Ph.D. Preliminary Examination

MICROECONOMIC THEORY

MAJORS

Fall 2012

The time limit for this exam is $3\frac{1}{4}$ hours.

Answer one question from each part, for a total of four questions.

You may use calculators to make calculations during the examination. However: in answering any question that requires you to justify your answer, if you do use a calculator, you must mention at what point in your answer you obtained results using a calculator, and what it was you were seeking to calculate.

Be sure you clearly define all **boldfaced/underlined** terms. Also, please be sure to define precisely any notation that you introduce.

Note: This examination should have 15 pages including this one (Check to make sure!)
Part I

Answer one question from Part I.
Question I.1

Consider the problem of finding a Pareto optimal allocation of aggregate resources $\omega \in \mathbb{R}_+^n$ in an economy with two agents:

$$\max_x \mu u^1(x) + (1 - \mu)u^2(\omega - x)$$

subject to $x \leq \omega$, $x \geq 0$,

where $u^i : \mathbb{R}_+^n \to \mathbb{R}$, for $i = 1, 2$, are agents' utility functions, and $\mu$ is the welfare weight of agent 1. $\mu$ lies in the interval $[0, 1]$. Let $x^*(\mu)$ be a solution (assumed unique).

(a) State a definition of utility function $u^i$ being supermodular in $x$. Give an example of a supermodular utility function other than the linear function. Justify your answer.

(b) Show that, if utility functions $u^1$ and $u^2$ are strictly increasing and supermodular in $x$, then $x^*(\mu)$ is non-decreasing in $\mu$. If you use a known mathematical theorem in your proof, make sure that you state that theorem clearly.
Question I.2

Consider two real-valued random variables $\tilde{y}$ and $\tilde{z}$ on some probability space. Random variables $\tilde{y}$ and $\tilde{z}$ have the same expectations, $E(\tilde{y}) = E(\tilde{z})$.

(a) State a definition of $\tilde{y}$ being more risky than $\tilde{z}$. Show that, if $\tilde{y}$ is more risky than $\tilde{z}$, then $\text{var}(\tilde{y}) \geq \text{var}(\tilde{z})$, where var() denotes the variance.

(b) Give an example of two random variables $\tilde{y}$ and $\tilde{z}$ (with the same expectations) such that $\text{var}(\tilde{y}) > \text{var}(\tilde{z})$, but $\tilde{y}$ is not more risky than $\tilde{z}$. Justify your answer.
Part II

Answer one question from Part II
Question II.1

Local television news media have recently reported that Delta Air Lines has been offering tickets on the same flights and same days at different prices to different customers, in particular showing higher prices sometimes to people that Delta knows have frequent flyer accounts and travel often (because this happened to people who are logged into their accounts when purchasing tickets and fly at least 50,000 miles per year credited to Delta). This question asks you to think about how one might analyze this in a general equilibrium model.

(a) How would you specify any systematic differences (what might be such differences, if any?) between frequent flyers and other consumers in your model?

(b) How would you model the idea of different prices for different customers formally? Be precise about the economic environment and any relevant assumptions in a formal general equilibrium model. To simplify, you may decide to neglect production and use a pure exchange economy since in the short run, Delta has already decided what products (routes, seats, meals, etc.) to produce and how much to produce.

(c) Discuss the implications for the existence of competitive equilibrium and both welfare theorems. Identify precisely which, if any, of the standard assumptions would not be valid in this situation. Define equilibrium and Pareto optimal allocations.

(d) Now suppose that instead of systematic price differences based on frequent flyer status, Delta is simply experimenting with pure randomized prices. How would you model this?

(e) Again for the case of random prices not correlated with frequent flyer status, what would be the implications for the existence of equilibrium and the welfare theorems? Define equilibrium and Pareto optimality in your basic model.
Question II.2

Consider a pure exchange economy with two commodities and three traders (indexed by subscripts 1, 2, and 3), each having the same consumption sets \(?R^2_+\) and the same initial endowment vector \( e_1 = e_2 = e_3 = (1, 1) \). For \( c \in IR_+ \) (so that the scalar \( c \geq 0 \) is a nonnegative constant), the first agent’s indifference curves are given by
\[
\{ (x, y) \in IR^2_+ | x = c \text{ or } x \geq c \text{ and } y = c \}\]
while the indifference curves of agents 2 and 3 are given by \( \{ (x, y) \in IR^2_+ | x + y = c \} \). For all agents, it should be understood that larger values of \( c \) correspond to higher (better) indifference curves.

