Ph.D. Preliminary Examination

International Trade and Payments Theory

Fall 2011

Answer ALL three parts

Please make your answers neat and concise. Make whatever assumptions you need to answer the questions. Be sure to state your assumptions clearly. Parts have equal weight. You have 4 hours and 30 minutes to complete the exam.
Part 1. Answer both questions

a. The allocation puzzle

Discuss what constitutes the ”allocation puzzle” (as defined by Gourinchas and Jeanne, 2009). Briefly outline a potential solution for it.

b. Efficient allocations with limited enforcement

Consider a one-good, two agents (or countries) economy in which preferences of each agent are given by the standard

\[ E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_{it}^{1-\sigma}}{1-\sigma} \]

Each agent (denoted by \( i \)) receives in each period an endowment \( y_{it} \). The stochastic process for endowment in both countries \( \{y_{1t}, y_{2t}\} \) is a Markov chain with \( N \) states and transition matrix given by \( \Pi \). In each period each country has the option to exit the economy and if it does so it stays in autarky forever. In period 0 both countries start with the same income.

1) Write down the equal weights planning problem that solves for the constrained efficient allocations in this environment (make sure to include enforcement constraints).

2) Write down the first order conditions for the planning problem.

3) Use those first order conditions to show that if there exists a state in which an enforcement constraint binds then the share of aggregate income consumed by each country is not constant across all possible realizations of \( \{y_{1t}, y_{2t}\} \).
Question 2

Consider a model with two countries. Suppose labor is the only factor of production and assume that the number of workers N is the same in each country. Each worker is endowed with one unit of time.

There are two sectors, manufacturing and services. In the service sector, one unit of labor produces one unit of services. (Note service productivity is the same in both countries).

The manufacturing sector follows Eaton and Kortum (2002). There are a continuum of differentiated manufacturing goods, \( j \in [0, 1] \). Let \( T_i \) be the productivity of country \( i \). This governs the distribution of productivity draws, so that the c.d.f. of productivity \( z \) in country \( i \) is

\[
F_i(z) = e^{-T_i z^{1+\theta}}.
\]

Suppose there is a CES aggregator of the differentiated manufacturing goods \( j \),

\[
Q = \left[ \int_0^1 q(j)^{\sigma-1} \, dj \right]^{\frac{\sigma}{\sigma-1}},
\]

where \( \sigma < 1 + \theta \), and \( Q \) is a level of the manufacturing composite, while \( q(j) \) is a quantity of differentiated good.

Finally utility of manufactured good composite and services is Cobb-Douglas,

\[
U = Q^\mu S^{1-\mu},
\]

where \( S \) is the quantity of services.

Let \( \tau \geq 1 \) be the iceberg cost of shipping manufactured goods between the two locations. Assume there is no iceberg cost to shipping services.

Let \( N^S_i \) and \( N^M_i \) be the quantity of labor working in each sector at country \( i \).

Let \( w_i \) be the wage at location \( i \), let \( P_i^M \) be the price index for the manufactured good composite at location \( i \). To simplify calculations, we review some of the results of EK that you can take as given. Define \( \Phi_1 \) and \( \Phi_2 \) by

\[
\Phi_1 = T_1 w_1^{-\theta} + T_2 w_2^{-\theta} \tau^{-\theta},
\]

\[
\Phi_2 = T_1 w_1^{-\theta} \tau^{-\theta} + T_2 w_2^{-\theta}.
\]

Then \( P_i^M \) equals

\[
P_i^M = \gamma \Phi_i^{-1/\theta},
\]
for a constant $\gamma$. Also, let $n \neq i$. Then the probability that country $i$ is the lowest cost provider to country $n$ equals

$$
\pi_{ni} = \frac{T_i w_i^{-\theta} \tau^{-\theta}}{T_n w_n^{-\theta} + T_i w_i^{-\theta} \tau^{-\theta}},
$$

if $n \neq i$, and the probability that $i$ is the lowest cost provider to itself is

$$
\pi_{ii} = \frac{T_i w_i^{-\theta}}{T_n w_n^{-\theta} \tau^{-\theta} + T_i w_i^{-\theta}}.
$$

1. Suppose first $T_1 = T_2$. Solve for the equilibrium levels of trade flows.

2. Suppose instead that $T_2$ is large relative to $T_1$ and that $\mu$ is close to one. Derive equations pinning down the equilibrium in this case.

