Ph.D. Preliminary Examination

MACROECONOMIC THEORY

Fall 2012

Majors and Minors: Answer ALL FOUR parts.

Please read the instructions before each part and make your answers neat and concise. Make whatever assumptions you need to answer the questions. Be sure to state your assumptions clearly. You have 5 hours to complete the exam.
Part 1. Please answer the following question

[Overlapping Generations] Consider an overlapping generations economy in which there is one good in each period and each generation, except the initial one, lives for two periods. The representative consumer in generation \( t, t = 1, 2, \ldots \), has the utility function

\[
\log c^t_i + \log c^{t+1}_{i+1}
\]

and the endowment \((w^t_1, w^{t+1}) = (3, 2)\). The representative consumer in generation 0 lives only in period 1, prefers more consumption to less, and has the endowment \( w^0_i = 2 \). There is no fiat money.

a) Define an Arrow-Debreu equilibrium for this economy. Calculate the unique Arrow-Debreu equilibrium.

b) Define a sequential markets equilibrium for this economy. Calculate the unique sequential markets equilibrium.

c) Define a Pareto efficient allocation. Prove either that the equilibrium allocation in part a is Pareto efficient or prove that it is not.

d) Suppose now that there is a continuum of measure 1 of two types of consumers in each generation \( t, t = 1, 2, \ldots \). Both types of consumers have the utility function

\[
\log c^t_{1i} + \log c^t_{2i}, \ i = 1, 2.
\]

Consumers of type 1 have the endowment \((w^t_{1i}, w^{t+1}) = (3, 2)\), while consumers of type 2 have the endowment \((w^t_{2i}, w^{t+1}) = (2, 2)\). The two representative consumers in generation 0 live only in period 1, prefer more to less, and have the endowment \( w^0_i = 2 \), \( i = 1, 2 \). There is no fiat money. Define an Arrow-Debreu equilibrium for this economy.

e) Define a sequential markets equilibrium for this economy.

f) In the equilibrium allocation is \( c^t_i = 3 \)? Explain carefully why or why not.
Part 2. Please answer the following question

Consider a stochastic cash credit goods in which households have preferences of the form $\sum_{t=0}^{\infty} \beta^t U(c_{1t}, c_{2t})$, where $c_{1t}$ and $c_{2t}$ denote consumption of cash and credit goods respectively, $U$ is strictly concave, differentiable and satisfies the Inada conditions, and $0 < \beta < 1$ is the discount factor. Households are endowed with $y$ units of a composite good which can be converted into cash and credit goods according to the resource constraint

$$c_{1t} + c_{2t} = y.$$ 

The endowment $y$ follows a first order Markov process and is the only source of uncertainty in the economy. The securities market meets at the beginning of the period. The household’s securities market constraint (for a deterministic version of the economy) is

$$M_t + B_t = (M_{t-1} - p_{t-1} c_{1t-1}) - p_{t-1} c_{2t-1} - p_{t-1} y + R_{t-1} B_{t-1} + T_{t-1}$$

where $M_t$ denotes cash balances, $B_t$ denotes holdings of one-period debt, $p_t$ denotes the price level, $R_t$ denotes the (gross) interest rate on debt and $T_t$ denotes lump-sum transfers by the government. The cash in advance constraint (for a deterministic version of the economy) is

$$p_t c_{1t} \leq M_t.$$ 

Assume real debt holdings are bounded below by a large negative number.

(a) Define a competitive equilibrium. In particular, be precise about what allocations depend on.

(b) Assume government policy is characterized by a sequence of constant interest rates, $R_t = R > 1$ for all $t$ and for all realizations of the exogenous uncertainty and that initial holdings of nominal assets are zero. Characterize the set of competitive equilibria. Is this set a singleton? What is the set of real allocations in such equilibria? Is this set a singleton? Does the economy have a unique equilibrium? Characterize the set of equilibria when initial nominal assets are positive. Prove your assertions.

(c) Now assume that $U(c_{1t}, c_{2t}) = \log c_{1t} + \log c_{2t}$. What is the sign of the correlation between the inflation rate and the rate of growth of output?
Part 3. Please answer both questions

Question 1. Asset pricing. Consider a Lucas economy in which there is an infinitely lived representative agent which owns a tree yielding a non storable fruit $d_t$ in every period. Preferences of the representative agent are given by

$$\sum_{t=0}^{\infty} \beta^t \log(c_t), 0 < \beta < 1$$

and the process for dividend is given by

$$d_{t+1} = d_t e^{\varepsilon_{t+1}}$$
$$\varepsilon_{t+1} \rightarrow N(\mu, \sigma^2), i.i.d.$$

- Solve for $p_t$, the price of the tree and for $q_t$, the price of a risk free bond which pays 1 unit of consumption next period
- Solve for the expected return from holding trees (stocks) and the expected return on risk free bonds.
- Discuss how a fall in the expected growth of the economy ($\mu$) affects bond and stock prices and give economic intuition for your result.

