Ph.D. Preliminary Examination

International Trade and Payments Theory

Fall 2012

Please Answer ALL FOUR questions

Please read the instructions before each part and make your answers neat and concise. Make whatever assumptions you need to answer the questions. Be sure to state your assumptions clearly. You have 5 hours to complete the exam.
Question 1. Monopolistic competition with heterogeneous firms and trade

Monopolistic competition with heterogeneous firms and trade

Consider an economy where the consumers have Dixit-Stiglitz utility functions and solve the problem

\[
\max (1-\alpha) \log c_0 + \frac{\alpha}{\rho} \log \int_0^m c(z)^\rho \, dz
\]

s.t. \( p_0c_0 + \int_0^m p(z)c(z)\,dz = w\ell + \pi \)

\( c(z) \geq 0. \)

Here \( 1 > \alpha > 0 \) and \( 1 > \rho > 0. \) Furthermore, \( m > 0 \) is the measure of firms, which is determined in equilibrium. Suppose that good 0 is produced with the constant-returns production function \( y_0 = \ell_0. \)

a) Suppose that the producer of good \( z \) takes the prices \( p(z'), \) for \( z' \neq z, \) as given. Suppose too that this producer has the production function

\[
y(z) = \max \left[ x(z)(\ell(z) - f), 0 \right].
\]

where \( x(z) > 0 \) is the firm’s productivity level and \( f > 0. \) Solve the firm’s profit maximization problem to derive an optimal pricing rule.

b) Suppose that good 0 is produced with the constant-returns production function \( y_0 = \ell_0. \) Suppose that firm productivities are distributed on the half line \( x \geq 1 \) according to the Pareto distribution with distribution function

\[
F(x) = 1-x^{-\gamma},
\]

where \( \gamma > 2 \) and \( \gamma > \rho/(1-\rho). \) Also suppose that the measure of potential firms \( \pi \) is fixed at \( \mu. \) Define an equilibrium for this economy.

c) Suppose that, in equilibrium not all potential firms actually produce. Write down and explain the equation that determines the productivity of the least productive firm that produces, that is, the productivity \( \bar{x} > 1 \) such that no firm with \( x(z) < \bar{x} \) produces and all firms with \( x(z) \geq \bar{x} \) produce. Relate the measure of firms that produce \( m \) to the measure of potential firms \( \mu \) and the cutoff \( \bar{x}. \)

d) Suppose now that the mass of potential firms, \( \mu, \) is endogenous. Potential firms pay a cost of taking a productivity draw from the distribution \( F(x) = 1-x^{-\gamma}. \) Explain carefully how the answers to parts b and c are altered.

e) Suppose now that there are two countries that engage in trade. Each country \( i, \) \( i=1,2, \) has a population of \( \ell_i \) and a measure of potential firms of \( \mu_i, \) which is again fixed. Firms’ productivities are again distributed according to the Pareto distribution,
$F(x) = 1 - x^{-\gamma}$. A firm in country $i$ faces a fixed cost of exporting to country $j$, $j \neq i$, of $f_e$ where $f_e > f_d = f$. Each country also imposes an *ad valorem* tariff $\tau$ on imports of differentiated goods from the other country. The revenue from these tariffs is redistributed in lump-sum form to the consumer in that country. Define an equilibrium for this world economy.
Question 2

We start by reviewing a standard Eaton Kortum model, only with three sectors. We then tweak things to make a prelim question out of it.

Three-Sector Eaton-Kortum

Consider a model with two countries. Suppose labor is the only factor of production and assume that the number of workers $N$ is the same in each country. Each worker is endowed with one unit of time.

There are three sectors, indexed by $k \in \{1, 2, 3\}$. Each sector follows Eaton and Kortum (2002). In each sector there are a continuum of differentiated goods on the unit interval. Let $T^k_i$ be the productivity parameter of country $i$ in segment $k$. This governs the distribution of productivity draws, so the c.d.f. of productivity $z$ in country $i$ and sector $k$ is

$$F^k_i(z) = e^{-T^k_i z - \theta}.$$ 

Suppose composite sector $k$ good is CES,

$$Q^k = \left[ \int_0^1 q^k(j) \frac{\sigma-1}{\sigma} dj \right]^{\frac{\sigma}{\sigma-1}},$$

where $\sigma < 1 + \theta$, where $q^k(j)$ is a quantity of differentiated good in sector $k$.

Utility of function for the composite goods is Cobb-Douglas, with spending shares of $\frac{1}{3}$ on each of the three sector goods.

Let $\tau \geq 1$ be the iceberg cost of shipping between the two locations.

