Part 1. Dynamic Heckscher-Ohlin model

Consider a two-sector growth model in which the representative consumer has the utility function

\[ \sum_{t=0}^{\infty} \beta^t \log(c_{1t}^{\alpha_0} c_{2t}^{\alpha_2}). \]

The investment good is produced according to

\[ k_{t+1} = dx_{1t}^{\alpha_1} x_{2t}^{\alpha_2}. \]

Feasible consumption/investment plans satisfy the feasibility constraints

\[ c_{1t} + x_{1t} = \phi_1(k_{1t}, \ell_{1t}) = k_{1t}, \]
\[ c_{2t} + x_{2t} = \phi_2(k_{2t}, \ell_{2t}) = \ell_{2t}. \]

where

\[ k_{1t} + k_{2t} = k_t \]
\[ \ell_{1t} + \ell_{2t} = 1. \]

The initial value of \( k_t \) is \( \bar{k}_0 \). All of the variables specified above are in per capita terms. There is a measure \( L \) of consumer/workers.

a) Define an equilibrium for this economy.

b) Write out a social planner’s problem for this economy. Explain how solution to this social planner’s problem is related to that of the one-sector social planner’s problem

\[ \sum_{t=0}^{\infty} \beta^t \log c_t \]
\[ \text{s.t. } c_t + k_{t+1} = dk_t^{\alpha_1} \]
\[ c_t, k_t \geq 0 \]
\[ k_0 = \bar{k}_0. \]
[You can write done a proposition or propositions without providing a proof or proofs, but be sure to carefully relate the variables in the two-sector model to the variables in the one-sector model.]

c) Solve the one-sector social planner’s problem in part b. [Recall that the policy function for investment has the form \( k_{t+1}(k_t) = Adk^\alpha \) where \( A \) is a constant that you remember or can determine with a bit of algebra and calculus.]

d) Suppose now that there is a world made up of \( n \) different countries, all with the same technologies and preferences, but with different constant populations, \( L' \), and with different initial capital-labor ratios \( \bar{k}_0 \). Suppose that goods 1 and 2 can be freely traded across countries, but that the investment good cannot be traded. Suppose too that there is no international borrowing. Define an equilibrium for the world economy.

e) State and prove versions of the factor price equalization theorem, the Stolper-Samuelson theorem, the Rybczynski theorem, and the Heckscher-Ohlin theorem for this particular world economy.

f) Let \( s_i = c_i / y_i \) where \( y_i = p_i k_i + p_{2i} = dk_i^\alpha \) is world GDP per capita. Transform the first-order conditions for the one-sector social planner’s problem in part b into two difference equations in \( k_t \) and \( s_t \). Use the first-order conditions for the consumer’s problem of the equilibrium in part d to show that

\[
\frac{y_i^t - y_i}{y_i} = \frac{s_t}{s_{t-1}} \left( \frac{y_i^{t-1} - y_i^{t-1}}{y_i^{t-1}} \right) = \frac{s_t}{s_0} \left( \frac{y_i^0 - y_0}{y_0} \right).
\]

g) Use the solution to the one-sector social planner’s problem in part c to solve for \( s_t \). Discuss the economic significance of the result that you obtain.
Part 2. Capital flows with a bubble

Consider a small open economy with many identical consumers with utility function

\[ \sum \beta^t \log c_t. \]

Each consumer receives a constant endowment \( e \) in each period and faces the world interest rate \( r = 1/\beta - 1 \). Let \( b_t \) denote the consumer’s net holding of bonds and \( q \) the bond price.

(i) Suppose that \( b_0 = 0 \), solve for equilibrium consumption and current account for all \( t \).

Now imagine that each consumer in the country owns \( \theta_0 \) worthless and unproductive units of land. Suppose that in period \( T \), suddenly and unexpectedly, all consumers (at home and abroad) start believing that each unit of land has value \( p_T > 0 \) and that the land’s value will evolve in all future periods according to the law of motion

\[ p_{t+1} = \beta^{-1} p_t. \]

Denote with \( \theta_t \) the holdings of land by the consumers

(ii) Write the consumer’s budget constraint including holdings of both bonds and land.

(iii) Derive the optimal path for consumption and for net wealth \( a_t = p_t \theta_t + b_t \).

Argue that the path for \( b_t \) and \( \theta_t \) is not unique but the path for \( a_t \) is unique.

(iv) Show that the country runs a current account deficit in all periods, either financed by borrowing (bond issues) or by land sales. Argue that, as this goes on, the world will be absorbing larger and larger amounts of either domestic bonds or domestic land.

(v) Suppose at some \( T' > T \) consumers in our economy and in the rest of the world realize that the bubble equilibrium is unsustainable and the bubble collapses, i.e., suddenly and unexpectedly we go to \( p_t = 0 \) for \( t \geq T' \). What happens to the current account of the country?

(vi) Suppose that in the bubble period some consumers sold bonds and some consumers sold land. Who is better off after the bubble collapses and why?
Part 3. Sudden stops and output drops

a. Carefully describe an economy in which a sudden tightening in a country’s borrowing constraint leads to a fall in output.

b. Establish your result in part a) as precisely as you can.

c. Now, describe an economy in which a sudden tightening in a country’s borrowing constraint leads to an increase in output.

d. Carefully explain the intuition behind why your result in part c) is different from your result in part a).

e. Try to draw some general lessons about what ingredients are needed in the model to generate the differing results.