Question 2. Permanent Income. Consider a consumer with the following quadratic utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$
$$u(c) = b_1 c - b_2 c^2, b_1 > 0, b_2 > 0$$

facing the following standard budget constraint

$$y_t + a_t (1 + r) = a_{t+1} + c_t$$

where the interest rate $(1 + r)$, satisfies $(1 + r)\beta = 1$. Assume a standard no Ponzi game condition and assume that income $y_t$ is given by the following process

$$y_t = z_t + \varepsilon_t$$
$$z_t = z_{t-1} + \eta_t$$

where $\eta_t \rightarrow N(0, \sigma_\eta), \varepsilon_t \rightarrow N(0, \sigma_\varepsilon)$

All serially and mutually uncorrelated

- Solve for $\Delta c_t = c_t - c_{t-1}$ and $\Delta a_{t+1} = a_{t+1} - a_t$ as a function of $\varepsilon_t$ and $\eta_t$
- Suppose an econometrician is interested in measuring the parameters $\sigma_\eta$ and $\sigma_\varepsilon$. Discuss what kind of data can be be used to identify these parameters and why.
Growth Models

Consider an economy with two equal size countries indexed by $i \in \{n, s\}$. Households in each country have measure one, live forever, and care each period about three things $\{c, \ell, h\}$, where $c$ is a traded good that can be produced in both countries, $\ell$ is a good that has to be consumed in the country that is produced (often described as tradables and nontradables respectively) and $h$ are our worked. Households like goods and do not like to work. Preferences are equal and given by the expected discounted value of $u(c_i, \ell_i, h_i)$.

The technology to produce good $c$ is the same in each country and is subject to country specific production shocks with the same transition $\Gamma^{xx'}$

$$F^c(z_i, c_i, K_i, H_i)$$

The technology to produce the local good is also the same and is not subject to shocks

$$F^\ell(K_i, H_i)$$

Capital deprecites at rate $\delta$, has to be installed one period in advance, and it can be reallocated freely across sectors within a period.

1. (10 points) Define the set of feasible allocations and a social planner problem.

2. (20 points) Define Recursive Competitive Equilibrium with complete markets. Make sure that you not only define the required objects but also state the conditions that such objects must satisfy. Will there be state contingent trades in equilibrium?

3. (10 points) Define now a Recursive Competitive Equilibrium where local firms in each sector own the capital and distribute the profits to the households of their same country. Households can borrow from each other in an uncontingent way in a world credit market.

4. (15 points) Imagine the economy has capital in its steady state values and the shock of country $n$ is at its unconditional mean while the shock of country $s$ is well below its mean. Please comment on what are the likely properties of the allocation that ensues. In particular, state some of the possible differences between the equilibrium allocations of the two market structures in the previous questions.

Monopolistic Competition

Imagine that preferences of a representative consumer in a static closed economy are given by

$$u(\{c(i)\}_{i \in [0, A]}, n) = \left( \int_0^A c(i)^\gamma \, di \right)^{\theta/\gamma} - \chi \frac{H^{1+\frac{1}{\nu}}}{1 + \frac{1}{\nu}}$$
Where \( 1 - n \) is leisure and \( n \) is time spent working. Output is produced with one unit of labor that is taken to be the numeraire.

5. (10 points) Give an expression for the price that each firm charges, as a function of the income of the consumer.

6. (10 points) What other expressions would you use to get a (perhaps implicit) formula for the price and quantity of each good produced?
There were two additions/corrections made during the test to the macroeconomics prelim in Fall 2012.

On page 3, part 2, the second equation should read:

\[ M_t + B_t = \left( M_{t-1} - p_{t-1}c_{1t-1} \right) + p_{t-1}c_{2t-1} + p_{t-1}y + R_{t-1}B_{t-1} + T_{t-1} \]

The equation originally read:

\[ M_t + B_t = \left( M_{t-1} - p_{t-1}c_{1t-1} \right) - p_{t-1}c_{2t-1} - p_{t-1}y + R_{t-1}B_{t-1} + T_{t-1}; \]

the third minus sign (between \( p_{t-1}c_{2t-1} \) and \( p_{t-1}y \)) should be a plus sign.

On page 5, part 4 the phrase “Assume investment goods are tradable” should be added to the first paragraph.