Let $w_i$ be the wage at location $i$, and let $P^k_i$ be the price index for sector $k$ in country $i$. To simplify calculations, we review some of the results of EK that you can take as given. Define $\Phi^k_1$ and $\Phi^k_2$ by

$$\Phi^k_1 = T_1 w_1^{-\theta} + T_2 w_2^{-\theta} \tau^{-\theta},$$

$$\Phi^k_2 = T^k_1 w_1^{-\theta} \tau^{-\theta} + T^k_2 w_2^{-\theta}.$$ 

Then $P^k_i$ equals

$$P^k_i = \gamma \Phi^k_i^{-1/\theta},$$

for a constant $\gamma$. Also, let $n \neq i$. Then the probability that country $i$ is the lowest cost provider to country $n$ in sector $k$ equals

$$\pi^k_{ni} = \frac{T^k_i w_i^{-\theta} \tau^{-\theta}}{T^k_n w_n^{-\theta} + T^k_i w_i^{-\theta} \tau^{-\theta}}.$$
if \( n \neq i \), and the probability that \( i \) is the lowest cost provider to itself is

\[
\pi_{i,i}^k = \frac{T_i^k w_i^{-\theta}}{T_n^k w_n^{-\theta} + T_i^k w_i^{-\theta}}.
\]

Finally, the overall price index in country \( i \) is

\[
P_{i,\text{overall}} = \omega \prod_{k=1}^3 (P_i^k)^{-\frac{1}{3}},
\]

for a constant \( \omega \).

**Turning it into a Question**

First, let’s simplify by assuming the technology parameters have the following structure,

\[
\begin{align*}
T_1^1 &= \alpha, & T_1^2 &= \beta, & T_1^3 &= \gamma \\
T_2^1 &= \beta, & T_2^2 &= \alpha, & T_2^3 &= \gamma,
\end{align*}
\]

for \( \alpha > \beta > 0 \), and \( \gamma > 0 \). Note that with this structure, country 1 has a comparative advantage in sector 1, while country 2 has a comparative advantage in sector 2.

(1) Solve for the equilibrium trade flows in each sector between country 1 and country 2. Explain how the within-sector and net across-sector trade flows depend upon \( \alpha, \beta, \gamma, \tau, \) and \( \theta \).

(2) Suppose the iceberg trade cost is initially \( \tau^0 \), but is reduced to \( \tau' < \tau^0 \). Derive a formula for the welfare gain from the lower trade cost.

(3) Does the welfare gain determined in part (2) depend on the parameter \( \gamma \)? Explain why or why not.
Question 3. Forward Premium Anomaly

In the data high interest rate currencies tend to appreciate so that

$$\text{cov}_t(i_t - i_t^*, E_t \log e_{t+1} - e_t) \leq 0$$

where $i$ and $i^*$ are home and foreign nominal interest rates and $e$ is the nominal exchange rate.

1. Develop an economy which could generate this negative covariance.

2. Explain as carefully as you can what features are necessary to generate it. Can the money supply have a unit root? Can it be i.i.d? Can we get it with log-linear approximations to the first order conditions of some economy?

3. Suppose that consumption and inflation are normally distributed around some mean with constant conditional variances. Can this economy generate this negative covariance.

4. What would this covariance be in a simple monetary model with homogenous consumers and in which money is neutral (as it is in the original Lucas two currency paper)?
Question 4. International Business Cycles and Diversification

Consider the following two countries, a good international real business cycle model.

Domestic households preferences are given by

\[ E \sum_{t=0}^{\infty} \beta^t \left( \log c_t - \phi \frac{l_t}{v} \right) \]

where \( c_t \) is consumption, \( l_t \) is labor, \( 0 < \beta < 1, \phi > 0 \) and \( v > 1 \) are parameters. Preference of foreign households are the same, with foreign consumption \( (c^*_t) \) and foreign labor \( (l^*_t) \) as arguments. Each country is endowed with a fixed amount of capital \( k \) which does not depreciate and that can be used for production together with local labor. So the world resource constraints in each period are given by

\[ A_t k^{\alpha} l_t^{1-\alpha} + A^*_t k^{\alpha} l^*_t^{1-\alpha} \geq c_t + c^*_t \]

where \( A_t \) and \( A^*_t \) represent stochastic productivity in the two countries and follow independent and uncorrelated AR(1) processes with mean 1 and variance \( \sigma^2 \). In period 0 each country starts with initial productivity equal to its long run mean i.e. \( A_0 = A^*_0 = 1 \).

1. Write down an equal weights planning problem and write down the first order conditions that characterize the planner’s allocations.

2. Suppose that in period 1 productivity in country 1 increases while productivity in country 2 stays at its long run mean. Characterize, qualitatively, the patterns of domestic and foreign consumption, labor and GDP in period 0 and period 1, in the planner’s allocation.

3. Now assume that capital stock in both countries is owned by local firms which operate it and pay dividends to stock owners. Stocks of these firms can be owned and traded by local and foreign consumers. Define a competitive equilibrium in this set-up (make sure you include notation for dividends and for domestic and foreign ownership of both stocks)

4. Show that there exist a competitive equilibrium in which consumers hold a constant portfolio of domestic and foreign stocks and where allocations in the competitive equilibrium are the same as allocations in the planner’s problem. Solve for the share of foreign stocks in this equilibrium.