculators to make calculations during the examination. However: in answering any question that requires you to justify your answer, if you do use a calculator, you must mention at what point in your answer you obtained results using a calculator, and what it was you were seeking to calculate.

BE SURE you clearly define all **boldfaced/underlined** terms.

NOTE: This examination should have 15 pages including this one (Check to make sure!)
Part I

Answer Question I.1 from Part I.
Question I.1

Consider a two-consumer, two-good pure exchange economy. Consumer 1 has a preference relation represented by the utility function:

\[ U^1(x_1, y_1) = x_1 y_1 + 10 \]

for \( x_1 \geq 0 \) and \( y_1 \geq 0 \). Consumer 2 has a preference relation represented by the utility function:

\[ U^2(x_2, y_2) = x_2^2 y_2^2 \]

for \( x_2 \geq 0 \) and \( y_2 \geq 0 \). Consumer 1’s initial endowments for each good are \( x_1^e = 0 \) and \( y_1^e = 15 \). Consumer 2’s initial endowments for each good are \( x_2^e = 20 \) and \( y_2^e = 15 \).

(a) Construct an Edgeworth box diagram (to scale, on the graph paper provided) showing (and labeling) the endowment allocation and typical indifference contours/sets including the directions of increasing preference for each consumer.

(b) Characterize the set of Pareto Efficient allocations and illustrate them on your diagram.

(c) Characterize the competitive (Walras) equilibrium allocation including the equilibrium prices and illustrate it on your diagram.
Part II

Answer one (1) question from Part II
Question II.1

Consider the function
\[ f(p_1, p_2, m) = \frac{m^{a_1+a_2}}{p_1^{a_1}p_2^{a_2}} \]

Where \( p_1 > 0, \ p_2 > 0, \) and \( m > 0 \) are the price of good 1, \( x_1, \) the price of good 2, \( x_2, \) and income; and \( a_1 \) and \( a_2 \) are constant parameters.

(a) Derive the Marshallian demands for good 1 and 2 assuming this function is a valid indirect utility function for an individual with a strictly convex, locally nonsatiated, continuous, and rational preference relation \( \succ \) on \( X = \mathbb{R}_+^2. \)

(b) List four properties that an indirect utility function must satisfy. What restrictions on \( a_1 \) and \( a_2, \) if any, are required to satisfy these properties?

(c) Assuming \( a_1 \) and \( a_2 \) satisfy the conditions necessary for \( f(p_1, p_2, m) \) to be a valid indirect utility function, derive the corresponding expenditure function and Hicksian demands for \( x_1 \) and \( x_2. \)

(d) Suppose the government is considering a unit tax of \( t \) on either \( x_1 \) or \( x_2. \) Derive conditions under which the consumer would be better off with a tax on good 1 instead of one on good 2. (Note: For these alternative tax policies, only consider the partial equilibrium effects and assume that the revenues from the tax are not used to benefit the individual in any way).
Question II.2

Consider a firm that produces a single output, $q \geq 0$, using two factors, $z_1 \geq 0$ and $z_2 \geq 0$ with an input requirement set that is regular, monotonic, and strictly convex. Assume the firm operates with competitive factor markets. The cost function for the firm is

$$c(r_1, r_2, q) = \left( r_1 + \sqrt{r_1r_2} + r_2 \right) q^2$$

where $r_1 > 0$ and $r_2 > 0$ are factor prices.

(a) Derive the firm’s conditional factor demands.

(b) What are four conditions that a valid cost function must satisfy? Verify that these conditions hold for the cost function above.

(c) Derive the profit function and unconditional factor demands assuming a competitive output market where the price of output is $p > 0$.

(d) What is the relationship between the own-price elasticity of the conditional and unconditional factor demands? What is the intuition underlying this relationship?
Question II.3

Suppose the utility function

\[ U(x_1, x_2) = \min \{x_1, 2x_2\} \]

represents a consumer’s rational and continuous preference relation \( \succeq \) on \( X = \mathbb{R}^2_+ \).