3. Provide a condition under which there is no trade in the service good, but there is trade in the manufactured good.

4. Provide a condition under which there is no trade in manufactured goods.

5. Next eliminate the service sector by setting $\mu = 1$. Let $X_{ni}$ be the spending of country $n$ on goods from country $i$ and let $\lambda = X_{11} / (X_{11} + X_{12})$ denote the share of country 1’s spending that is on goods produced in country 1, i.e. the domestic expenditure share. Let $\varepsilon$ be defined by

$$
\varepsilon \equiv \frac{\partial \ln (X_{12}/X_{11})}{\partial \ln \tau}.
$$

Let the wage in country 1 be the numeraire, $w_1 = 1$. Let $W$ denote real income in country 1. Let $\tau^0$ be an initial level of the trade friction and let $\tau'$ be a new level. Show that the ratio of welfare (or real income) in the two cases can be written as

$$
\frac{W'}{W^0} = \left( \frac{\lambda'}{\lambda^0} \right)^{\frac{1}{\varepsilon}}.
$$

6. In a recent paper, Arkolakis, Costinot, and Rodriguez-Clare show that in wide class of models, including versions of the Armington, Krugman, and Melitz models of trade, the welfare gains from trade can be written in the same form as just derived for the Eaton and Kortum model. Briefly comment on the potential significance or interpretation of this result.
Question 3. Monopolistic competition with heterogeneous firms and trade

Consider an economy where the consumers have Dixit-Stiglitz utility functions and solve the problem

$$\max_{c} \left(1 - \alpha \right) \log c_{o} + \frac{\alpha}{\rho} \log \int_{0}^{m} c(z)^{\rho} dz$$

subject to

$$p_{o} c_{o} + \int_{0}^{m} p(z) c(z) dz = w \ell + \pi$$

$$c(z) \geq 0.$$ 

Here $1 > \alpha > 0$ and $1 > \rho > 0$. Furthermore, $m > 0$ is the measure of firms, which is determined in equilibrium. Suppose that good 0 is produced with the constant-returns production function $y_{0} = \ell_{0}$.

a) Suppose that the producer of good z takes the prices $p(z')$, for $z' \neq z$, as given. Suppose too that this producer has the production function

$$y(z) = \max \left[ x(z) \left( \ell(z) - f \right), 0 \right].$$

where $x(z) > 0$ is the firm’s productivity level and $f > 0$. Solve the firm’s profit maximization problem to derive an optimal pricing rule.

b) Suppose that good 0 is produced with the constant-returns production function $y_{0} = \ell_{0}$. Suppose that firm productivities are distributed on the interval $x \geq 1$ according to the Pareto distribution with distribution function

$$F(x) = 1 - x^{-\gamma},$$

where $\gamma > 2$ and $\gamma > \rho/(1 - \rho)$. Also suppose that the measure of potential firms is fixed at $\mu$. Define an equilibrium for this economy.

c) Suppose that, in equilibrium not all potential firms actually produce. Find an expression for the productivity of the least productive firm that produces. That is, find a productivity $\bar{x} > 1$ such that no firm with $x(z) < \bar{x}$ produces and all firms with $x(z) \geq \bar{x}$ produce. Relate the measure of firms that produce $m$ to the measure of potential firms $\mu$ and the cutoff $\bar{x}$.

d) Suppose now that there are two countries that engage in trade. Each country $i$, $i = 1, 2$, has a population of $\ell_{i}$ and a measure of potential firms of $\mu$. Firms’ productivities are again distributed according to the Pareto distribution, $F(x) = 1 - x^{-\gamma}$. A firm in country $i$ faces a fixed cost of exporting to country $j$, $j \neq i$, of $f_{i}$ where $f_{e} > f_{d} = f$. Each country also imposes an ad valorem tariff $\tau$ on imports of differentiated goods from the other country. The revenue from these tariffs is redistributed in lump-sum form to the consumer in that country. Define an equilibrium for this world economy.

e) Suppose that the two countries in part d are symmetric in the sense that $\ell_{1} = \ell_{2} = \ell$ and $\mu_{1} = \mu_{2} = \mu$. Explain how to characterize the equilibrium production patterns with a cutoff value, or values, as in part c. [You should explain carefully how to
calculate any cutoff values, but you do not actually need to calculate it. Compare this value, or these values, with that in part c. Draw a graph depicting what happens when a closed economy opens to trade.

f) Discuss the strengths and limitations of this sort of model for accounting for firm-level data on exports.