(a) Are the preferences of these traders monotone? Strictly monotone? Convex? Strictly convex?

(b) Find a utility representation for each trader’s preferences.

(c) Define competitive equilibrium in this economy. (Be sure to define any notation that you introduce.)

(d) Find all competitive equilibrium allocations in this economy.

(e) Find all competitive equilibrium price vectors in this economy.

(f) State a theorem on the existence of competitive equilibrium and explain why it does or does not apply to this economy. (I.e., does this economy satisfy the assumptions for the theorem you stated? Why or why not?)

(g) SKETCH a proof of the theorem you stated in part (f), including giving complete statements of any major mathematics theorem(s) that are key to the proof. DO NOT GIVE A COMPLETE PROOF—it will NOT receive credit here.

Question II.2 continues on the next page.
Question II.2 continued:

(h) Can you modify the demand relation of the first agent so that the first agent’s excess demand correspondence does not cause the economy to fail to satisfy the assumptions of the Very Easy Existence Theorem? In other words, you are being asked how the Very Easy Existence Theorem can be applied—by modifying the demand relation of each agent in an innocuous way—to guarantee that an economy consisting (only) of $N$ identical copies of agent 1 has a competitive equilibrium. (Ignore whether adding the excess demands of agents 2 and 3 to obtain aggregate excess demand for this economy causes any assumptions to fail to be satisfied.) Explain your answer. What does this indicate about the first agent’s potential competitive equilibrium allocations in this economy? Explain.
Part III

Answer one question from Part III
Question III.1

A play in an extensive form game (EFG) is a complete history, from the initial node to one of the final nodes. An EFG is said to be linear if every information set is crossed only once in every play.

(a) Give an example of an EFG which is not linear.

(b) Compare linear games and games with perfect recall. Which set is a subset of the other? Prove your answer.
Question III.2

(a) Prove that the strategy profile inducing a sequential equilibrium is a sub-game perfect equilibrium.

(b) Are the two set of equilibria different? Show with an example that they are or prove that they are not.
Part IV

Answer one question from Part IV
Question IV.1

A seller is designing a selling mechanism to sell an indivisible object to a buyer. The seller has no use of the object, and aims at maximizing expected revenue. The buyer has quasilinear preference, whose valuation of the object, $t$, is known only to buyer himself. The seller knows only that $t$ is either 2 or 3, with equal probabilities. It is commonly known that $t$ is correlated with a random variable $\tilde{\tau} \in \{\tau, \tau'\}$. Conditional on $t = 3$, the probability that $\tilde{\tau} = \tau$ is $1/4$. Conditional on $t = 2$, the probability that $\tilde{\tau} = \tau$ is $\pi$. While the seller and the buyer agree that $\pi > 1/4$, they disagree on the exact value of $\pi$. While the seller thinks that $\pi = 3/4$, the buyer thinks that $\pi = 1/2$, and these heterogeneous beliefs are common knowledge among them. Since reasonable people can agree to disagree on probability assessments, the seller maintains her own belief even though she knows perfectly well that the buyer’s belief is different from hers. What is the seller’s optimal selling mechanism?
Question IV.2

An auctioneer is designing an auction mechanism to auction an indivisible object among two bidders (1 and 2). The auctioneer has no use of the object, and aims at maximizing expected revenue. The bidders have quasilinear preferences, with their valuations of the object denoted by $t_i$, $i = 1, 2$. The values of $t_i$, $i = 1, 2$, are drawn independently from the uniform distribution over $[0, 1]$. Bidder 1 knows his valuation $t_1$ before deciding whether to participate the auction, whereas bidder 2 will learn his valuation $t_2$ only after he participates. The value of $t_i$ will remain bidder $i$’s private information afterward. What is the auctioneer’s optimal auction mechanism?
Question IV.3

Consider a repeated game with discounting given by $\delta$.

(a) Define the operator $SP^\delta$, giving for every set of feasible payoffs the continuation values in subgame perfect equilibria.

(b) Prove that the set of sub-game perfect equilibrium payoffs is a fixed point of $SP^\delta$. 