(a) Is the consumer’s preference relation monotone? Justify your answer.

(b) Is the consumer’s preference relation convex? Justify your answer.

(c) Is the consumer’s preference relation homothetic? Justify your answer.

(d) Given the competitive prices \( p_1 > 0 \) and \( p_2 > 0 \) for \( x_1 \) and \( x_2 \) and income \( m \), derive the consumer’s Marshallian demands and indirect utility function.

(e) Use the Slutsky Equation to derive the effect of a change in \( p_1 \) on the Hicksian demand for \( x_1 \). Explain the intuition of your result in the context of the preference relation represented by \( U(x_1, x_2) \).
Part III

Answer one (1) question from Part III
Question III.1

Consider an economy with one private good and one public good. There are two consumers \((i = 1, 2)\) with Cobb-Douglas utility functions

\[ u^i(x_i, z) = x_iz, \]

where \(x_i \geq 0\) denotes consumption of the private good and \(z \geq 0\) consumption of the public good. Public good can be produced using private good as input according to the production function \(z = y\), where \(y \geq 0\) is the input quantity of private good. Initial endowments of private good are \(\omega^1 = 12\), \(\omega^2 = 8\).

Consider a strategic game, called subscription game, in which consumers simultaneously choose their contributions to the production of public good and consume the resulting quantity of public good and whatever remains of their endowments of private good.

(a) Find a Nash equilibrium of the subscription game. Justify that the strategies you find are indeed equilibrium strategies. Explain clearly what assumptions about consumers’ knowledge of each other characteristics (such as endowments and utility functions) have you made in your derivation of equilibrium strategies.

(b) Are the equilibrium strategies strictly dominant for each consumer?

(c) Is the equilibrium allocation of public and private goods Pareto optimal? Justify your answer.
Consider the following optimal insurance problem: An agent with preferences represented by expected utility function (strictly increasing and differentiable) has a deterministic initial wealth $w$ and faces a risk of losing $L > 0$ with probability $\pi$ or not losing it with probability $1 - \pi$. The agent can purchase insurance against the loss by paying the premium of $p$ per dollar of coverage.

(a) Show that if the agent is strictly risk averse and the premium equals the probability of loss, i.e., $p = \pi$, then the agent’s optimal level of insurance is full insurance (i.e., coverage in the amount of $L$).

(b) Show that the agent is strictly risk averse and the premium (strictly) exceeds the probability of loss, i.e., $p > \pi$, then the agent’s optimal level of insurance is (strictly) less than full insurance.
Part IV

Answer one (1) question from Part IV
There are two risk neutral decision makers, indexed by the subscripts \( i = 1, 2 \), who can observe random variables \( \tilde{s}_1 \) and \( \tilde{s}_2 \) respectively, where

\[
\tilde{s}_1 = \begin{cases} 
    H & \text{with probability } \pi_1 \\
    T & \text{with probability } 1 - \pi_1
\end{cases}
\]

and

\[
\tilde{s}_2 = \begin{cases} 
    H & \text{with probability } \pi_2 \\
    T & \text{with probability } 1 - \pi_2
\end{cases}
\]

Each decision maker will have the opportunity to purchase one unit of a risky financial asset (i.e., to pay to make a bet) at a price of $100 having the (risky) payoff \( r(\tilde{s}_1, \tilde{s}_2) \), where

\[
\begin{align*}
r(H, T) &= r(T, H) = 0 \\
r(H, H) &= $100 \\
r(T, T) &= $300
\end{align*}
\]

(a) Assume that the random variables \( \tilde{s}_1 \) and \( \tilde{s}_2 \) are perfectly correlated. How much would decision maker \( i \) be willing to pay to observe \( s_i \) before he or she must decide whether to participate in the bet? How much more would \( i \) be willing to pay to also observe the other random variable \( s_{i'} \) in addition to \( s_i \), where \( i' \neq i \) before deciding whether to participate in the bet?

