Preliminary Examination

International Trade

Fall 2009

Answer a total of THREE (3) questions. Each question must come from a DIFFERENT part. (There are three parts, and some parts have more than one choice)
Part I

Answer the question from Part I
Question I.1

Consider an economy with two countries (home and foreign, with * denoting a foreign country variable) and two perishable goods (a and b). Representative consumers in country 1 only consume good a while consumers in country 2 only consume good b. Lifetime utility is given by

\[
\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \log c(s^t)
\]

for home consumers and by

\[
\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \log c^*(s^t)
\]

for foreign consumers, where \(\pi(s^t)\) is the time 0 probability of state \(s^t\), \(\beta\) is the common discount factor and \(c\) and \(c^*\) are home (composed of good a) and foreign (composed of good b) consumption. Consumers in each country are also endowed with 1 unit of time that they inelastically (they have no disutility from working) supply on the domestic competitive labor market.

There are two sectors in each country (one sector producing good a and one sector producing good b) so labor in each country is allocated between the two sectors.

The technology for producing the two goods in each country is constant return to scale and is represented by the following four equations

\[
a(s^t) = \alpha z(s^t) l_a(s^t)
\]
\[
b(s^t) = z(s^t) l_b(s^t)
\]
\[
a^*(s^t) = z^*(s^t) l^*_a(s^t)
\]
\[
b^*(s^t) = \alpha z^*(s^t) l^*_b(s^t)
\]

where \(a(s^t)\), \(b(s^t)\) and \(a^*(s^t)\), \(b^*(s^t)\) are the quantities of the two goods produced in each sector in the two countries, \(l_a(s^t) \geq 0, l_b(s^t) \geq 0\) and \(l^*_a(s^t) \geq 0, l^*_b(s^t) \geq 0\) are the amount of labor used in each sector in the two countries, \(z(s^t) > 0\) and \(z^*(s^t) > 0\) are exogenous, stochastic, country specific but common across sectors, productivity shocks and \(\alpha\) is a fixed parameter that determines the relative productivity of each country in each good. Assume that \(\alpha > 1\) so that the home country is more productive in producing good a and the foreign country is more productive in producing good b. In each state and in each date there are no restrictions to international trade in goods.
IMPORTANT: Answer all four questions in Section (A) (worth 10 points each), but answer only one question from Section (B) (worth 20 points).

(A) Answer all four questions in this section.

(1) Define a competitive equilibrium for this economy and write down the conditions that characterize it.

(2) Define an equal weights planning problem for this economy, write down the conditions that characterize it and show its solution can be decentralized as a competitive equilibrium. Show that the planning solution is static, in the sense, that you can solve for the endogenous variables as a function of current shocks only.

(3) Show that in the planning problem there are no realizations of the shocks $z(s^t)$ and $z^*(s^t)$ for which both countries produce strictly positive quantities of both goods

(4) Characterize the interval for the ratio of the shocks

$$\left( \frac{z(s^t)}{z^*(s^t)} \right)$$

for which, in a given period in the planning problem defined above, there is full specialization (Home country only produces good $a$ and foreign country only produces good $b$)

(B) Answer one of the three questions in this section.

(5) Define the terms of trade and the consumption based real exchange rate. Show that in the case of full specialization these prices are not unique. Solve for these prices under full specialization and under partial specialization (i.e. one country produces both goods)

(6) Show that in this economy all trade is intertemporal (there are not states under which both countries export). Are net exports in this model counter or procyclical? How about in the US data? Comment.

(7) Show that if the support of the shocks is such that the economy is always in full specialization then the allocation arising under financial autarky is equivalent to a complete markets allocation. What does this tell you about the relation between the importance of international financial markets and the volatility of country specific shocks?
Part II

*Answer one of the three questions from Part II*
Question II.1

Supporting Positive Debt

Bulow and Rogoff considered an environment in which if agents defaulted on their debts they were prohibited from borrowing again but they still could save in a state-contingent fashion at the existing rates. They claimed that in such an environment it is not possible to support positive debt. Hellwig and Lorezoni took a similar environment and claimed to show that it is possible to support positive debt.

(a) Sketch out a model and develop a version of the Bulow Rogoff argument that it is not possible to support positive debt.

(b) Sketch out an alternative version of that model in which it is possible to support positive debt.

(c) Discuss in detail the differences in assumptions.

(d) Are the assumptions needed for the Hellwig Lorenzoni argument to apply supported in the data? Explain.
Question II.2
Less Developed Economy Savings

The simplest theories with well-functioning credit markets would imply that an economy that is currently poor but knows that it soon will be rich will borrow—not save—with the rest of the world, if the rest of the world is growing more slowly. Some developing economies, like China, have been saving funds on net with the rest of the world.

(a) Sketch out a model that can potentially get a less-developed economy to save from the rest of the world instead of borrow from it.

(b) What happens in your model if you now make the financial markets function perfectly?