(b) Now assume for the rest of this question that \( \tilde{s}_1 \) and \( \tilde{s}_2 \) are independent. Suppose that the two decision makers can pay to build a communication link between them and, if the link is built, then they are both required to share their information freely and honestly—i.e., to show the other decision maker \( i' \) the true value of one’s own random variable \( s_i \) \((i' \neq i)\) without any exchange of money. If the link is built (i.e., if the total that the decision makers are willing to pay exceeds the cost of the link), the Question IV.1 continued on the next page
Question IV.1 continued:

actual cost of building the link is shared equally between the two decision makers. The decision regarding whether to bet is made after the information is shared and they do not bet with or against each other. [Do not worry about strategic misrepresentations of the amount that decision makers are willing to pay.] What would decision maker $i$ be willing to pay to build a communication link before observing $s_i$?

(c) Suppose that decision maker $i$ observes $s_i$ and then must state his or her willingness to pay for the communication link, assuming that $\pi_1 = \pi_2 = 1/2$. (As above, after the information is shared—if the communication link is built—the decision makers decide whether to participate in the bet with a third party.

(d) Notice that the answers to parts (b) and (c) imply that seeing the other decision maker’s willingness to pay for the communication link tells you what the decision maker has observed. Find cost ranges for the communications link (where the link is built if and only if the total willingness to pay equals or exceeds the cost) that would enable the decision makers to make their decisions regarding the bet based only on whether the link is built, so that actual information sharing is not needed.

(e) Write a short paragraph to compare and contrast this situation to rational expectations equilibrium.
Question IV.2

Parts of this question could be viewed as relating to the current health care reform debate. However, do not express your opinions on this issue in answering. You are asked to use economics to analyze a very specific simple model.

Consider a situation of a competitive model having three types of consumers: The healthy, denoted by subscript $H$, are ill with probability zero, the middle group, denoted by subscript $M$, are ill with probability one half, and the sick group, denoted by subscript $S$, are ill with probability one. Each consumer knows to which group he or she belongs (but insurers do not know this information) and all groups are of the same size. Ill individuals require treatment at cost $T > 0$, and all individuals have income or wealth $W > T$. No one derives utility or disutility from either illness or treatment.

(a) If all consumers are required to purchase actuarily fair full (i.e., 100% coverage for the treatment cost of $T$ if ill) insurance, what will the premium be?

(b) Will any group or groups (if so, which ones?) strictly or weakly (specify) prefer not to purchase the insurance under some assumptions? For each group in your answer, specify the minimal assumptions that are needed in order for them to prefer not to buy the insurance.

(c) If any groups that strictly prefer to forego this insurance do not purchase it, what is the new actuarially fair premium? Now is there a remaining group that would prefer not to purchase full insurance at the new price? What minimal assumptions are needed for this result?

(d) More generally, suppose all consumers know their own health status (but it cannot be observed by the insurer) and anyone who strictly prefers no insurance can avoid purchasing actuarially fair full insurance. What can you say about the group of consumers who do buy the insurance under all of the assumptions you listed in part (b)?

Question IV.2 continues on the next page.
Question IV.2 continued:

(e) Now return to the initial situation in part (a) and suppose that consumers can choose to pay $C < T/3$ for a certificate that accurately reveals their group. If they then present their certificates to the insurer, they are allowed to purchase an actuarially fair full insurance policy that is only offered to their own group. Specify which groups, if any, purchase the certificate and which groups, if any, purchase insurance. Note conditions under which a group is indifferent between getting insurance or not. For each group that purchases insurance, specify the premium. Briefly explain your reasoning.

(f) In a brief essay (perhaps a paragraph of about 1/2–1 page, or 50–200 words) compare and contrast the situation in part (e) to Spence’s signalling model.

(g) Finally, suppose that the insurer can costlessly observe the group of each consumers and instead of full insurance, all policies require $x\%$ coinsurance, where $x \in [0,100]$ but the policies are still actuarially fair. Under what assumptions (specify them) will each group strictly or weakly (specify which) prefer to purchase such insurance for at least some $x \in (0,100)$? Briefly explain your answer.