(c) What happens if you make the less developed economy grow at a faster rate relative to the world?
Question II.3

Borrowing with Enforcement Constraints

Consider a simple pure exchange economy with two countries and identical consumers in each country and two goods per period (apples and oranges). Suppose that contracts enforced only in a limited way. Specifically, if an agent reneges on a contract than that agent is not allowed to borrow or lend in any future period, but that agent can engage in static trade (i.e. the agent can trade apples and oranges at any given date and state).

(a) Show the competitive equilibrium of this economy solves a certain type of planning problem, often referred to as the conditional efficiency problem.

(b) Write down the natural notion of constrained efficiency for this environment by way of writing down another planning problem.

(c) Discuss when the competitive allocations will be not be constrained efficient.
Part III

Answer one of the two questions from Part III
Question III.1
A Ricardian Model with Transportation Costs and Tariffs

Consider a world with two countries. There is a representative consumer in each country who has preferences over the interval of goods $X = [0, 1]$ given by the utility function

$$\int_X \log c(x) \, dx.$$ 

In each country there is a single factor, labor. Endowments are $\bar{\ell}_1 = \bar{\ell}_2 = \bar{\ell}$. Production functions are linear but differ across countries:

$$y_j(x) = \ell_j(x)/a_j(x)$$

$$a_1(z) = e^{\alpha z}$$

$$a_2(z) = e^{\alpha(1-z)}.$$ 

Initially, there are no transportation costs or tariffs.

(a) Define an equilibrium for this model.

(b) Characterize as much as possible the patterns of specialization and trade in the equilibrium.

(c) Suppose now that there are uniform iceberg transportation costs. In particular, suppose that it now requires $(1 + \tau) a_i(x)$ units of labor in country $i$ to produce one unit of good $x$ for delivery in country $j \neq i$. Explain how your definition of equilibrium is altered and characterize as much as possible how the new equilibrium differs from that in items (a) and (b).

(d) Suppose now that the countries engage in a trade war in which each imposes an ad valorem tariff of $\tau$ on imports of good $x$ from the country. Explain how your definition of equilibrium is altered and characterize as much as possible how the new equilibrium differs from that in item (c).

(e) Calculate gross domestic product and the real income index

$$v_j = \exp \int_0^1 \log c_j(z) \, dz$$

as functions of $\tau$. Supposing that in the base period $\tau = 0$, find real GDP—that is, GDP in base period prices—as well as GDP in current prices. Compare the calculations for the equilibrium in item (d) with those in items (b) and (c).
Question III.2
Monopolistic Competition with Heterogeneous Firms and Trade

Consider an economy where the consumers have Dixit-Stiglitz utility functions and solve the problem

$$\max (1 - \alpha) \log c_0 + \frac{\alpha}{\rho} \log \int_{0}^{m} c(z)^{\rho} dz$$

subject to

$$p_0 c_0 + \int_{0}^{m} p(z) c(z) dz = w \ell$$

$$c(z) \geq 0.$$ 

Here $1 > \alpha > 0$ and $1 > \rho > 0$. Furthermore, $m > 0$ is the measure of firms, which is determined in equilibrium. Suppose that good 0 is produced with the constant-returns production function $y_0 = \ell_0$.

(a) Suppose that the producer of good $z$ takes the prices $p(z')$, for $z' \neq z$, as given. Suppose too that this producer has the production function

$$y(z) = \max \left[ x(z) (\ell(z) - f), 0 \right].$$

where $x(z) > 0$ is the firms productivity level and $f > 0$. Solve the firm’s profit maximization problem to derive an optimal pricing rule.

(b) Suppose that good 0 is produced with the constant-returns production function $y_0 = \ell_0$. Suppose that firm productivities are distributed on the interval $x \geq 1$ according to the Pareto distribution with distribution function

$$F(x) = 1 - x^{-\gamma},$$

where $\gamma > 2$ and $\gamma > \rho/(1 - \rho)$. Also suppose that the measure of potential firms is fixed at $\mu$. Define an equilibrium for this economy.

(c) Find an expression for the productivity of the least productive firm that produces. That is, find a productivity $\bar{x} > 1$ such that no firm with $x(z) < \bar{x}$ produces and all firms with $x(z) \geq \bar{x}$ produce. Relate the measure of firms that produce $m$ to the measure of potential firms $\mu$ and the cutoff $\bar{x}$. 
(d) Suppose now that there are two symmetric countries that engage in trade. Each country \( i, i = 1, 2 \), has a population of \( \bar{\ell} \) and a measure of potential firms of \( \mu \). Firms’ productivities are again distributed according to the Pareto distribution, \( F(x) = 1 - x^{-\gamma} \). A firm in country \( i \) faces a fixed cost of exporting to country \( j, j \neq i \), of \( f_e \) where \( f_e > f_d = f \) and an iceberg transportation cost of \( \tau - 1 \geq 0 \). Define an equilibrium for this world economy.

(e) Explain how to characterize the equilibrium production patterns with a cutoff value, or values, as in item (c). [You should explain carefully how to calculate any cutoff values, but you do not actually need to calculate it.] Compare this value, or these values, with that in item (c).

(f) Briefly discuss the strengths and limitations of this sort of model for accounting for firm-level data on exports.